INFORMS TutORials in Operations Research

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

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To cite this entry: Damian R. Beil. Procurement Auctions. *In* INFORMS TutORials in Operations Research. Published online: 14 Oct 2014; 248-259.
https://doi.org/10.1287/educ.1100.0079

TUTORIALS IN
OPERATIONS RESEARCH
INFORMS 2010

 | ISBN 978-0-9843378-0-4
DOI 10.1287/educ.1100.0079

Procurement Auctions

Damian R. Beil
Stephen M. Ross School of Business, University of Michigan, Ann Arbor, Michigan 48109,
dbeil@umich.edu

Abstract This tutorial introduces the basic ingredients of procurement auction research. After discussing the classic market-pricing logic of the open-descending auction, we demonstrate how simple modeling embellishments can extend the market-pricing auction logic to also capture key operational realities and trade-offs encountered in procurement practice. This tutorial should be of value to anyone interested in studying competitive bidding in supply chains.

Keywords competitive bidding; auctions; procurement; sourcing; supply chain

1. Introduction

Sourcing (or procurement) refers to the process of obtaining goods and services required by the firm. As a result of continued trends toward outsourcing over the past few decades, on average U.S. manufacturers spend about half their revenue on goods and services from external suppliers (U.S. Department of Commerce [7]). These external suppliers are increasingly located abroad, as developing and emerging economies offer attractive cost-saving offshoring opportunities for buyer firms. These realities make sourcing strategically important for firms: Staggering dollar values directly impact the firms' profitability; buyer–supplier interactions in long supply chains provide numerous opportunities for negotiation of price and terms, to determine what will be purchased from whom and at what price; and the globalization of these supply chains provides many opportunities and risks for buyers, as low-cost potentials increase and visibility into the supply chain worsens.

The scope for research opportunities in sourcing has grown as this business function has complexified over the years. Vertically integrated firms that were prevalent several decades ago procured less from external suppliers. These firms' sourcing (or procurement) departments coordinated the acquisition of goods and services from internal suppliers. Today, sourcing departments in firms are more likely to rely on external suppliers, multiplying the challenges of acquiring materials. Sourcing managers must now decide which supplier will provide the best value and how prices should be negotiated with suppliers. Competitive bidding is the process by which multiple suppliers submit competing offers to supply the goods or services requested by the firm, which then awards business to the supplier(s) based on these offers. This tutorial will cover sourcing research and models that speak to the issue of negotiation with suppliers, in particular the use of competitive bidding.

In focusing on competitive bidding, this tutorial by no means represents an exhaustive view of sourcing-related research topics. For example, important questions can be posed and answered around issues as make-or-buy decisions, determining what is needed, evaluating supplier performance, etc. Although these other topics are important, we will focus our discussion on competitive bidding for two reasons. First, competitive bidding is widely deployed. For example, the U.S. government uses competitive bidding (sealed tenders) as the standard way of awarding government contracts to suppliers. This standard practice makes the award of contracts based on best value and adds transparency to the process. For similar reasons, in private industry, competitive bidding is also frequently used. In private industry, the methods of competitive bidding include the sealed tenders approach, as well as other approaches. The emergence of Internet-enabled communications between firms has enabled them to organize competitive bidding events that are open bid (suppliers can see competitors' bids) and dynamic (suppliers can respond to others' bids by adjusting their own bid).

The second reason we study competitive bidding is that this an important and prevalent aspect of sourcing that is amenable to an operations researcher's toolkit, namely, analytical modeling. Game theory models of competitive bidding have been studied for over 40 years in the economics literature on auction theory, dating back to Vickrey's seminal work ([8]). Auction theory provides operations researchers with a powerful framework for analyzing competitive bidding in supply chains. As we will explore in this chapter, these models provide a framework in which a variety of important and operationally relevant questions on strategic sourcing can be asked. We will demonstrate in this chapter how this is done by examining several variants. In doing so we will see that in many cases the associated analyses are particularly appropriate for operations researchers' skills, involving a good deal of probability analysis (this is rooted in the fact that the sourcing manager does not know the supplier's true costs, and so she applies probability to predict the auction price outcomes under various conditions).

The rest of this chapter is organized as follows. In §2 we introduce basic theoretical predictions of auction outcomes. In §3 we embed these predictions in models tackling issues that arise in the underlying business environment of procurement. In §4 we conclude.

2. Theoretical Predictions of Competitive Bidding Outcomes

An auction format is simply a set of rules governing how bids will be submitted and how the winner and payments will be subsequently determined based on the bids. Auctions come in many formats. Two important dimensions along which formats differ are whether the bidding is "open" (bidders can view and respond to competing bids from other bidders) or "sealed," and whether the bidding is "dynamic" (occurs over several rounds of bidding) or "one shot." Government procurement is an example of a sealed-bid, one-shot auction: each bidder directly submits their bid to the government, who, upon opening all bids determines the contract winner by applying some predefined evaluation criteria. In industry, this type of competitive bidding is often referred to as "sealed tenders," while the term "auction" refers to dynamic bidding events. The format most associated with the word procurement "auction" is the open-descending auction, which we describe in more detail below. The first-price, sealed-bid auction (i.e., what auction theorists call sealed tenders) and the open-descending auction are the most widely used formats in industrial procurement (field studies have documented their use, e.g., Beall et al. [1]), and no matter what they are called in practice, from an auction theorist's perspective, both are auction formats. There are other auction formats used in practice (e.g., the Dutch auction); see Krishna [5]. In this and the following section we will focus our attention on the open-descending auction format because it is analytically more tractable than other formats and serves to quickly illustrate our main points.

2.1. Basic Model: Open-Descending Auction

The open-descending auction is a dynamic format that operates as follows. The buyer announces a calling price that she lowers as the auction progresses.[1] Suppliers signal their willingness to accept the contract at the current calling price, and the buyer lowers the

[1] In practice the price is either lowered continuously (following a price "clock") or according to fixed decrements.

calling price as long as at least two bidders are willing to accept the contract at that price. The auction ends as soon as only one supplier remains at a price denoted by p_0.[2]

2.1.1. Predicting Auction Outcome. Given this setup, the most basic question a buyer can ask is, "What is my expected payment p_0?" To answer this question, the buyer needs a model of the supplier's strategic bidding behavior in the auction. We will address this below, using a bare-bones auction-theoretic approach.

Supplier bidding behavior. First, the buyer needs to understand suppliers' objectives. Label the suppliers $i = 1, \ldots, N$.[3] We will assume that supplier i's true cost of fulfilling the buyer's contract is c_i and that if awarded the contract at price p he will enjoy a payoff equal to $p - c_i$. Second, we assume the supplier is rational and seeks to maximize its expected payoff.

Finally, given these supplier objectives, in light of the open-descending auction rules we conclude that as long as the auction price p is above c_i, then supplier i will remain in the auction—this is true because winning at price $p > c_i$ would yield a positive payoff, which is preferable to dropping out, which would yield zero payoff. Moreover, the supplier would drop out if p drops below c_i—this is because dropping out and earning zero payoff would be preferable to winning the auction at a price that would result in a negative payoff. To summarize, we see that the buyer has used a model of suppliers' objectives together with the auction rules to determine the supplier's strategic bidding behavior: each supplier finds it optimal to stay in the auction until winning the auction or until the price reaches the supplier's true cost, which happens first.

The bidding strategy described above forms a bidding equilibrium.[4] This equilibrium is appealing because it is easy to understand and captures the essential idea of competitive bidding: the winner is determined competitively based on cost, using a last-man-standing type of market logic. With this bidding equilibrium, the auction winner will be the supplier with the lowest cost. If we let (i) denote the index of the ith-lowest-cost supplier, then the rank-ordered list of costs is $c_{(1)} \le c_{(2)} \le \cdots \le c_{(N)}$. Supplier (1) wins the auction and the auction ending price (payment received by supplier (1)) is equal to $c_{(2)}$, the cost of the second-best supplier (the cost at which supplier (2) finally drops out of the auction).

Auction ending price. Returning to our original question, the buyer can apply the above analysis of supplier bidding behavior to conclude that $p_0 = c_{(2)}$. That is, the buyer's question (what is the auction payment?) is answered by computing the second-lowest supplier cost. If the buyer knew the supplier's costs $\{c_i\}_{i=1}^{n}$ and deployed an open-descending auction, then the buyer would anticipate the ending price as precisely $c_{(2)}$. The buyer may wish to deploy an auction even if she knows the suppliers' costs, because using competitive bidding (the auction) to establish the contract price is in some cases necessary to convince the supply base of what the "fair" market price for the contract truly is. More generally, however, the buyer utilizes an auction precisely to discover the fair market price. Rather than being a forgone conclusion, the buyer cannot precisely predict the eventual auction ending price, and so prior to the auction settles for computing an expectation, $E[p_0] = E[c_{(2)}]$.

To compute the expected second-lowest supplier's cost, $E[c_{(2)}]$, we need to introduce a probabilistic model governing all supplier's costs. Suppose that suppliers' costs are independent and identically distributed according to some cumulative probability distribution F

[2] For simplicity we focus on auctioning an indivisible contract that will be awarded to a single supplier.

[3] Note that N need not be large—in industrial procurement auctions it is not unheard of for there to be only two suppliers. This tends to be true when qualifying suppliers to bid is difficult for the sourcing manager, e.g., because of needed testing or because the internal customer (such as engineering) drags their feet on qualifying new suppliers.

[4] More precisely, it is a "dominant strategy" equilibrium because it holds regardless of the supplier's beliefs about his competitors.

whose probability density is given by f. Then $E[c_{(2)}]$ is simply the expected second-lowest order statistic of N random draws from distribution F. Written formally,

$$E[c_{(2)}] = \int_0^{\infty} cNf(c)(1-F(c))^{N-1}\,\mathrm{d}c,$$

where $Nf(c)(1-F(c))^{N-1}$ is the probability density of the second-lowest among N draws from distribution F. For example, if F is the uniform distribution over interval $[0,1]$, then

$$E[c_{(2)}] = \frac{2}{N+1}.$$

In general, closed-form order statistics only exist for only a few families of probability distributions, notably the exponential, and power-function families (the uniform distribution is a member of the latter).

2.2. Adding Operational Considerations to the Basic Model

In the open-descending auction format discussed in §2.1, bidders were differentiated solely on contract price. In that setting, we saw that the auction served to determine a "fair market price" for the contract. By doing nothing more than finding a fair market price, this reverse auction (in which an item is purchased) is merely the dual of a traditional forward auction (in which an item is sold). Forward auctions have been the mainstay of auction-theoretic research dating back over 40 years, which has examined almost every imaginable variant of assumptions on bidder preferences, beliefs, types, number of items, etc.

In light of this vast research heritage, it comes as good news for those interested in making a research contribution in procurement auctions that purchasing engenders issues that are not present when selling. One of the most obvious is that sales transactions tend to be consummated rather easily—the winning bidder simply pays the seller and the item changes hands—whereas purchasing transactions can be extremely complex. For example, for a contract for six months of supply of printed circuit boards, the supplier must be paid, the supplier must produce the boards, the buyer must arrange for and take delivery, the boards must be determined to live up to the terms agreed upon in the contract, etc.

Industrial buyers often address such complexities by considering factors beyond a supplier's bid price. From a practical perspective, these nonprice factors often capture "operational" features of the underlying problem. One way a modeler can conceptualize procurement auction research is that traditional auction theory informs the basic market-price-discovery logic while additional embellishments can capture operational realities that are crucial in procurement settings. In the next subsections we will explore the following embellishment to the traditional open-descending auction: rather than being differentiated solely on price, the buyer considers in §2.3 the "cost of doing business" with each supplier and in §2.4 the supplier's "qualification status."

2.3. Embellishment I: Open-Descending, Total-Cost Auction

When considering the total cost of doing business with a supplier, the contract payment is only one factor. This is particularly true when the buyer's margins are already razor-thin—as an example, in the PC industry, commodity suppliers place unit price bids in tenths-of-a-cent increments. For buyers in such competitive industries, transportation costs, commissions, and logistics costs can be quite important to the buyer (Pedersen [6]). The buyer can capture these as nonbiddable costs that she considers when making a total-cost decision. For example, if the buyer is responsible for shipping items from the supplier (as she would be under Ex works incoterms[5]), then the buyer could add a markup to each supplier's bid that accounts for the shipping cost the buyer expects to incur throughout the life of the

[5] Internationally agreed upon terminology for various shipping cost arrangements between buyers and suppliers is described at http://www.iccwbo.org/incoterms.

contract. The shipping cost would be valuated by the buyer and would be something she considers when making her total-cost decision, but the cost would not be "biddable" by the supplier.

As a concrete illustration, consider a buyer procuring 80,000 units of an item. If the buyer assumes responsibility for shipping the goods from the supplier's location, the supplier's location determines the buyer's transportation costs, but is not a biddable factor controlled by the supplier. If it would cost the buyer \$0.3 per unit to ship the items from the supplier location, the buyer will add (\$0.3 × 80,000 =) \$24,000 to the supplier's contract execution bid price to account for the transportation costs.

For each supplier $i = 1, \ldots, N$ we will augment the model introduced in §2.1 with a nonbiddable, nonprice cost a_i capturing the additional cost the buyer would expect to incur over the life of the contract when doing business with supplier i. Thus, the buyer's expected *total cost* of doing business with supplier i at contract price p would be $p + a_i$, namely, the contract price plus the additional cost associated with the supplier. We will refer to a_i as supplier i's cost adjustment.

We now introduce the total-cost, open-descending auction format. This auction proceeds as follows. The buyer announces a calling total-cost bid, which she lowers as the auction progresses. Suppliers signal their willingness to accept the contract at the current calling total-cost bid, and the buyer lowers the calling total-cost bid as long as at least two bidders are willing to accept the contract at that total cost. The auction ends as soon as only one supplier remains, at a total-cost bid denoted by t_0, and the auction winner, say, supplier i, is awarded the contract and paid $t_0 - a_i$.

2.3.1. Predicting Auction Outcome. Given this setup, the most basic question a buyer can ask is, "What is my expected total cost t_0?" To answer this question, the buyer needs a model of the supplier's strategic bidding behavior in the auction.

Supplier bidding behavior. As before, the first step is modeling the supplier's objectives. To do so, we reuse the structure from §2.1, implying that supplier i's true cost of fulfilling the buyer's contract is c_i; if awarded the contract at total-cost bid t, he will enjoy a payoff equal to $t - a_i - c_i$; and the supplier is rational and seeks to maximize its expected payoff. With these assumptions, arguing as we did for the simple price-only, open-descending auction, we can conclude that each supplier finds it optimal to stay in the auction until winning the contract or the auction's total-cost calling bid reaches $c_i + a_i$, which from the buyer's perspective is the supplier's true total cost.

Auction ending price. Letting (i) denote the index of the supplier having the ith-lowest total cost, such that $(c+a)_{(1)} \leq (c+a)_{(2)} \leq \cdots \leq (c+a)_{(N)}$, we conclude that supplier (1) will win the auction, and the buyer's total cost will be $(c+a)_{(2)}$. In other words, the lowest total-cost bidder wins, and the buyer incurs the second-lowest total cost. Formally, if the supplier's production costs are unknown to the buyer, then the buyer computes $E[t_0] = E_{c_1, \ldots, c_N}[(c+a)_{(2)}]$. Note that the expectation is over the supplier's production costs, because $\{a_i\}_{i=1}^N$ is known to the buyer prior to the auction. Supplier i's total cost is represented by random variable $C_i + a_i$, where C_i denotes the fact that, from the buyer's perspective, the supplier's production cost c_i is unknown. Note that even if we assume as before that suppliers' production costs are i.i.d. according to distribution F, the random variables $C_i + a_i$ will no longer be i.i.d. Thus, $E_{c_1,\ldots,c_N}[(c+a)_{(2)}]$ is the expected second-lowest among N draws of *nonidentically* distributed random variables. In general, it is difficult to find closed-form expressions for the order statistics when draws are from nonidentical random variables.

In conclusion, we see that compared with the basic open-descending model, with the total-cost embellishment the supplier's strategic bidding behavior remains quite tractable. However, finding a closed-form expression for the buyer's expected payment is no longer as straightforward.

2.4. Embellishment II: Open-Descending, Total-Cost Auction with Postqualification

While total cost is an important consideration, a buyer will typically not award business to a supplier unless the buyer is satisfied that the supplier is indeed *qualified* for the business. A supplier is called qualified if the buyer feels that, with a reasonable degree of certainty, the supplier would be able to fulfill its end of the contract as spelled out in the contract terms. Qualification can span many aspects, including capabilities (does the supplier have the machinery and expertise necessary to produce the goods?), financial status (is the supplier at risk of going bankrupt before the contract is complete?), and business relationships (does the buyer firm's management team see eye to eye with the supplier's management team on critical issues like lean principles?).

Unfortunately, a supplier's qualification status is often difficult, costly, and time consuming for the buyer to assess. The due-diligence process of performing *supplier qualification screening* can include activities such as ordering and testing supplier products, auditing the supplier's finances, and visiting the supplier site to tour its facilities and meet its employees. If the supplier passes the supplier qualification screening process, it is considered qualified and is eligible for contracting. On the other hand, if the supplier fails the supplier qualification screening process, the buyer considers it unqualified for the contract and will not consider awarding business to the supplier.

In this subsection we will consider an auction with suppliers who differ from each other in terms of their qualification status. To simplify our discussion, we consider an auction with two suppliers—an incumbent supplier and an entrant supplier. The incumbent supplier, who has previously fulfilled contracts for the buyer, is known to be qualified for the business being auctioned off. In contrast, the entrant's qualification status is unknown to the buyer. Suppose it would cost the buyer K dollars to perform qualification screening on the entrant, and with probability $\beta \in (0,1)$ the entrant would be deemed qualified. We will denote the incumbent with subscript i and the entrant with subscript e. This basic model captures the basic tension present in many procurement settings where the buyer has an incumbent whose contract is up for renewal. Seeking a cheaper price, the buyer pits the incumbent against an entrant in an auction.

The buyer has the incumbent and entrant compete in an open-descending, total-cost auction with postqualification. The process unfolds as follows. First, the buyer sets cost adjustments $a_i = 0$ and $a_e = K/\beta$. Then the buyer holds a total-cost auction between the incumbent and entrant, as described in §2.3, where again t_0 denotes the ending price of the total-cost auction. If the incumbent wins the auction, he is awarded the contract and receives payment $t_0 - a_i = t_0$ from the buyer. If the entrant wins, the buyer performs qualification screening on the entrant: If the entrant is deemed qualified (which occurs with probability β), the buyer awards the contract to the entrant and pays the entrant $t_0 - a_e = t_0 - K/\beta$; if the entrant is deemed unqualified (which occurs with probability $1-\beta$), the buyer awards the contract to the incumbent and pays the incumbent $t_0 - a_i = t_0$.

This approach is referred to as "postqualification" in the sense that the buyer first runs the auction and subsequently, if necessary, determines the qualification status of the auction winner. The buyer adds a "markup" of $a_e = K/\beta$ to the entrant's bid to account for the fact that postqualification on the entrant costs the buyer K and succeeds with probability β. Note that $a_i = 0$. That is, the incumbent has zero markup, because the incumbent is already known to be qualified. The markup a_e can be thought of as the buyer's "switching cost."

2.4.1. Predicting Auction Outcome. Given this setup, the most basic question a buyer can ask is, "What is my expected total (payment plus qualification) cost?" To answer this question, the buyer needs a model of the supplier's strategic bidding behavior in the auction.

Supplier bidding behavior. Let c_i and c_e, respectively, denote the incumbent's and entrant's true costs of fulfilling the contract. If the incumbent (entrant) wins the contract at price t_0, his payoff is $t_0 - a_i - c_i$ ($t_0 - a_e - c_e$). We begin by examining the entrant's bidding strategy. Because the incumbent is already qualified, the entrant's bidding problem is akin to that studied in §§2.1 and 2.3: the entrant finds it optimal to bid in the auction until the incumbent drops out or the total-cost bid reaches its true total cost $c_e + a_e$. This is because the entrant must win the auction to win the contract.

We now turn to the incumbent, who knows that he could lose the auction but still win the contract. This occurs if the entrant fails postqualification, resulting in the buyer de facto awarding the contract to the incumbent defacto. Thus the incumbent faces a tricky problem: if he bids hard and tries to beat the entrant on price alone, he is less likely to forfeit the contract to the entrant and go away empty-handed. However, if he drops out early and simply hopes the entrant fails postqualification, he can potentially win the contract at a very high price.

This model and the incumbent's bidding equilibrium was proposed and analyzed in Wan et al. [10], which proves the following result. Suppose that $c_i \sim F_i$ over $[l, R]$, $l < 1 \le R$ and $c_e \sim F_e$ over $[0, 1]$. The supplier's costs c_i and c_e are private information, but F_i, F_e, K, and β are common knowledge. Let $\underline{p}(c_i)$ denote the price at which the incumbent will drop out of the auction if the entrant has not yet dropped out.

Theorem 1 (Wan et al. [10]). *Given any F_i, F_e, $\beta \in (0, 1)$, l, and R, for an incumbent with cost c_i, there exists a static optimal bid-down-to level $\underline{p}(c_i)$ such that the incumbent should stay in the auction until the price reaches $\underline{p}(c_i)$ or the entrant drops out, whichever happens first. There exist two thresholds c_B and c_W, such that $c_W \le c_B$, $c_B < R$, $c_W \le K/\beta$, and*

(i) $\underline{p}(c_i) = R$ *if and only if* $c_i \ge c_B$,
(ii) $\underline{p}(c_i) = K/\beta$ *if and only if* $c_i \le c_W$, *and*
(iii) $x_i < \underline{p}(c_i)$, $K/\beta < \underline{p}(c_i) < R$, *and* $\underline{p}(c_i)$ *strictly increases in* c_i *if* $c_W < c_i < c_B$.

Theorem 1 predicts that the incumbent's bidding behavior is markedly different than what was seen earlier in this tutorial. The incumbent finds that bidding until the auction ends or the entrant drops out is optimal only when the incumbent's cost is extremely low, meaning he knows he is likely to be able to beat the entrant on price alone (this is captured in Part (ii) of the theorem). When his cost is very high the incumbent drops out at the reserve price and essentially boycotts the auction (captured in Part (i) of the theorem), and when his cost is moderate the incumbent employs an intermediate strategy whereby he tries to beat the entrant on price alone but eventually abandons the effort and gives up if the entrant stays in the auction too long (captured by Part (iii) of the theorem).

Auction ending price. Given the bidding strategy from Theorem 1, Wan et al. [10] show that the buyer's expected total cost in the open-descending auction with postqualification is

$$E\left[\max\left\{\min\left\{c_e + \frac{K}{\beta}, R\right\}, \underline{p}(c_i)\right\}\right]. \tag{1}$$

This cost comes from the fact that if $c_e + K/\beta > R$, then the auction will not take place (the entrant will drop out immediately and the contract will be awarded to the incumbent at price R), and otherwise the auction will take place and the ending total cost will be $c_e + K/\beta$ if the incumbent wins or $\underline{p}(c_i)$ if the entrant wins, whichever is smaller.

2.5. Lessons

Summarizing our discussion on predicting the outcomes of competitive bidding events, we see that

1. The basic open-descending auction leads to very simple and intuitive equilibrium bidding behavior, and the auction ending price is easy to formulate as an order statistic.

This basic auction-theoretic building block captures the salient idea of using competitive bidding to find a "fair" market price.

2. Embellishing the basic model by including additive cost adjustments for suppliers does not greatly complicate the analysis. However, to predict the total-cost auction outcome, one must derive an expected order statistic of nonidentically distributed random variables.

3. Embellishing the basic model by including postqualification makes both the bidding analysis and the expected total-cost computation significantly more complex.

In summary, the basic auction-theoretic, price-only model can be enriched to capture more complex realities of industrial procurement. Depending on the embellishment, the resulting research challenges can be mostly probabilistic in nature (as in the case of computing the ending price in a total-cost auction) or game theoretic (as in the case of computing the equilibrium bidding strategy in the auction with postqualification). For an operations researcher considering working on competitive bidding topics, the lesson is that, depending on the type of model chosen, skills in areas such as probability and game theory can be leveraged to varying extents depending on what type of competitive bidding model is chosen. In the next section we will explore how we can apply the competitive bidding outcome predictions introduced above to answer relevant research questions in industrial sourcing.

3. Putting Outcome Predictions to Work

In this section, we explore how §2's predictions of competitive bidding outcomes can be applied to address research questions about sourcing. For brevity we will focus on research problem formulations.

3.1. Basic Model: Choosing the Number of Suppliers

In §2.1 we took the number of suppliers, N, as exogenously given. However, if the buyer is sourcing a new item for which she has few preexisting suppliers, a simple but fundamental question facing her is, "How many suppliers should be invited to the auction?" This is a meaningful question, because finding potential suppliers can be costly for the buyer, particularly if the items she is sourcing are unusual or require specific supplier skills. Moreover, to keep the auction "low bid wins" (rather than "low qualified bid wins," as in §2.4), some buyers will only allow fully qualified suppliers into their auctions (see Beall et al. [1]), which means the buyer must bear qualification costs on each supplier she allows to bid in the auction (whether or not this approach to qualification timing is an optimal policy is the topic of §3.3 below).

To model this, let K denote the cost of qualifying a new supplier. Thus, recruiting N suppliers for an auction would cost the buyer $N \cdot K$ dollars. The main conclusion of §2.1 was that the buyer's expected contract payment, $E[p_0]$, is given by $E[p_0] = E[c_{(2)}]$, where $c_{(2)}$ is the cost of the second-best supplier in the supply base. If we notice that $E[c_{(2)}]$ is indeed a function of N, we see that the buyer's optimal number of suppliers can be found by solving the following:

$$\max_{N \geq 2} \{E[c_{(2)}] - N \cdot K\}.$$

For example, if supplier costs are uniformly distributed over the unit interval, then the buyer solves

$$\max_{N \geq 2} \left\{ \frac{2}{N+1} - N \cdot K \right\}.$$

We see here that the contract price reduction the buyer expects from each additional supplier is decreasing in N, while the cost of acquiring each new supplier is fixed at K. Thus, the buyer's optimization problem in quite straightforward.

3.2. Total-Cost Model: Designing a Supply Base

This subsection is based on content from Wan and Beil [9]. Buyers in competitive industries with fast-changing technologies often use short-term supply contracts and re-source frequently (e.g., quarterly) to get the most up-to-date competitive pricing from the supply market (Beall et al. [1]). Given the costs associated with locating and qualifying new suppliers, such buyers often reauction to the same cohort of suppliers who have been prequalified to produce the items being auctioned off. This group of prequalified suppliers is known as the "supply base" for that part. For example, buyers we interacted with commonly had two, and sometimes up to four or five, suppliers for a particular part, depending on the supply market and the difficulty of qualifying suppliers. In this section, we will take the supply base size N to be fixed; finding the optimal N would involve trade-offs akin to those introduced in §3.1.

3.2.1. Supply Base Design Affects Costs. Consider the total-cost auction model of §2.3, and suppose a_i represents the buyer's cost of transporting goods from the supplier. The cost of transporting goods from a supplier is largely driven by the supplier's geographic location, and consequently suppliers located in the same geographic region will present similar transportation costs for the buyer. For example, consider a buyer in Europe whose supply base consists of two suppliers in China. Let y_{China} denote the cost of shipping goods from China to Europe. Then $a_1 = a_2 = y_{\text{China}}$. Note that with this supply base, the buyer is vulnerable to shipping price shocks along the China–Europe route. A recent example is the 1,000% increase in shipping insurance premiums for Asia to Europe ocean transport, as freighters funneling through the Suez Canal face pirates off the Somali coast (Costello [2]). With all its suppliers located in China, the buyer would be forced to incur these increased costs. Ideally, a buyer could respond to transportation cost shocks by instantly augmenting her supply base with new suppliers from unaffected regions. However, doing so can be impractical (for all but the most catastrophic scenarios) because it can take up to half a year to find and qualify a new supplier. (This was the case at a manufacturer we interacted with.)

Realizing her vulnerability to large transportation costs when all her suppliers are colocated, a buyer might consider managing this risk by choosing a supply base with suppliers from different geographic regions. Consider the situation where supplier 1 was located in China while supplier 2 was located in Brazil. Then $a_1 = y_{\text{China}}$ and $a_2 = y_{\text{Brazil}}$, where y_{Brazil} is the cost of shipping from Brazil to Europe. In this case, if faced with Somali pirate attacks along the Asia–Europe shipping corridor, the buyer could potentially source from Brazil instead.

While diversifying appears attractive on the surface, the problem is not necessarily as straightforward as it seems: because the suppliers' pricing for the contract is determined by competitive bidding in an auction, it is possible that a supplier who enjoys a regional cost advantage (as Brazil would over China in the above example) might simply take windfall profits by demanding a very high contract price in the auction. Thus, even if the buyer has access to a supplier from whom transportation costs are minimal, the buyer may still end up paying a high total price if her next-best option is a supplier whose transportation costs are large. In other words, when the buyer relies on competitive bidding for price concessions, cost parity among suppliers is important for generating strong competition and low prices.

3.2.2. Supply Base Design Model. We now formalize the buyer's strategic problem of designing her supply base for a part. The supply base design problem consists of the following stages. Let r denote the index of geographic regions (e.g., China, Brazil, etc.). In stage 0 the buyer selects n_r suppliers from region r, where $n_1 + n_2 + \cdots + n_R = N$. The buyer then auctions off supply contracts among this supply base. These auctions happen periodically (e.g., quarterly). Call the time periods covered by these quarterly contracts periods 1, 2, 3, etc. For example, period 1 might be January through March, and period 2 might be April through June.

At the outset of a future period, call it period k, the following will happen. Each supplier i's cost of production during period k is revealed (privately), and for each geographic region r the transportation costs that are expected to be incurred by the buyer during the period k contract is revealed (publicly). The buyer then holds a total-cost, open-descending auction among the N suppliers. The lowest total-cost bidder is awarded the period k contract and the buyer's total cost in period k is equal to the second-lowest total period k cost among the supply base (as seen in §2.3). Suppose that for $i = 1, \ldots, N$, supplier i's period k production cost c_i^k is distributed according to F_k, and for $r = 1, \ldots, R$ region r's period k expected transportation cost y_r^k is distributed according to G_k. To keep things simple, suppose that the distributions F_k and G_k are stationary across periods and independent.

The buyer wishes to design the supply base so as to minimize her expected discounted total (production plus transportation) procurement cost in periods $k = 1, \ldots$. It suffices to design her supply base in period 0 so as to minimize her expected total cost in any given period k. For convenience we drop the subscript k, and the buyer's supply base design problem can be stated as

$$\begin{aligned} \min_{n_1, \ldots, n_R} \quad & \underset{(c_1, \ldots, c_N, y_1, \ldots, y_R)}{E} [(c + a)_{(2)}] \\ \text{s.t.} \quad & R \in \{1, \ldots, N\}, \quad n_i \in \mathbb{N}\ \forall i \in \{1, \ldots, R\}, \quad n_1 + \cdots + n_R = N, \quad \text{and} \\ & a_i = y_r \quad \text{if supplier } i \text{ is located in region } r. \end{aligned}$$

Detailed analysis of this problem is provided in Wan and Beil [9]. The point we wish to make here is simply that a very basic model of total-cost, open-descending auctions can be embedded into a larger important and strategic problem faced by industrial buyers, namely the design of a supply base for a part. By choosing where to locate her suppliers, the buyer selects the amount of cost correlation among them (since suppliers located in the same region have perfectly correlated transportation costs). Notice that the above math program roughly boils down to the following statistical question: To minimize the expected second-lowest value among N draws of random variables, how much correlation is optimal to have across these random variables? Very often, the fact that auction outcomes are described by order statistics means that complex statistical and probability questions arise in research problems on procurement auctions.

3.3. Total-Cost with Postqualification: Choosing Qualification Timing

This subsection is based on content from Wan et al. [10]. In §2.4 we assumed that the buyer only qualified the entrant if they won the auction against the incumbent. Using postauction qualification screening can be attractive for the buyer because the buyer avoids spending money qualifying an entrant who does not wind up with an attractive cost in the auction. However, as shown in Theorem 1, using postauction qualification gives the incumbent an incentive to hold back on bidding. Unfortunately, this strategic incumbent behavior can essentially forestall the bidding competition, making the auction useless.

As an alternative, the buyer might consider attempting to qualify the entrant prior to the auction. If the entrant survives the qualification screening (is deemed qualified), then the buyer can hold an open-descending auction with the incumbent and entrant and tout the fact that only the lowest bid will win the contract. In this case, which occurs with probability β, the incumbent would have no incentive to strategically drop out of the bidding early, and the expected auction payment is $E[\max\{c_i, c_e\}]$. On the other hand, the preauction qualification strategy can backfire on the buyer—if the entrant fails qualification (is deemed unqualified), which happens with probability $1-\beta$, then the buyer would have to directly award the contract to the incumbent, at price R.[6]

Which strategy is best—pre- or postauction qualification? We can answer this question by comparing the buyer's expected total (contract payment plus qualification) cost under each strategy. We already computed the buyer's expected total cost under postqualification (Equation (1)). Given the arguments in the above paragraph, we see that the buyer's expected cost under preauction qualification is

$$\beta \cdot E[\max\{c_i, c_e\}] + (1-\beta) \cdot R + K. \tag{2}$$

Comparing Equations (1) and (2) provides us with the buyer's optimal qualification timing decision. This problem is analyzed in Wan et al. [10].

3.4. Lessons

The focus of much of auction theory has been on understanding how different auction formats perform in different situations. In procurement auctions, the complexities of the underlying business environment creates a myriad of interesting research questions that arise even when the auction format is simply taken as a given. We demonstrated this by looking at three different procurement research topics that all applied the open-descending auction format, which is reasonable to assume since it is so commonly used in practice. Of course, one can also ask how different auction formats would affect the answers we arrive at under the open-descending format. (For example, this is done in Wan and Beil [9], Kostamis et al. [4].) Thus the scope for interesting procurement auction research is virtually limitless.

4. Conclusions

This tutorial provided a brief discussion of procurement auctions. We focused our discussion around variants of the open-descending auction format. This auction format is commonly used in practice, captures the salient ideas of using competitive bidding to find a market price, and in many instances yields simple strategic bidding behavior that facilitates analysis. In light of the complexities of the underlying procurement business environment, this model can serve as a useful starting point for richer modeling, leading to analytical challenges that are ripe for operations researchers (such as challenging involving probability and order statistics). Additionally, when embellished with operational realities the open-descending auction model can also require somewhat complex game-theoretic analyses, meaning the scope for work in the area of procurement auctions can certainly encompass important game-theoretic issues as well.

This chapter by no means presented an exhaustive overview of topics in procurement auctions. Instead, by focusing on a few simple auction models and demonstrating procurement auction research topics employing them, this chapter hopes to galvanize by example further work in procurement auctions by operations researchers. Readers interested in learning more about these topics can consult the excellent survey on procurement and competitive bidding by Elmaghraby [3] and the introductory text on auctions by Krishna [5].

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