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Optimization Modeling and Techniques for Systemic Risk Assessment and Control in Financial Networks

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Abstract Since the financial crisis of 2007–2008, the assessment and control of the systemic risk in financial networks has become one of the most important and active research areas in economics and finance. In this tutorial, we will give a basic introduction to various optimization models. These include linear optimization (LO) problems with uncertain data on both sides of the constraints, linear optimization with nonconvex quadratic constraints, mixed integer linear optimization, and stochastic optimization. Our discussion focuses on the following three topics in the risk analysis of financial networks: (i) vulnerability analysis of a financial network when only limited information of the network is available and the assets are subject to market shocks, (ii) identification of the least stable network structures for fixed assets and exploration of the cascading failures in them, and (iii) policies and strategies for risk mitigation.

Specifically, we will demonstrate how classical LO theory can be used to assess the contagion in the network when complete information about the network is available, and we introduce new optimization theory to deal with the scenario when only limited information of the financial network is exposed. We show how conventional optimization techniques, such as alternative convex search and successive linear approximation, can help identify the least stable network structure, and we discuss under what circumstances these conventional optimization techniques may fail and what new optimization techniques need to be developed. We will also discuss how to combine advanced analysis in continuous optimization and graph modeling to identify the network structure. Open questions and challenges from both an optimization and financial perspective will be discussed as well.

Keywords financial network • systemic risk • sensitivity analysis • (in)feasibility analysis • worst-case linear optimization • financial contagion

1. Introduction

A typical financial network comprises multiple financial institutions such as firms, traders, and banks. They interact with each other directly through loan contractual obligations or interconnect through overlapping portfolios. Such tight linkages among financial institutions have various consequences in the global financial market. On the one hand, it speeds up the transaction process and affects the asset prices by acquiring and processing the related information more efficiently. On the other hand, it also forms a channel through which the failure of a single institution can quickly spread to the entire system, resulting in a cascade of failures, a catastrophic disaster. This is usually referred as the so-called *systemic risk*. The financial

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crisis of 2007–2008 is one example of this type of disaster that triggered events in not only the entire U.S. financial industry but also multiple financial markets around the world (Murphy [50]). The European sovereign debt crisis is another example, which led to a serious loss of confidence in European financial business (Lane [47]). As a consequence of these crises, new programs and financial regulations have been implemented by various governments and agencies to stabilize the financial system. For example, in October 2008, the U.S. government created TARP, the Troubled Asset Relief Program, which is called a *bailout plan* (Webel [57]), and Basel III accord was created to strengthen its standard on bank's minimum capital ratios and introduce requirements on liquid asset holdings and funding stability to mitigate the risk.

The catastrophic disaster of such financial crises has drawn intensive attention from various researchers, and a large literature has been established in the study of systemic risk. Most existing works in the literature fit into one of the following three streams: (a) the assessment of the risk, (b) the stability and resilience of financial networks, and (c) policies and strategies to mitigate the risk. Stream (a) includes works in the development of various risk measurements and models such as Acharya et al. [3], Adrian and Brunnermeier [4], Billio et al. [15], Cont et al. [26], Eisenberg and Noe [28], Elsinger et al. [34], and Huang et al. [42] and works on empirically measuring risk based on market data such as Boss et al. [17], Cocco et al. [25], Furfine [37], Mistrulli [49], Nier [51], Upper and Worms [55], and Sheldon and Maurer [54]. Works in stream (b) concentrate on exploring the impact of market shocks and network structure on the stability of a financial system. For example, Acemoglu et al. [1, 2], Chen et al. [22], Cont et al. [26], Elsinger et al. [30, 31, 32, 33], Glasserman and Young [41], and Liu and Staum [48] study the contagions in a financial system under different settings. Acemoglu et al. [1, 2], Allen and Gale [5], Amini and Minca [6], Amini et al. [7, 8], Battison et al. [11, 12, 13], Capponi et al. [21], Chen et al. [23], Feinstein et al. [34], Freixas et al. [36], Gai and Kapadia [38], Gai et al. [39], Glasserman and Young [41], and Khabazian and Peng [46] explore the impact of network structure on the resilience of a financial network. The last stream includes Bernard et al. [14], Capponi and Chen [20], Kallio and Khabazian [44], and Pokutta et al. [52], where different policies/strategies are proposed to mitigate the risk in the system.

Several books and survey papers (Caccioli et al. [18], Capponi [19], Glasserman and Young [41], Hurd [43], and Visentin et al. [56]) have also been published on systemic risk, where the authors summarize recent exciting developments on various perspectives of systemic risk. For example, Capponi [19] reviews various approaches in the literature to assess the systemic risk based on market data, policies to mitigate the risk, and their economic impact. Different from the above-mentioned works, in this survey we will focus on various optimization models and techniques in the literature. To start, we mention that a seminal work in this direction is the paper by Eisenberg and Noe [28], who introduce the clearing payment model to assess the systemic risk in interbanking networks. In this survey, we concentrate on optimization models based on the clearing payment model and its variants, which have been widely studied in the literature (Acemoglu et al. [2], Bernard et al. [14], Capponi and Chen [20], Chen et al. [23], Elsinger et al. [33], Glasserman and Young [41], and Liu and Staum [48]).

The rest of the paper is organized as follows. In Section 2, we first describe the basic clearing agent model and review existing works on assessing systemic risk using the basic model and market data. Then we present several variants of the basic model and discuss their financial implications. In Section 3, we review existing works on contagions in financial networks (i.e., the impact of market shocks on the stability of financial networks). We also discuss the effect of asset inequality on the stability of financial networks. In Section 4, we review existing works on the exploration of the relationship between the network structure and the stability/resilience of the network. In Section 5, we describe several optimization models and techniques for mitigation policies proposed in the literature. Finally, we close this survey by discussing open questions and challenges from both an optimization perspective and a financial perspective in Section 6.

Table 1 provides a list of notations and parameters used in this survey.

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Table 1. Notations and Parameters

Parameter	Definition
n	The number of financial institutions in the network
$\mathcal{I}$	The set of all the financial institutions in the system
$\mathcal{I}_1$	The index set of default nodes
$\mathcal{I}_2$	The index set of solvent nodes
$\mathcal{B}$	The set of merged institutions in the system
α_i	The non-interbank assets of financial institution i ($i = \{1, \ldots, n\}$)
l_{ij}	The liability of financial institution i toward financial institution j
p_i	The total liability of node i ($i = \{1, \ldots, n\}$)
E_i	The equity of financial institution i
γ	The fraction of the nominal value of the asset after liquidation
β	The fraction of the nominal value of interbank asset after liquidation
C_{ij}	A fraction of organization i owned by organization j
D_{ik}	The value of asset k held by organization i
V_i	The equity value of organization i
v_i	The market value of organization i
V_i	The book value of organization i
d_i	The outside liabilities of institution i
s_i	The amount of market shock for institution i
b_i	The amount of bailout that insolvent institution i will receive
$\bar{b}$	The bailout budget
κ_i	A merging cost for financial institution i

We should remind the readers that our notations might be slightly different from what is used in the literature.

2. The Basic Clearing Agent Model and Its Variants

In this section, we describe the basic clearing agent model and some of its variants proposed in the literature to assess the systemic risk in financial networks.

2.1. The Basic Clearing Agent Model by Eisenberg–Noe

The basic clearing agent model was introduced by Eisenberg and Noe [28], who consider a financial network with n banks (represented by n nodes) interconnected to each other. A clearing agent is in charge of the process of settling the liabilities among these nodes. The ability of one node to settle its obligations depends on the repayment of other nodes to this node and also its own assets. Let α be the non-interbank asset, and let $L \in \Re^{n\times n}$ be the interbank liability matrix where l_{ij} is the liability of node i toward node j. Because each nominal claim is nonnegative and no node has a nominal claim against itself, we have $l_{ij} \geq 0$ and $l_{ii} = 0$, $\forall\, i, j = 1, \ldots, n$. The total liability of node i is equal to $p_i = \sum_{j=1}^{n} l_{ij}$. Based on this, the equity of bank i is given by

$$E_i = \sum_j l_{ji} + \alpha_i - p_i. \tag{1}$$

Note that the above expression corresponds to the book value of the equity of bank—that is, $E_i < 0$ indicates that bank i's assets are less than its liabilities, or in other words, bank i is not be able to repay its liabilities fully. We also say a financial network is well balanced if $p = Le = (L^T e)$, where e is the n-vector of all ones (i.e., for every node in the network, its total liability equals its total claim within the network). Based on (1), if the financial network is well balanced, $E_i = \alpha_i, \forall\, i = 1, \ldots, n$. According to Eisenberg and Noe [28], the clearing model should satisfy the bankruptcy law provisions that include (i) limited liability, (ii) absolute priority, and (iii) proportionality. They propose a clearing algorithm, called the fictitious default algorithm, which starts from the candidate clearing payment vector and generates

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a sequence of payment vectors until convergence is reached. To illustrate this, let $\alpha^{(k)}$ denote the asset value at iteration k. Based on this, the fraction of bank i's total liability that it can repay to its creditors at iteration k (with $\alpha^{(0)} = \alpha$), denoted by $x_i^{(k)}$, can be obtained as follows:

$$x_i^{(0)} = \frac{1}{p_i} \min\{\max(\alpha_i^{(0)}, 0), p_i\}.$$

Here, the algorithm starts with the payment ratio $x^{(0)}$, and based on this, the new asset vector can be obtained:

$$\alpha^{(1)} = \alpha^{(0)} + L^T x^{(0)}.$$

Therefore, at the kth iteration, the repayment ratio reads

$$x_i^{(k)} = \frac{1}{p_i} \min\{\max(\alpha_i^{(k)}, 0), p_i\}, \tag{2}$$

where $\alpha^{(k)} = \alpha^{(0)} + L^T x^{(k-1)}$. Notice that at each iteration, we have that $x^{(k)} \geq 0$, and therefore, the sequence of repayment ratios is nondecreasing (i.e., $x^{(k)} \leq x^{(k+1)}$). From (2), one can see that the repayment ratio is bounded above (i.e., $x^{(k)} \leq 1$). As shown in Eisenberg and Noe [28], the above-mentioned clearing algorithm converges to the clearing payment ratio vector (x^*), which is the optimal solution to the following optimization model (denoted by E-N):

$$\begin{aligned} \max_x \quad & p^T x \\ \text{s.t.} \quad & (P - L^T)x \leq \alpha; \\ & 0 \leq x \leq e, \end{aligned} \tag{3}$$

where $P = \operatorname{diag}(p)$. In this framework, $p_i x_i$ is the payment made by bank i, where x_i represents the fraction of bank i's total liabilities it can repay to its creditors once the system is cleared. Based on this, for a bankrupt financial institution that cannot pay back any of its liabilities, this value is 0 ($x_i^* = 0$); for a defaulted institution that can pay back only a fraction of its liabilities, the value is less than 1 ($0 < x_i^* < 1$); and for a solvent institution that can fulfill its liabilities, the value is always 1 ($x_i^* = 1$).

The clearing algorithm presents one way for financial institutions to make their payments. Note that a financial institution may use other strategies such as liquidation to pay its liabilities. Several different clearing algorithms have been studied by Barucca et al. [9], Kai and Kapadia [38], and Rogers and Veraart [53]. For more details in this direction, we refer to the monograph by Hurd [43] and references therein.

Eisenberg and Noe [28] also explore various properties at the optimal solution of the E-N model. We combine several theoretical results in Eisenberg and Noe [28] to summarize the key properties of the E-N model in the following proposition.

Proposition 1. *Suppose that the financial network is fully connected. Then we have that*
(i) *the clearing vector (x^*) exists and is unique;*
(ii) *at the optimal solution of problem (EN), for every $i = 1, \ldots, n$, it holds that either $[(P - L^T)x^*]_i = \alpha_i$ or $x_i^* = 1$; and*
(iii) *the optimal solution to the E-N model can be obtained by solving n decomposed problems as follows:*

$$\begin{aligned} \max_x \quad & x_i \\ \text{s.t.} \quad & (P - L^T)x \leq \alpha; \\ & 0 \leq x \leq e. \end{aligned} \tag{4}$$

Several essentially equivalent variants of the E-N model (3) have been studied in Chen et al. [23], Elsinger et al. [31, 32], and Liu and Staum [48]. For example, Elsinger et al. [31, 32] discuss how to empirically assess the risk in financial networks using the data from the UK and Austrian interbank markets. In their study, the joint impact of two major sources of risk, the correlated

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exposure and domino effect, is considered. As pointed out in Elsinger et al. [31, 32], the data from banks usually reveal partial information regarding the interbank liabilities, and the assets are subject to market fluctuation. To address this issue, they first suggest to compute the interbank liabilities by solving an entropy optimization problem based on the so-called *Kullback–Leibler* (*KL*) *divergence*. Then they fix the liability matrix L, cast the asset vector as a random vector, and use stochastic optimization and scenario generation to assess the risk. The survey by Elsinger et al. [33] discusses various approaches in risk assessment and challenges up to that time.

Liu and Staum [48] apply the standard sensitivity analysis in linear optimization to the E-N model to estimate the impact of the market shock received by a single financial institution. They observe that, based on the decomposed problems (4), the contagion risks (or the partial derivatives of the repayments with respect to the assets) are precisely the shadow prices for problem (4). From this observation, they estimate the contagion risk under the assumptions that the complete information of a financial network is available, and the market shock will not change the set of defaulted banks and the set of solvent banks. However, they did not discuss the case with reasonably large shocks that may cause some solvent nodes to become insolvent.

Chen et al. [23] consider a new optimization model where the asset of an institution includes external investment, liquid assets, and illiquid assets. In their framework, a financial institution can use all of its interbank assets, external investments, and liquid assets with the face value to meet its liabilities. It may also need to sell some amount of its illiquid assets for its repayment, which can be converted to cash at its market price. Under the assumption that the set of solvent and insolvent nodes remains invariant, the authors apply sensitivity analysis to investigate the impact of a market shock on the repayment ability and asset price for each institution and to examine the benefit of two policy interventions (i.e., capital injection and direct purchase of the illiquid assets).

It should be pointed out that the E-N model not only provides a simple way to determine the clearing payment vector but also convincingly demonstrates how insolvency can be transmitted from one node to another one, amplifying the effect of the initial market shock. Nevertheless, the E-N model has several limitations. One is that in the E-N model, other sources of financial contagion such as the cost of bankruptcy, the cost of liquidation, and fire sales have not been taken into account. Second, as pointed out in Elsinger et al. [33], the E-N model is built on the assumption that the complete information on the financial network is available, whereas in reality only partial information regarding the financial network is exposed. Third, the E-N model does not take into account the impact of the leverage ratio requirement on the stability of the system. In the remaining subsections, we will review several important variants of the E-N model proposed in the literature to address these issues.

2.2. Variants of the Eisenberg–Noe Model

One variant of this model is developed by Rogers and Veraart [53], who extend the E-N model by taking into account bankruptcy and liquidation costs. They use a coefficient γ to quantify the fraction of the nominal value of the asset after the liquidation and another coefficient β to quantify the fraction of the nominal value of interbank asset after the liquidation. So when institution i defaults in the system, it recovers only a fractional asset ($\gamma\alpha_i$) and interbank asset ($\beta[L^Tx]_i$), and therefore, the cost of its failure can be obtained as follows:

$$(1-\gamma)\alpha_i + (1-\beta)\left[L^T x\right]_i.$$

From this, they modify model (3) as follows (denoted by R-V):

$$\begin{aligned} \max_x \quad & p^T x \\ \text{s.t.} \quad & [(P - L^T)x]_i \le \alpha_i - \big((1-\gamma)\alpha_i + (1-\beta)[L^T x]_i\big)\mathbb{I}_{x_i<1}, \quad \forall i = \{1, \ldots, n\}; \\ & 0 \le x \le e, \end{aligned} \tag{5}$$

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where $0 < \gamma, \beta < 1$, and $\mathbb{I}_{x_i<1}$ is a binary indicator variable taking value 1 if $x_i < 1$ and value 0 otherwise. Rogers and Veraart [53] show that the existence and uniqueness of the clearing payment vector also holds for the extended model (5) (see theorem 3.1 in Rogers and Veraart [53]).

Different from Rogers and Veraart [53], Glasserman and Young [40] define the insolvency cost of a default node in the system via a function proportional to the amount of default. Specifically, they introduce a multiplier $\gamma \geq 0$ and define the loss in the assets of a node i (when it defaults) by

$$\gamma\big(p_i - \big(\alpha_i + \big[L^T x\big]_i\big)\big).$$

Consequentially, they derive the following extended model (denoted by G-Y):

$$\begin{aligned} &\max_x \; p^T x \\ &\text{s.t. } (P - L^T)x \leq \alpha - \frac{\gamma}{1+\gamma}(e - x)p; \\ &\qquad 0 \leq x \leq e. \end{aligned} \tag{6}$$

It is easy to see that when $\gamma = 0$, the G-Y model is equivalent to the E-N model. Otherwise, this factor increases the loss generated as a result of the insolvency in the system. The existence and uniqueness of the clearing payment vector is also studied in Glasserman and Young [40].

It is interesting to note that based on (6), the leverage ratio for node i is defined by

$$r_i^{GY} = \frac{[\alpha - (P - L^T)x]_i - \gamma(1 - x_i)p_i/(1+\gamma)}{(1 - x_i)p_i}.$$

On the other hand, in the E-N model where the insolvency loss is ignored, the leverage ratio for node i is defined by

$$r_i^{EN} = \frac{[\alpha - (P - L^T)x]_i}{(1 - x_i)p_i}.$$

After the financial crisis in 2007–2008, Basel III imposed a minimum leverage ratio requirement at 3% (Basel Committee on Banking supervision [10]). In July 2013, the U.S. Federal Reserve announced that the minimum leverage ratio would be 6% for eight systemically important financial institutions.[1] If we incorporate the minimum ratio (denoted by r_{min}) requirement into the E-N model, then we derive the following extended model:

$$\begin{aligned} &\max_x \; p^T x \\ &\text{s.t. } (P - L^T)x \leq \alpha - r_{min}(e - x)p; \\ &\qquad 0 \leq x \leq e. \end{aligned} \tag{7}$$

We remark that the above model is very close to model (6), indicating that model (6) implicitly imposes some minimum leverage ratio requirement.

2.3. The Elliott–Golub–Jackson Model

An alternative to the E-N model is the E-G-J model introduced by Elliot et al. [29], where the authors consider cross-holdings as direct claims on the value of organizations. They consider a network of organizations holding assets and shares in each other. The market value v_i of organization i can be obtained as follows:

$$v_i = \sum_j C_{ij} V_j - \sum_j C_{ji} V_i + \sum_k D_{ik} p_k,$$

where C_{ij} is the fraction of organization i owned by organization j, D_{ik} is the value of asset k held by organization i, and V_i is the equity value of organization i. In the E-G-J model, if the

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value of a node i falls below a certain threshold ($\underline{v}_i$) (i.e., $v_i < \underline{v}_i$), node i fails and needs to liquidate its assets. They assume that the liquidation cannot be partial and is irreversible. Such failure (liquidation) costs are subtracted from its cash flow, which depends on the market value of node i (i.e., $\beta_i(p) = \lambda_i p_i$). Based on this, the book value of node i becomes

$$V_i = \sum_{j \neq i} C_{ij} V_j + \sum_k D_{ik} p_k - \beta_i(p) \mathbb{I}_{v_i < \underline{v}_i},$$

where $\mathbb{I}_{v_i < \underline{v}_i}$ is a binary indicator variable taking value 1 if $v_i < \underline{v}_i$ and 0 otherwise. They observe that the discontinuities in the model can lead to cascading failures and also the presence of multiple equilibria.

Elliot et al. [29] also introduce the level of integration and diversification of the financial system as the level of exposure of institutions toward each other and the number of institutions directly interacting with one another through their cross-holdings, respectively. For the mathematical interpretation of the integration and diversification, they introduce the adjacency matrix G where $G_{ij} = 1$ if organization i has a claim on organization j and $G_{ij} = 0$ otherwise. From this, they define the in-degree $d_j^{in} = \sum_i G_{ij}$ and out-degree $d_i^{out} = \sum_j G_{ij}$. Under the assumption that a fraction c of each organization is held by other organizations, we have

$$C_{ij} = \begin{cases} \dfrac{cG_{ij}}{d_j}, & \text{if } d_j > 0; \\ 0, & \text{otherwise.} \end{cases}$$

In this framework, for fixed c, if we increase out-degree d_j, diversification increases, and for the case that matrix G is fixed, if we increase c, integration of the network will increase. From this, Elliot et al. [29] investigate how the level of integration and diversification can affect the stability of the system. Some of their key results, as a combination of propositions 2 and 3 in Elliott et al. [29], are listed in the following proposition.

Proposition 2. *The E-G-J model has the following properties:*

(i) *A more integrated network can help to avoid the first failure in the system, and it can also exacerbate the contagion effect of failure given that the first failure occurs.*

(ii) *If either integration or diversification is extreme, then there can be no substantial contagion.*

(iii) *If both integration and diversification are intermediate, then contagion can occur.*

Birge [16] further extends the E-G-J model considering the endogenous investment choices that are assumed to determine the level of cross-holdings of the institutions. In this model, the cross-holding matrix C is endogenous, which results from incentive-compatible choices of the institutions.

3. Vulnerability Analysis of Financial Networks Under Uncertain Assets

3.1. The Framework by Acemoglu et al.

Acemoglu et al. [1] cast a financial network as a weighted directed graph consisting of n banks, where the edge weights represent interbank liabilities among banks. They assume that bank i borrows only a given maximum amount from bank j, and such a relationship does not need to be symmetric. In addition to interbank liabilities, each bank has outside liabilities of magnitude $d > 0$, which is assumed to have seniority. They employ a framework where there are three time periods. At time $t = 0$, banks borrow funds from one another or the outside financier to invest in a project. The project can be liquidated prematurely at time $t = 1$ or kept until maturity to $t = 2$. They also introduce two discrete measurements, the inverse of the expected number of defaults and the inverse of the

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maximum number of possible defaults, to define the stability and resilience of a financial system, respectively.

They study the stability and resilience of a financial system under three different network structures: (i) a complete network, (ii) a ring network, and (iii) a γ-convex combination of the complete and ring network whose interbank liabilities can be obtained as a linear combination of interbank liabilities of the complete and ring. In Acemoglu et al. [2], they consider a well-balanced network under the following condition.

Assumption 1. *All the banks have the same assets and the same total liability:* $\alpha_i = \bar{\alpha}$, $p_i = \bar{p}, \forall\, i = 1, \ldots, n$.

As pointed out in Glasserman and Young [41], Assumption 1 is very restrictive and typically will not be satisfied in real financial networks. Under such an assumption, they estimate the impact on the stability of a financial network with a small shock and a large shock. In their work, a shock is considered large if its size is larger than the total available assets after the senior liabilities are fulfilled (excess liquidity) (i.e., $n(\bar{\alpha} - d)$). From this, they show that as long as the size of the shock is small, financial networks with a more equal distribution of interbank liabilities are more stable. However, if the magnitude of the shock is large, more interconnections lead to fragility (see propositions 2 and 3 in Acemoglu et al. [1]).

Acemoglu et al. [1] also consider a generic scenario with multiple negative shocks in the system. In this case, they propose a threshold, defined by the excess liquidity divided by the number of negative shocks in the system, and suggest using such a threshold to separate small and large shocks. They show that the number of negative shocks plays a similar role as the size of the shocks. More specifically, when the number of negative shocks and size of the shocks are small enough, a more connected financial network will enhance the stability of the system. However, when the size of the shock or the number of negative shocks is large enough, a weakly connected network is more stable (see proposition 4 in Acemoglu et al. [1]).

3.2. The Framework by Glasserman and Young

Different from the analysis developed in Acemoglu et al. [1], Glasserman and Young [40] propose to estimate the contagious effect of failure using probability analysis. They analyze the contagious effect in the network based on a full-fledged shock distribution ($s_i \in [0, \alpha_i]$) and estimate how much of the losses are directly linked to the network connectivity. In their analysis, they use the following three pieces of information, which can be obtained from the balance sheets of the financial institutions:

i. the net worth of bank i, the difference between its total assets and its total liabilities;

ii. the financial connectivity of bank i, the fraction of its liabilities held by other financial institutions:

$$R_i^c = \frac{p_i - d_i}{p_i},$$

where d_i represents the outside liabilities of node i, and $p_i = \sum_j l_{ij} + d_i$; and

iii. the leverage ratio, the fraction of bank i's assets over its net worth:

$$R_i^l = \frac{\alpha_i}{E_i}.$$

They derive upper bounds for the probability that a default of one bank can cause defaults at the other banks in the system without knowing the exact details of the system's topology, which is presented in the following.

Proposition 3. *Suppose that only node i receives a shock, and the system is solvent before the shock. Then, we have the following:*

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(i) *The probability that the shock* (s_i) *causes all nodes* $j, \forall j \neq i$, *to default in the system is at most*

$$Pr\Big(s_i \geq E_i + \frac{1}{R_i^c}\sum_{j\neq i} E_j\Big).$$

(ii) *Contagion from* i *to* $j \neq i$ *is impossible if*

$$\sum_{j\neq i} E_j > E_i R_i^c \big(R_i^l - 1\big).$$

From this, they introduce $E_i R_i^c (R_i^l - 1)$ as a contagion index that determines how the failure of one insolvent institution will cause other institutions to fail through contagion. Their result shows that when a financial institution has a higher leverage ratio, larger net worth, and higher financial connectivity, it is more likely that the failure of that institution cascades through the system compared with the case when financial institutions directly default. Under the same assumption, they also show that when all the financial institutions have the same asset, the contagion will be weak from any financial institution to any other set of institutions, regardless of the network structure.

The concept of node depth is also introduced to characterize the size of the amplification in a financial system. They show that the contagion effect becomes more significant when other channels of contagion (e.g., the bankruptcy cost and market-to-market losses from credit quality) are taken into account.

3.3. The Framework by Khabazian and Peng

Another line of study on contagions in financial networks is in applying the classical sensitivity analysis for linear optimization to the E-N model or its variants. Works in this direction include Liu and Staum [48] and Chen et al. [23], where they assume that the market shock will not change the set of defaulted banks and the set of solvent banks. Most results in Glasserman and Young [40] are also built on such an assumption.

Note that the assumption that the sets of defaulted banks and solvent banks remain invariant implicitly implies that the market shock is insignificant or small, which is very different from what happened during the financial crisis of 2007–2008 when the large market shock led to the bankruptcy of financial institutions such as Lehman Brothers. Clearly, the small shock assumption cannot be used in the analysis of bankruptcy in a financial network. A challenge here is how to assess the systemic risk of a financial network when only limited and incomplete information regarding the financial network is available and the market shock is significant.

To address the above challenge, Khabazian and Peng [45] develop a new theoretical framework to analyze the vulnerability of a financial system under mild assumptions on the market shocks. For this, they first propose to relax the E-N model by removing the nonnegativity constraints in it:

$$\begin{aligned} \max_x \quad & p^T x \\ \text{s.t.} \quad & (P - L^T)x \leq \alpha, \\ & x \leq e. \end{aligned} \tag{8}$$

At the optimal solution of (8), it is possible that there exists some index i satisfying $x_i^* < 0$. A negative value of x_i would mean that payments due from bank i instead flow in the opposite direction, which is clearly not relevant in practice. Several properties of the relaxed model are studied in Khabazian and Peng [45]. The following proposition lists some key properties of model (8), which is a combination of proposition 2.1, corollary 2.1, and theorem 2.1 in Khabazian and Peng [45].

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Proposition 4. *Let $x^{(1)}$ be the optimal solution of problem (3), and let $x^{(2)}$ be the optimal solution of problem (8). Then, we have the following:*

(i) $x^{(2)} = x^{(1)}$;

(ii) $[(P - L^T)x^{(2)}]_i = \alpha_i$ *or* $x_i^{(2)} = 1$, $\forall i = 1, \ldots, n$; *and*

(iii) *if the underlying network is fully connected, then problem (8) is infeasible if and only if*

$$\sum_i \alpha_i < 0. \tag{9}$$

Based on this proposition, the optimal solution of problem (3) can be used as a clearing payment vector for a financial system, and the total assets can be used as an indicator for the infeasibility of problem (8), which corresponds to the bankruptcy in a financial system. It is interesting to note that the large shock defined in Acemoglu et al. [2] satisfies the relation (9).

On the basis of problem (8), Khabazian and Peng [45] analyze the vulnerability of a financial network with an uncertain asset vector and two scenarios of market shock: a single shock and multiple shocks. For the special case of a single shock received by node i, they propose to first set $x_i = 0$ or $x_i = 1$ and then solve the following modified variant of problem (8):

$$\begin{aligned} \max_x \quad & p^T x \\ \text{s.t.} \quad & (P - L^T)x \le \alpha + s; \\ & x \le e, \end{aligned} \tag{10}$$

where s has only a single nonzero element $s_i < 0$, representing the market shock received only by node i. On the basis of the solutions from variants of problem (10), they characterize the conditions under which bank i will be solvent, defaulted, or bankrupted. They provide an estimate on the minimum amount of negative shock (s_i) such that bank i can survive bankruptcy (denoted by Δ_i) and an estimate of the minimum amount of positive shock or market gain for an insolvent bank i to become solvent (denoted by Γ_i) (see theorems 3.1 and 3.2 in Khabazian and Peng [45]). The estimated number of shocks depends on the current market information, which allows the financial institution to estimate the worst-case scenario it can survive or the asset increment they need to become solvent. By combining the results in theorems 3.1 and 3.2 in Khabazian and Peng [45], they introduce the default window as

$$\big(\max\big(e^T\alpha, \Delta_i - \alpha_i\big), p_i + \Gamma_i - \alpha_i\big),$$

which can be used as an indicator for the resistance of a default bank i to the market shock. They also estimate the magnitude of a single shock under which some nonreceiving node will be bankrupted. The results can be used to compare the sustainability of some nonreceiving node and receiving node in the system.

For the generic scenario with multiple shocks, Khabazian and Peng [45] assume that the total assets is fixed and study the impact of asset inequality on the stability of the system. For this, they introduce the following definitions for the asset inequality and the stability of a financial system.

Definition 1. A node (i) in the financial system is said to be a *strictly solvent* node if

$$x_i^* = 1, \quad \big[\big(P - L^T\big)x^*\big]_i < \alpha_i,$$

where x_i^* is the optimal solution of (3) or (8). Correspondingly, the gap between the assets of one strictly solvent node and some insolvent node in the system is called an asset inequality.

Definition 2. Consider two financial systems with the same network structure L with asset vectors α^1 and α^2 satisfying $e^T\alpha^1 = e^T\alpha^2 = \bar{\alpha}$. The first system is said to be less stable than the second one if

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$$p^T x^*(\alpha^1) < p^T x^*(\alpha^2),$$

where $x^*(\alpha^1)$ and $x^*(\alpha^2)$ denote the optimal solutions of (3) with $\alpha = \alpha^1$ and $\alpha = \alpha^2$, respectively.

In Khabazian and Peng [45], it is shown that the asset inequality in the financial network has a negative effect on the stability of the system. From this, they study the stability of the system with a monopoly node. In their framework, a node i in a financial network is said to be a monopoly node if

$$\alpha_i = \bar{\alpha}, \quad \alpha_j = 0, \quad \forall j = 1, \ldots, i-1, i+1, \ldots, n.$$

To identify the worst-case asset distribution, Khabazian and Peng propose to solve the following optimization problem:

$$\begin{aligned} \min_{\alpha} \max_{x} \quad & p^T x \\ \text{s.t.} \quad & (P - L^T)x \leq \alpha, \\ & x \leq e, \\ & e^T \alpha = \bar{\alpha}, \quad \alpha \geq 0. \end{aligned} \tag{11}$$

They show that the optimal solution to the above problem can be attained at some network with a monopoly node (see proposition 4.2 in Khabazian and Peng [45]).

For the scenario of multiple shocks, Khabazian and Peng develop a probability analysis to assess the vulnerability of a well-balanced financial network under the following assumption.

Assumption 2. *All the shocks follow the same independent normal distribution with a zero mean and variance σ^2 (i.e., $s_i \sim \mathcal{N}(0, \sigma^2)$).*

They show the following.

Theorem 1. *Given a well-balanced financial network with a fixed total asset, if Assumption 2 holds, then the following conclusions hold.*

(i) *The network with a monopoly node has the highest probability of insolvency and is the most vulnerable one. Moreover, it holds that*

$$Pr(\exists i : x_i^* < 1) \geq 1 - (0.5)^{n-1}.$$

(ii) *The system is most stable when the assets are evenly distributed (i.e., $\alpha_i = \frac{\bar{\alpha}}{n}$, $\forall i = 1, \ldots, n$).*

Khabazian and Peng [45] also derive a lower bound for the probability that some bank will be bankrupted in the system under Assumption 2 (see theorem 4.2 in Khabazian and Peng [45]). Motivated by the results in Theorem 1, Khabazian and Peng further study the effect of bankruptcy in the most vulnerable network: a financial network dominated by some monopoly node (i) in it. They consider a special scenario where the monopoly node is liability-free (i.e., $l_{ij} = 0, \forall j = 1, \ldots, n, j \neq i$) and show that there is an extremely high risk in such a network. Their results are summarized in the following theorem.

Theorem 2. *Given a fully connected and well-balanced financial network with a monopoly node, we have the following.*

(i) *If the monopoly node is bankrupted, then all other nodes in the system will be bankrupted as well.*

(ii) *Suppose that the network has a tridiagonal structure.*[2] *If a nonmonopoly node is bankrupted, then all the nodes following it will be bankrupted as well.*

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4. Resilience of Financial Networks

In this section, we examine the impact of network topology on the stability of the financial system. As pointed out in a recent survey by Elsinger et al. [33], the full liability matrix L is usually not exposed, and only partial information such as the total liability and the total claim of a bank (corresponding to the summations of all the elements in a row and a column of L) is available. To estimate the systemic risk in the system, most works in the literature first compute the liability matrix by solving some entropy optimization problem based on the KL divergence, and then they analyze the contagion risk based on the estimated liability matrix. As observed in Elsinger et al. [33], this has led to a significant underestimation of the risk in the financial system. In this section, we will review the results in three recent works from different researchers (Acemoglu et al. [2], Capponi et al. [21], and Khabazian and Peng [46]) to address this issue.

4.1. Systemic Resilience, Network Structure, and Size of the Shock

Along the line of Acemoglu et al. [1], which is discussed in Section 3.3, Acemoglu et al. [2] develop a theoretical framework to study the network's role in shock propagation and amplification. They consider a similar network model as in Acemoglu et al. [1] appended with liquidation cost. In their framework, they introduce social surplus to measure the stability and resiliency of the financial network, which is a function of the number of negative shocks, the size of the shocks, and the network structure. Based on their definition, stability and resiliency capture the expected and worst-case performances of the financial network.

First they consider a well-balanced network. They show that when a single node receives the shock, an increase in interbank liabilities decreases the stability of the system. They compare the stability and resilience of two financial networks based on the relationship between the two liability matrices in the system by using the concept of majorization. On the basis of their definition (see definition 4 in Acemoglu et al. [2]), given two subsets of banks S^1 and S^2, the financial network $\tilde{L}$ is an (S^1, S^2, M)-majorized version of the well-balanced financial network L if the following relation holds:

$$\tilde{l}_{ij} = \begin{cases} \sum_{k \notin S^1} m_{ik} l_{kj}, & \forall\, i \notin S^1,\ j \in S^1; \\ l_{ij}, & \text{if } i, j \in S^2, \end{cases}$$

where M is a doubly stochastic matrix of the appropriate size.[3] From the definition, one can see that the liabilities between the banks in S^2 remain unchanged, whereas the interbank liabilities between the banks in S^1 and its complement become more evenly distributed. They consider the same assumption to characterize the small shock and large shock as mentioned in Section 3.1. We list some key results in the following theorem.

Theorem 3. *Given a well-balanced financial network, under Assumption 1, we have the following results.*

(i) *Under the small shock regime, if the interbank liabilities among default nodes and the rest of the nodes in the system become more evenly distributed, the number of default nodes will not increase.*

(ii) *Under the large shock regime, if the interbank liabilities among default nodes in the system become more evenly distributed, the number of default nodes will not decrease.*

Using the concept of majorization, they also assess the contagion risk with multiple shocks in the system. They use the concept of harmonic distance (see definition 6 in Acemoglu et al. [2]) to study the resiliency of the banks. They show that when the banks are closer to insolvent banks in terms of their harmonic distance, they are more vulnerable to default (see proposition 8

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in Acemoglu et al. [2]). Such a result highlights the fact that the bank that has a minimum harmonic distance to other banks in the system is the most "systemically important" bank.

4.2. Liability Concentration and Systemic Losses

Capponi et al. [21] utilize the concept of majorization to study the stability of the financial system in terms of liability concentration. They consider the G-Y model and define the loss generated by node i as follows:

$$\delta_i = p_i - p_i x_i^{GY},$$

where x^{GY} is the optimal solution of the G-Y model (6). From this, they consider the systemic loss, which is the total loss generated in the system to measure the stability of the system. They introduce the notion of balancing and unbalancing financial networks in terms of their equity value as follows.

Definition 3. *Suppose that all the nodes can pay their full liabilities, then we have the following:*
(i) *A financial network is balancing if each node i with smaller liability is associated with larger equity; that is,*

$$\alpha_i + \left(L^T e\right)_i - p_i.$$

(ii) *A financial network is unbalancing if each node i with smaller liability is associated with smaller equity, which is computed by reducing each node's i liability by a level (ξ), that is,*

$$\alpha_i + \left(\underline{L}^T e\right)_i - \underline{p}_i,$$

where $\underline{p}_i = p_i - \xi$, *and* $\underline{l}_{ij} = \frac{\underline{p}_i}{p_i} l_{ij}$.

They show that there is a dominance relation between the two clearing payment ratios and the systemic losses in two financial systems corresponding to the liability concentration. To compare two network topologies with different liability configurations, the notion of majorization for vectors is generalized to majorization for matrices (see definition 6 in Capponi et al. [21]).

Definition 4. *Given two liability matrices L^1 and L^2, if there exists a doubly stochastic matrix M such that $L^1 = ML^2$, then matrix L^1 is majorized by matrix L^2.*

Definition 4 implies that in L^1, the liabilities are more evenly distributed across the nodes in the network compared with L^2, where the liabilities are more concentrated. From this, they obtain the following result.

Theorem 4. *Suppose that L^1 is majorized by matrix L^2, and the financial network is unbalancing. Then, we have that the loss in the system corresponding to L^1 is less than the loss in the system corresponding to L^2; that is,*

$$\sum_i p_i(1 - x^{GY}(L^1)_i) < \sum_i p_i(1 - x^{GY}(L^2)_i),$$

where $x^{GY}(L^1)$ and $x^{GY}(L^2)$ denote the optimal solution of (6) when the liability matrix is L^1 and L^2, respectively.

They also show that the opposite result holds in balancing systems (see theorems 1 and 2 in Capponi et al. [21]). Note that, using a generic doubly stochastic matrix M, we have $(L^1)^T e \neq (L^2)^T e$ if all the nodes in the second system have different total liabilities (i.e., $(L^2)^T e \neq te$) for some scalar t, which indicates that the total liabilities and total claims of the corresponding financial system are not the same.

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4.3. The Worst-Case and Best-Case Network Structure

Recently, Khabazian and Peng [46] have developed a new approach to assess the systemic risk in a financial network based on the relaxed E-N model (8) under the assumption that only incomplete partial information about interbank liabilities is available. The main motivation of their work is to identify the worst-case and best-case network structures in terms of the overall payment in the system. To measure the stability of a financial system, they adopt a similar measurement as in Definition 2 as follows.

Definition 5. *Consider two financial systems with the same asset vector* (α) *with liability matrices* L^1 *and* L^2 *satisfying* $e^TL^1 = e^TL^2$ *and* $L^1e = L^2e$. *The first system is said to be less stable than the second one if*

$$p^Tx^*(L^1) < p^Tx^*(L^2),$$

where $x^*(L^1)$ *and* $x^*(L^2)$ *denote optimal solutions of* (3) *when* $L = L^1$ *and* $L = L^2$, *respectively.*

They propose to solve the following worst-case linear optimization model:

$$\begin{aligned} \min_{\Delta L \in \mathcal{U}_L} \max_{x} \quad & p^Tx \\ \text{s.t.} \quad & (P - L^T - \Delta L^T)x \le \alpha; \\ & x \le e, \end{aligned} \tag{12}$$

where L is known, and ΔL is the uncertain part of the liability matrix in the uncertainty set defined as follows:

$$\mathcal{U}_L = \{\Delta L : \Delta Le = \Delta L^Te = 0, \Delta l_{ii} = 0, \forall i = 1, \ldots, n, -l_{ij} \le \Delta l_{ij}, \forall i, j\}.$$

We remark that the first constraint in the uncertainty set follows from the assumption that the total claim and the total liability for each node are known and must remain invariant. The second one is because $l^+_{ii} = l_{ii} = 0$ ($L^+ = L + \Delta L$), and the last constraint follows from the assumption that $l^+_{ij} \ge 0$.

To solve problem (12), Khabazian and Peng [46] first apply the duality theorem for the linear optimization problem to the subproblem in (12) and rewrite it as follows:

$$\begin{aligned} \min_{\lambda, \Delta L} \quad & (\alpha^T - e^T(P - L))\lambda + e^Tp \\ \text{s.t.} \quad & (P - L - \Delta L)\lambda \le p; \\ & \lambda \ge 0, \quad \Delta L \in \mathcal{U}_L. \end{aligned} \tag{13}$$

Then, they develop an integrated approach for problem (13) that combines three different schemes to update L and λ iteratively. We next describe briefly these update schemes.

Alternative Convex Search. Alternative convex search (ACS) is a popular approach for generic bilinear optimization problems. Khazabian and Peng [46] develop a new ACS method for problem (13) that updates L and λ alternatively to improve the objective function value. They also discuss how to update the liability matrix $L^+ = L + \Delta L$ to keep the dominance relationship between the two payment vectors before and after one update of the liability matrix L.

Successive Linear Approximation. Successive linear approximation (SLA) has been widely used in nonlinear optimization. Let $\mathcal{I}_1$ be the index set of default nodes, and let $\mathcal{I}_2$ be the index set of solvent nodes. Khabazian and Peng [46] observe that if all the nodes in $\mathcal{I}_2$ are strictly solvent and ΔL is sufficiently small, then the optimal solution to problem (13) can be represented as

$$\lambda_{\mathcal{I}_1} = (P - L - \Delta L)^{-1}_{\mathcal{I}_1} p_{\mathcal{I}_1}.$$

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On the basis of such an observation, Khazabian and Peng derive a linear approximation to the objective function in (13) and propose to find a search direction ΔL by solving the following linear optimization problem:

$$\min_{\Delta L} \; (\alpha^T - e^T(P-L))_{\mathcal{I}_1}(P-L)^{-1}_{\mathcal{I}_1}\Delta L_{\mathcal{I}_1}(P-L)^{-1}_{\mathcal{I}_1}p_{\mathcal{I}_1} \tag{14}$$

$$\begin{aligned} \text{s.t.} \;\; & \Delta L \in \mathcal{U}_L; \\ & \|\Delta L\|_1 \leq 2\min\{l_{ij} > 0, \forall i,j \in \mathcal{I}\}. \end{aligned} \tag{15}$$

Then they use line search to find a step size (β) and update $L^+ = L + \beta\Delta L$.

New Algorithm for Special Case. Noting that neither the ACS or SLA approach works if there is only one default node, Khabazian and Peng [46] then propose a new algorithm to update the liability matrix or verify that the current solution is optimal.

Khabazian and Peng [46] establish the convergence of the integrated algorithm and study various properties at the obtained solution. They further identify some network structures corresponding to the obtained solution such as the triangular structure mentioned earlier in this survey. They also develop a similar approach to identify the best-case network structure in which the overall payment in the system is maximal. The identified best-case network structure can be used to improve the resilience of the financial system, as it does not change the total liabilities and the total claims of all the financial institutions in the network. As observed in Khabazian and Peng [46], the subnetworks induced by the default nodes in the identified worst-case network and the best-case network share some common structure. We note that a similar phenomenon is exhibited in Acemoglu et al. [2], where the authors show that the complete network is the most stable one for small shock and the least stable network for large shock.

5. Optimization Models and Techniques for Risk Mitigation

As pointed out in the introduction, new programs and financial regulations have been implemented by various governments and agencies to stabilize the financial market after the financial crises of 2008–2009. Accordingly, various risk mitigation policies and strategies have been proposed by numerous researchers. In this section, we review several optimization models for risk mitigation and discuss the resolution techniques for these models.

5.1. Extensions of the E-N Model for Bailout Policies

Pokutta et al. [52] extend the basic E-N model to identify strategies under various bailout policies. They first assume that there is no limit on the bailout budget, and they propose three different policies: the full bailout model (FBM), the minimum capital bailout model (MCB), and maximum stability bailout model (MSB) (where only select important financial institutions receive bailout), all based on some minor modification of the basic E-N model. For example, the FBM model minimizes the total bailout cost necessary to prevent any defaults:

$$\begin{aligned} \min_{b,x} \;\; & \sum_i b_i \\ \text{s.t.} \;\; & (P - L^T)x \leq \alpha + b; \\ & x = e; \\ & b \geq 0. \end{aligned} \tag{16}$$

Similar to the E-N model, the optimal solution to the FBM model exists and is unique. Similar results are obtained for the MCB and MSB models (see theorems 2.1 and 2.9 in Pokutta et al. [52]). Pokutta et al. [52] also propose a mixed integer linear optimization model to maximize the stability of the underlying financial network with a restricted bailout budget and determine whether a bank should receive a bailout. Dong et al. [27] assume that a bankrupted bank may not pay any of its liabilities, and they propose the following binary optimization model:

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$$\begin{aligned} \max_{b,x} \quad & \sum_i p_i x_i \\ \text{s.t.} \quad & (P - L^T)x \le \alpha + b; \\ & x \in \{0,1\}^n; \\ & \sum_{i=1}^n b_i = \bar{b}, \quad b \ge 0, \end{aligned} \tag{17}$$

where $\bar{b}$ denotes the bailout budget. Dong et al. [27] discuss how to solve the above mixed integer programming model. They first show that model (17) is NP-hard and develop an effective polynomial procedure to solve a static variant of model (17) when b is fixed. From this, they also propose a Lagrangian-type method for (17), where they construct a new penalty function by adding a penalized term associated with the budget constraint to the objective function in (17). They show that the penalized problem without the budget constraint can be solved in polynomial time and that the value of the budget constraint function at the optimal solution of the penalized problem is monotone in terms of the penalty parameter. Then they propose a simple bisection search algorithm to locate a suitable penalty parameter and show that, under mild conditions, the exact solution to problem (17) can be recovered exactly from the solution of the penalized problem.

5.2. Rescue Consortium

Rogers and Veraart [53] discuss how to form a rescue consortium to help the failing banks in the system. In their framework, the consortium consists of only solvent banks and is formed by merging multiple solvent banks. They also assume that if bank i is involved in merging, a merging cost κ_i will occur. They define a bank merger as follows.

Definition 6. *Let $\mathcal{I} = \{1, \ldots, n\}$ be the set of all the banks, and let $\mathcal{B} \subset \mathcal{I}$ denote the set of merged banks in the system. The merger of all the banks in $\mathcal{B}$ is a new financial system with a new liability matrix $\hat{L}$ and the asset vector $\hat{\alpha}$, which is indexed by $\hat{\mathcal{I}} := \{0\} \cup \mathcal{B}^c$:*

$$\hat{\alpha}_0 = \sum_{i \in \mathcal{B}} \alpha_i - \kappa_i, \quad \hat{\alpha}_i = \alpha_i, \forall i \in \mathcal{B}^c.$$

The new liability matrix $\hat{L}$ is obtained as follows: the liability between the merger and a nonmerging bank in the network is simply the summation of the liabilities between the nonmerging bank and all the banks in the consortium, whereas the liabilities between the merged banks are cancelled.

Rogers and Veraart [53] next define the rescue incentive and rescue ability considering the equity value of the nodes before and after the clearing, the merger cost, and the bailout cost in the system. For this, let E_i^* denote the equity of node i after the clearing; that is,

$$E_i^* = \left(L^T x^{RV}\right)_i + \hat{\alpha}_i - p_i,$$

where x^{RV} is the optimal solution of the R-V model. Here, the bailout cost $\sum_{i=1}^n b_i$ can be obtained as follows:

$$\sum_{i=1}^n b_i = \sum_{i=1}^n \max\{0, -\hat{E}_i\},$$

where $\hat{E}_i = (L^T e)_i + \hat{\alpha}_i - p_i$. Based on this, a rescue consortium is defined as follows.

Definition 7. *Suppose that the set of insolvent nodes $\hat{\mathcal{I}}_1 = \{i : (L^T e)_i + \hat{\alpha}_i - p_i < 0\}$ is nonempty. A* rescue consortium *is a set $\mathcal{A} \subseteq \mathcal{I} \backslash \hat{\mathcal{I}}_1$ such that the following two conditions hold.*

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(i) *Rescue incentive*:

$$\sum_{i\in\mathcal{A}}\left[(\hat{E}_i)^+ - \left(E_i^*\right)^+\right] > \sum_{j=1}^{n} b_j + \sum_{k\in\mathcal{A}\cup\hat{\mathcal{I}}_1} \kappa_k.$$

(ii) *Rescue ability*:

$$\sum_{i\in\mathcal{A}}(\hat{E}_i)^+ > \sum_{j=1}^{n} b_j + \sum_{k\in\mathcal{A}\cup\hat{\mathcal{I}}_1} \kappa_k.$$

As mentioned earlier, in their model, Rogers and Veraart [53] consider the liquidation cost for both the outside asset and interbank asset. They also show that when such failure costs are not considered, there is no incentive for solvent banks to rescue the insolvent banks in the system.

5.3. Bail-ins and Bailouts

In a recent paper, Bernard et al. [14] propose three intervention policies that a social planner may use to stabilize the system, including bailout, bail-in, and subsidized bail-in. They consider the network contagion framework proposed by Eisenberg and Noe [28], which is appended with the bankruptcy cost as in Rogers and Veraart [53] and Battison et al. [11]. In their framework, banks can partially liquidate their assets to cover their liabilities, and there is a cost to early liquidation of interbank claims. Let us consider that α_i^l is the amount of asset of node i that is liquidated to meet its liabilities. Thus, we have

$$\alpha_i^l = \min\Big(\frac{1}{\gamma}\big(p_i - c_i - \sum_j l_{ij}\big)^+, \alpha_i\Big),$$

where γ is the recovery rate when the illiquid asset is liquidated, and c_i is the cash holding of bank i. Based on this, the clearing payment vector can be obtained as follows:

$$\begin{aligned}
\max_x \quad & p^T x \qquad\qquad (18)\\
\text{s.t.} \quad & (P - L^T)x \le \gamma\alpha^l + c - \big((1-\beta)[L^T x]_i\big)\mathbb{I}_{x_i<1};\\
& \alpha^l \le \alpha;\\
& \alpha^l \le \frac{1}{\gamma}\big(P - c - L^T e\big)^+;\\
& 0 \le x \le e,
\end{aligned}$$

where $0<\gamma, \beta<1$, and $\mathbb{I}_{x_i<1}$ is an indicator variable taking value 1 if $x_i<1$ and value 0 otherwise. For the case that $x_i^* = 0$, the depositors suffer a loss of

$$\delta_i = \big(c_i + \gamma\alpha^l + \beta\big[L^T x^*\big]_i\big)^-.$$

From this, they define the welfare losses w as the weighted sum of the losses because of default costs as follows:

$$w = (1-\gamma)\sum_{i=1}^{n}\alpha_i^l + (1-\beta)\sum_{i\in\mathcal{I}_1}\big[L^T x^*\big]_i\Big) + \theta\sum_{i\in\mathcal{I}_1}\delta_i,$$

where the weight θ captures the importance that a social planner assigns to the depositor's losses, and the set $\mathcal{I}_1 = \{i : x_i^* < 1\}$ shows the set of insolvent nodes. From this, they analyze

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the performance of three intervention policies—(i) bailout, (ii) bail-in, and (iii) subsidized bail-in—in terms of welfare losses in the system.

Bernard et al. [14] define *bailout* as the amount of money that a social planner makes up to cover the shortfall of the fundamentally defaulted banks. On the basis of their assumption, when bank i liquidates all its illiquid assets (α) and it still cannot fulfill its liabilities, it will default, and the amount of bailout (denoted by b^o) will cover its shortfall, which can be obtained as

$$b_i^o = p_i - c_i - \sum_j l_{ji} - \gamma\alpha_i.$$

In Bernard et al. [14], *bail-in* is defined as the amount of debt of fundamentally defaulting banks that is purchased by banks to ensure the stability of the system. Let b_k^i denote the amount of debt of bank k that is purchased by bank i. Based on their assumption, b^i can be obtained as follows:

$$\sum_i b_k^i = p_k - c_k - \sum_j l_{jk} - \gamma\alpha_k.$$

The third intervention policy is *subsidized bail-in*, which corresponds to a mix of bailout and bail-in policies (b^o, b^i). In other words, the social planner provides liquidity assistance to incentivize the formation of a bail-in consortium.

Bernard et al. [14] consider a sequential game consisting of three stages to study the economic incentives behind the determination of the above-mentioned intervention policies. In the first stage, the social planner proposes a subsidized bail-in (b^o, b^i). In the second stage, each bank decides whether to accept the social planner's proposal. If the proposal is accepted by all banks, the game will end; otherwise, it moves to the third stage, where the social planner is faced with three choices: (i) purchase the debt that a bank that rejects the proposal was supposed to buy, (ii) purchase the entire debt (public bailout), or (iii) abandon the rescue, which is called the social planner's threat of no intervention. From this, they characterize the optimal proposal of the social planner and its equilibrium welfare losses.

Proposition 5. *Let $i_1, \ldots, i_n$ be a nonincreasing order of the banks in terms of their ability to increase the social welfare through (b^o, b^i). Let w_P and w_N denote the welfare losses when we have public bailout and no intervention, respectively. Then we have the following:*

(i) *if $w_P < w_N$, then the unique equilibrium outcome is a public bailout by the social planner; or*

(ii) *if $w_N \leq w_P$, then the unique equilibrium outcome is a subsidized bail-in.*

They analyze the impact of the shock size, the size of recovery rate, and the level of interbank connectivity on the performance of the intervention policies (see propositions 4.6, 4.7, and 4.8 in Bernard et al. [14]).

It should be pointed out that in the works by Pokutta et al. [52] and Rogers and Veraart [53] incentive compatibility is ignored, the government is always assumed to have full commitment powers, and banks are assumed to be unresponsive to any policy change. However, the question of banks' response to resolution policy actions by the government is considered in Bernard et al. [14].

Kallio and Khabazian [44] propose a different strategy to mitigate the risk in the network by formulating several coalitions of banks such that all the banks in the same coalition collaborate with each other and pay the same fraction of their liabilities. Their approach can serve as a decision support mechanism for such negotiations. For mathematical modeling of solvency analysis, the authors rely on the seminal paper by Eisenberg and Noe [28] appended with the default costs similar to those proposed by Glasserman and Young [40]. In their study, they consider two incentives to initiate the cooperation: the threat tax by the government and the

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threat of credit and deadweight losses in case cooperation fails. They also estimate the lower bound for the threat tax rate under which the cooperative solution exists. Furthermore, given such initial motivation exists, fair division principles of cooperative game theory (the *nucleolus*, calling for the characteristic function of the game) are employed to create an incentive for banks to stay with and participate in the grand rescue coalition. Such fair division results in allocations of subsidies paid and received by each bank, and it therefore determines how bailout costs are shared among banks. For demonstration of the cooperative bailout, the data of 22 largest European banks used by the European Banking Association in 2016 European Union-wide stress testing are employed.

6. Conclusions

In this article, we review various optimization models and techniques for systemic risk assessment and control in financial networks. Our discussion concentrates on the E-N model and its variants. However, it should be pointed out that there are several other important risk models in the literature. For a more comprehensive introduction to systemic risk, we refer to Fouque and Langsam [35], Glasserman and Young [41], and the references therein.

On the other hand, although significant progress has been made in the study of systemic risk, there still exist several open issues. For example, when a bank has to liquidate some of its assets to pay its liabilities, there is usually a loss caused in the liquidation process. Note that in such a case, the bank itself may be solvent after the liquidation. However, it still incurs a liquidation loss. Moreover, based on works in asset liquidation (see Chen et al. [24] and the references therein), the liquidation loss is usually nonlinear in terms of the amount of liquidated assets. This is very different from the literature in systemic risk study (Glasserman and Young [40] and Rogers and Veraart [53]), where linear functions are adopted to measure the loss, which implies that the market impact on the liquidation has been neglected. More study is needed to address such an issue. Another issue is that most risk mitigation strategies in the literature depend on bailouts or subsidies from the federal government. It will be interesting to investigate whether we can improve the stability and resilience of the financial network without or with minimal governmental intervention.

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Endnotes

[1] *Wikipedia, s.v.* "Basel III," last modified May 29, 2018, 10:06, http://en.wikipedia.org/wiki/Basel_III.

[2] A financial network is said to have a tridiagonal structure if it satisfies the following relations: $l_{ij} > 0$, $\forall (i,j) \in \{(i,j) : |i-j| \leq 1.\ \forall i,j = 1,\ldots,n\}$.

[3] A square matrix is said to be doubly stochastic if it is element-wise nonnegative and each of its rows and columns add up to 1.

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