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The Analyst's Bookshelf

Reviews

NIGEL HOWARD, *Paradoxes of Rationality—Theory of Metagames and Political Behavior*, MIT Press, Cambridge, Mass., 1971, 348 pages, $12.95.

THIS BOOK clearly belongs to a small select list of major works on game theory. Its contents are highly controversial, but HOWARD has based his conclusions on arguments that will require careful consideration by game theorists whether they agree or disagree with the general thrust of the author.

The three paradoxes that provide the book with its title can all be explained in the context of two-person, nonzero-sum games. We suppose that player 1 has strategies $s_1, \cdots, s_p$; that player 2 has strategies $t_1, \cdots, t_q$; and that if player 1 chooses s_i and player 2 chooses t_j, then the outcome is O_{ij}. We suppose further that u_{ij}, v_{ij} provide ordinal preference measures of O_{ij} for players 1 and 2, respectively. The matrix of ordered pairs $[(u_{ij}, v_{ij})]$ is called the *game matrix*.

The decision underlying choice of a strategy is said to be (1) *subjectively rational* if it is the best that can be made based on information available to the decision maker; (2) *objectively rational* if it is the best that can be made based on the true state of affairs (including the opponents decision). An objectively rational decision is obviously *stable* in the sense that it will not be changed by any additional information.

The three breakdowns of rationality can now be stated:

First Breakdown: In some games it is impossible for both players to be objectively rational [p. 10].

Second Breakdown: In some games both players do better if irrational than if rational [p. 45].

Third Breakdown: In some games following a sure-thing (objectively rational) strategy capitulates entirely to the opponent.

In Howard's words [p. 181], "If, however, there is any possibility of conflict between stable outcomes; that is, if there are two or more possible compromises, of which the one most favored by player 1 is not the one most favored by player 2; then to choose a sure-thing strategy is to be a 'sucker' that capitulates entirely to the other side."

The three breakdowns are illustrated, respectively, by the games known as Matching Pennies, The Prisoner's Dilemma, and Chicken. Game matrices for these are given in Table I.

The first breakdown has been well known for many years and, indeed, was an important barrier to the initiation of a theory of games. The celebrated minimax theorem of VON NEUMANN in the late 1920's provided a way of dealing with this problem and thus marked the true beginning of game theory.

The root of the breakdown is, of course, the logical contradiction involved in having simultaneous moves with each player knowing the other's strategy before he selects his own.

In Matching Pennies '1' means 'lose' and '2' means 'win'; the row player wishes to match and the column player wishes to differ.

In the Prisoner's Dilemma the strategies are '*C*' for 'cooperate' and '*D*' for 'defect.' For each player, *C* is a sure-thing strategy, i.e., *C* is better than *D* regardless of which strategy the opponent chooses. Thus rational players end at (D,D) with payoff $(2,2)$ whereas if both players select the irrational strategy C they end up at $(3,3)$, which both prefer.

We defer discussion of the third breakdown.

Howard's main contribution is his concept of *metagame*, which he presents as a remedy to the second breakdown. He associates with each game a hierarchy of games called metagames; metarationality is defined to be rationality in some metagame.

Let G be a game with matrix $A=[(u_{ij}, v_{ij})]$; then Howard defines a new game $1G$ called the 1-metagame (of G) by changing the rules of G so that player 1 moves after being told the choice of player 2. The metagame $2G$ and iterates $k_s k_{s-1} \cdots k_1 G$ are defined analogously. For example, the game $2G$ for the Prisoner's Dilemma is

$$\begin{array}{cc} & \begin{array}{cccc} C/D & D/D & C/D & D/C \end{array} \\ \begin{array}{c} C \\ D \end{array} & \left[\begin{array}{cccc} (3,3) & (1,4) & (3,3) & (1,4) \\ (4,1) & (2,2) & (2,2) & (4,1) \end{array}\right], \end{array}$$

Here the symbol X/Y stands for the metagame strategy (for player 2) of choosing X against C and Y against D. The game $1,2G$ has 16 strategies for player 1 and 4 for player 2; the symbol $W/X/Y/Z$ represents the strategy W against C/D, X against D/D, Y against C/D, and Z against D/C. In a game G with r players each permutation $k_r, \cdots, k_1$ of $1, \cdots, r$ defines a metagame $k_r \cdots k_1 G$. The $r!$ metagames thus obtained are the *complete*, (because each index appears at least once), and *primitive* (because no index is repeated) *metagame* descendants of G. Howard shows (1) that no equilibrium is lost in passing from a game to one of its metagames (called descendants), and (2) that any equilibrium obtained in a metagame of G also exists in some primitive descendant of G. These results justify focussing attention on the set of complete primitive metagames.

TABLE I

$\begin{array}{cc} & \begin{array}{cc} H & T \end{array} \\ \begin{array}{c} H \\ T \end{array} & \left[\begin{array}{cc} (2,1) & (1,2) \\ (1,2) & (2,1) \end{array}\right] \end{array}$	$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \left[\begin{array}{cc} (3,3) & (1,4) \\ (4,1) & (2,2) \end{array}\right] \end{array}$	$\begin{array}{cc} & \begin{array}{cc} A & C \end{array} \\ \begin{array}{c} A \\ C \end{array} & \left[\begin{array}{cc} (3,3) & (2,4) \\ (4,2) & (1,1) \end{array}\right] \end{array}$
Matching Pennies	Prisoner's Dilemma	Chicken

It is the possibility of obtaining new equilibria that brought the metagame concept widespread attention. For example, in the Prisoner's Dilemma both $12G$ and $21G$ have equilibria with payoffs $(3,3)$. In G itself this is not an equilibrium outcome although it has been widely regarded as representing a desirable social outcome, so in a sense the metagame concept provides a resolution to the Prisoner's dilemma.

There are four important classes of equilibria: basic (E, p. 28), symmetric ($\sum$, p. 122), metagame (T, p. 120), and general (Γ, p. 120). These are nested in the sequence

$$E \subseteq \sum \subseteq T \subseteq \Gamma.$$

Howard shows (1) that every metagame has an equilibrium (p. 27), (2) that in every complete metagame (of an ordinal game G) each Pareto optimum is a metagame equilibrium (p. 154). In a sense these results, respectively, 'rescue' rationality from its first two breakdowns (which can be restated in the respective forms 'E may be empty,' and 'E may contain no Pareto optimum'). But proving that T is nonempty and that its elements are Pareto optima still leaves open the question of symmetry, i.e., the nonemptiness of $\sum$. In the Prisoner's Dilemma the outcome (3,3) is symmetric as well as being a Pareto optimum and for the reviewer a major contribution of metagame theory is that it provides a transition from the ordinary Prisoner's Dilemma to complete metagames whose outcomes have these two properties.

We return now to the game of Chicken and the third breakdown. In the game $12G$ the only sure thing strategy yields the metarational outcome (2,4), which is most favored by player 2, and is next to the worst for player 1; in $21G$ the corresponding sure thing outcome is (4,2). The lack of concordance between these two complete metagames is the basis for the third breakdown.

Thus the social problem presented by games like Chicken is more severe than those resembling the Prisoner's Dilemma.

Since Howard proposes the metagame structure as a descriptive rather than as a normative theory, it must be judged by its empirical success and not by its logical structure. One positive feature of his approach is that he is willing, yea eager, to put his model to experimental testing. He reports some experimental results in Appendix B and has since gone much further. However, the OR practitioner will understandably yearn for a normative version of the theory.

In my opinion the weakest feature of the theory is its limitation to ordinal utilities. In defense of this he claims that cardinal utilities cannot be practically constructed; this places Nigel Howard in sharp opposition to the claims and practice of RONALD HOWARD (and other decision theorists). Of course, a corollary to rejection of cardinal utility is the rejection of mixed strategies.

As now developed metagame theory requires the use of normalized as opposed to characteristic function form. This may be justified in the two-person case but the lack of adequate treatment of coalitions and side payments seems a major flaw if metagames are to be taken seriously beyond the two-person case (which is precisely where its major triumphs lie).

I found the book stimulating, but the lack of formal structure made reading unnecessarily difficult. Many important theorems and definitions (indeed, even precise statements of the breakdowns of rationality) were not identified by name or number. I also feel that the impact of the book would be greater if less were claimed (e.g., in the n-person case) so that the reader could concentrate more clearly on the major contributions.

In summary, I recommend *Paradoxes of Rationality* . . . for any serious student of decision theory; whether or not he agrees with many of its arguments he should find reading it, and having it available for reference, valuable.

ROBERT M. THRALL
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JATI K. SENGUPTA, *Stochastic Programming—Methods and Applications*, North Holland/American Elsevier, New York, N. Y., 1973, 313 pages, $11.00.

AS THE TITLE suggests, this book deals with linear and nonlinear programming problems upon which some type of random conditions have been imposed. Two ways of including randomness in programming problems are by considering some (or possibly all) of the structural data to be random, and through the concept of chance constraints. In the latter case one requires that various constraints be satisfied with specified probability. Employing a frequency interpretation of probability makes this concept equivalent to having the specified constraints satisfied in a specified percentage of the total set of problems being considered. Various conceptional difficulties with such an interpretation are not considered in the book, nor should they be. SENGUPTA provides an exposition of technique, illustrated by some applications. Possible questions of interpretation or applicability to various situations are, quite properly, omitted. The possible random character of the structura- data can be of two distinct forms. Theoretical distributions can be assumed for the data, together with such features as statistical independence, to provide a model whose analysis leads to derived distribution information about the random solution. Alternatively data can be interpreted as representing a statistical sample from distributions that are, to varying degrees, unknown. This interpretation leads to estimation or hypothesis testing analyses.

Stochastic Programming consists of five chapters of varying length and significance. Chapter 1 presents some of the major concepts that are given more detailed treatment in the remainder of the book. A number of interesting and useful ideas are somewhat incompletely presented, including probabilistic concepts and characterizations of stochastic programming models. Much of the material from Chapter 1 is given better treatment later but it is probably worthwhile to read as an introduction. Chapter 2 is the most significant chapter, being a good presentation of stochastic linear programming methodology. Among the several major concepts developed in this chapter is the system reliability approach, in which satisfaction of chance constraints represent proper functioning of the model. This view allows definition of reliability of the complete programming model (i.e., system) and leads to techniques of analysis. There is considerable use of probability and particularly nonparametric technique in Chapter 2 that contributes considerably to its interest. Chapter 3 deals with nonlinear and dynamics stochastic programming. The material is reasonably presented, but because of the nature of the problems the methodology is not nearly so well developed as for linear stochastic programming. As one might suppose, approaches based on geometric programming and sequential unconstrained minimization technique are described.

Applications are presented in Chapter 4 which serve to illustrate the material from the previous chapters and also to indicate possible areas of utility for stochastic programming concepts. Among the selected applications one finds a crop-mix problem and consideration of dynamic investment allocation. Throughout the book Sengupta has stressed that the subject of stochastic programming is largely in a characterization phase. In Chapter 5, which is very short, some future developments in the subject are described. A good set of references, which is often referred to throughout the text, concludes the book. There is no index, but the nature of the material hardly calls for one in view of a rather detailed Table of Contents.

Stochastic Programming would not be suitable for a text in a standard course. The material draws upon a wide range of subjects, there are very few illustrative examples, and no problems. It would be very good as the basis for an advanced course in which the students already had a knowledge of probability, linear, and nonlinear programming. A particularly worthwhile use for Sengupta's book is that of bringing a number of concepts together in approaching problems of interest to a wide variety of people. In particular, it turns out to be much more of a book on applied probability than a book on mathematical programming. Those trained in either of these disciplines can obtain a number of stimulating ideas from the rich intersection of subjects presented in this book.

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L. KOSTEN, *Stochastic Theory of Service Systems*, Pergamon Press, New York, N. Y., 1973, xii + 168 pp., $11.50.

E. PAGE, *Queuing Theory in OR*, Crane, Russak & Co., New York, N. Y., 1972, 187 pp., $11.00.

HERE ARE two books on queuing theory written for practitioners of operations research and graduate students of applied science interested in the theory as applied to practical problems. Both books pre-suppose a knowledge of calculus and elementary probability theory. They treat standard queuing systems in steady state, the main results being queue-length and waiting-time averages.

The outlines of Kosten's book are as follows. The introductory chapter describes the basic queuing model. The next two chapters are concerned with the $M/M/s$ loss system with finite as well as infinite source, and the $M/M/s$ delay system under the queue-disciplines first-come, first-served, last-come, first-served, and random order. The $M/G/s$ loss and $M/G/1$ delay systems are analyzed in Chapter 4 by the method of supplementary variables, first used by the author but usually attributed to D. R. Cox. Chapter 5 investigates the nonstationary (transient) behavior of a Markov process with a finite number of states and applies the results to the $M/M/s$ loss system. Heuristic combinatorial methods are used in Chapter 6 to derive various averages in a priority system with head-of-the-line discipline, and also in a model for machine breakdowns. Chapter 7 treats models with restricted availability, while Chapter 8 is concerned with bulk queues. The final Chapter 9 describes various aspects of simulation as applied to queues.

In Page's book an introductory chapter describes queues arising in industry and elements of the queuing model. Chapter 2 treats the system $M/M/s$ and Chapter 3 the system $M/D/s$. D. G. Kendall's imbedded Markov chain analysis of the queue $GI/M/s$ is described in the special case of deterministic input in Chapter 4 (however, the term 'imbedded chain' is not explicitly mentioned). In Chapter 5 more general queuing models are discussed in outline—$E_k/E_l/1$ and several multiserver systems. The so-called machine interference problem (leading to $M/M/s$ and $M/D/1$ queues with finite source) is treated in Chapter 6. Finite capacity systems $M/M/s$ and $D/M/s$, and the loss system $M/M/s$ are discussed in Chapter 7 and the $M/M/s$ queue with balking in Chapter 8. The final Chapter 9 gives a summary of results for some priority queues.

Kosten has written his book from the engineer's point of view and uses some notions of functions of a complex variable and Laplace transforms. His objective is to present a relatively simple analysis of his systems based mainly on the birth-and-death process formulation. However, this book presents queuing theory literature up to the 1950's, and even this it does much less satisfactorily than the 1960 book by R. Syski. This reviewer does not see the need in 1974 for a book such as this, and questions the publishers' wisdom in undertaking this venture.

Page treats a wider variety of queuing models and uses a relatively more 'modern' approach than Kosten. He is somewhat more concerned with transient behavior of his processes—thus busy period results are described in some detail. Theoretical discussion is supplemented with numerical computations, which give a pleasant 'practical' flavor to the presentation. Several numerical examples are worked out and there are a number of problems for solution (these are conspicuous by their absence in Kosten's book). In addition the appendix contains 12 numerical tables for various quantities of interest described in the text. Unlike some previously published texts on applied queuing theory, Page's book encourages its readers to adopt a respectful attitude towards the mathematical theory and makes a genuine attempt to demonstrate the application of the theory to practical problems. As a result this book will be very useful to OR practitioners. This reviewer congratulates the author for writing a truly applied book—probably the first of its kind. May his tribe increase!

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GORDON F. NEWELL, *Approximate Stochastic Behavior of n-Server Service Systems with Large n*, Springer-Verlag, New York, N. Y., 1973, 118 pages, $7.20 (paper).

THIS MONOGRAPH is Volume 87 in Springer-Verlag's series, *Lecture Notes in Economics and Mathematical Systems,* whose aim is "to report new developments . . . quickly, informally and at a high level." Manuscripts may present "a new angle on a classical field" and may be "unfinished or tentative." In this monograph Professor Newell suggests some tentative methods of approximation in queuing theory to answer questions about the (in general, nonstationary) $G/G/n$ queue for which either there are no solutions available or the solutions are so complicated that they are not useful for applications. The task Newell sets for himself is to develop a methodology for the description of queuing phenomena that is sufficiently accurate and easy to use so as to become a practical design technique. This is a very worthy task but, unfortunately, the accuracy of the methodology Newell proposes remains to be evaluated and, even if it should prove sufficiently accurate, the purported simplicity may not compare well with the popular and conceptually simple technique of simulation.

An indication of the flavor of the monograph is perhaps best conveyed by letting the author speak. From the preface:

> The methods of analysis exploit a postulate that $n \gg 1$, that all relevant counts of customers are made on a scale which is also large compared with 1 (typically of order n), and that stochastic fluctuations in arrival counts of order n are of order $n^{1/2}$. Graphs of cumulative counts of customer arrivals and available servers are used to represent the evolution of stochastic realizations of the system. A combination of graphical and analytic methods are then used to estimate queue distributions for various typical types of behaviors.
>
> If the traffic intensity (arrival rate/service rate), $\rho(t)$, increases toward a maximum, the system behaves like an ∞-channel system until $1-\rho(t)$ is of order $n^{-1/2}$. When $\rho(t)$ come sufficiently close to 1 or exceeds 1 so that queuing becomes virtually certain, the system then behaves essentially like some effective single-channel service system. While $\rho(t)$ is between these two extremes, when it is uncertain whether or not a queue exists, several different types

of behavior can exist depending upon how long the system stays in this transition state, and how $\rho(t)$ behaves during this time.

The monograph consists largely of a classification of types of behavior and descriptions of methods that assertedly could be used to obtain more precise (that is, quantitative) results. It contains very few detailed results, although some numerical comparisons are made with theoretical values for the stationary $M/M/n$ and $M/D/n$ queues. The author asserts, however, that "... for $n \gtrsim 10$, any person skilled in the art of making rough calculations should be able to estimate average lengths of queues, delays, etc., to within about 10% in most situations he is likely to encounter...."

The text is difficult to read, and although the methods are heuristic, a fairly high mathematical level is required to follow the arguments Newell makes in support of his methods. Also, there is no index and a glossary is badly needed.

In summary, Newell's monograph is essentially a proposal to investigate the methodology outlined therein. If this methodology should prove to be accurate and easy to apply, it will be an important contribution. Despite the fact that this monograph appears in a series that communicates "unfinished or tentative" work, some further effort at evaluation would have been very helpful.

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