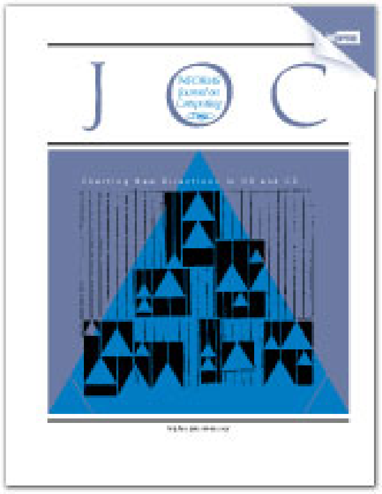

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Guest Editors' Introduction

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Guest Editors

This special issue of the *ORSA Journal on Computing* is dedicated to the subject of *computational geometry*—the algorithmic study of geometric problems. In addition to the wealth of applications in computer graphics, robotics, and solid modeling, the rapidly growing field of computational geometry has addressed many problems of interest in operations research. Several new algorithmic and analytical techniques have been developed for a variety of optimization problems. Conversely, many well established methods of operations research have contributed to the progress in geometric algorithms. This special issue is designed to stimulate the interplay between computational geometry and operations research and to enhance the application of techniques from computational geometry to problems arising in operations research.

Computational geometry is the design and analysis of efficient algorithms for geometrical problems. For example, a typical (now classic) problem in the field is to compute the convex hull of a given finite set of n points. The objective is usually to obtain the best possible algorithm, together with its analysis, where the measure of goodness is the worst-case asymptotic dependence on n. For n points in the plane, for instance, one can compute the convex hull in time $O(n \log n)$, and this is best possible, according to some standard models of computation.

Computational geometry evolved into a cohesive discipline in the mid to late 1970s, prompted, to a large extent, by the publication of the Ph.D. thesis of M. Shamos in 1978. During its first decade, the field grew quite rapidly. Its methods were applied to a variety of problems in diverse disciplines, and many new tools and techniques were developed. While it grew, the field naturally consolidated too, as it was discovered that many of the fundamental structures, such as arrangements, convex hulls, and Voronoi diagrams, are closely interrelated. The last several years have been a period of significant maturing. The field has increased its mathematical depth and its applicability to other disciplines.

The papers in this issue address several topics within computational geometry and its applications, including optimization, facility location, machine vision, and manufacturing.

The first paper, *Dynamic Three-Dimensional Linear Programming*, by David Eppstein, addresses a problem of fundamental relevance to the field of operations research. It is known that a linear program in *fixed dimension d* can be solved in time *linear* in the number of constraints. Eppstein studies the linear programming problem in *three* dimensions, and obtains efficient methods to maintain a representation of the feasible region under the insertion and deletion of constraints, so that optimization queries can be answered very quickly at any time, for any given objective function. Several related applications are given, including an efficient solution to the 2-center problem from facility location theory.

The second paper, *Recognizing Voronoi Diagrams with Linear Programming*, by David Hartvigsen, shows how one can test in polynomial time whether or not a given tesselation of d-space is a Voronoi diagram for some set of sites. An interesting feature of this paper is that the author applies classical linear programming methodology to solve a computational geometry problem.

The third paper, *Matching Points into Pairwise-Disjoint Noise Regions: Combinatorial Bounds and Algorithms*, by Esther M. Arkin, Klara Kedem, Joseph S. B. Mitchell, Josef Sprinzak, and Michael Werman, studies a problem in two-dimensional pattern matching: Given a set of n points and another set of n regions, assumed pairwise disjoint, can the set of points be transformed (e.g., translated) so that there is exactly one point in each region? The authors give bounds on the worst-case number of possible distinct matches, and they give efficient algorithms to compute all possible matches.

The fourth paper, *Fast Algorithms for Geometric Traveling Salesman Problems*, by Jon Louis Bentley, studies several TSP heuristics, devising very efficient methods for implementing them for geometric instances of the problem in which one desires tours in multidimensional point sets, under a variety of distance metrics. Bentley shows how techniques of computational geometry, such as "K-d trees" (K-dimensional binary trees), can be applied to speed up known heuristics, such as the nearest neighbor rule, in geometric versions of the problem. His results have successfully solved enormous, million-city TSP instances to within a few percent of optimality.

The fifth paper, *The Voronoi Partition of a Network and its Implications in Location Theory*, by S. Louis Hakimi, Martine Labbé, and Edward Schmeichel, deals with a problem in facility location. A set of points in a network defines a Voronoi partitioning of the network, in exactly the same way as a set of points in the plane induces a planar Voronoi diagram. The authors give an algorithm for computing a Voronoi partition, and discuss the problem of locating facilities in a network in order to minimize the size of the largest Voronoi region.

The sixth paper, *Efficient Algorithms for the Capacitated 1-Median Problem*, by Hossam ElGindy and J. Mark Keil, addresses another problem in facility location: Given a set of weighted demand destinations, where should one locate a single *capacitated* facility so that it can serve as many of the demand destinations as possible, subject to the constraint that the sum of the weighted distances from the facility to the demand destinations does not exceed the capacity. The authors devise efficient solutions for the cases of demand points in the plane, under rectilinear distances, and for demand points in a tree network.

The seventh paper, *1-Segment Center Problems*, by Hiroshi Imai, D. T. Lee, and Chung-Do Yang, considers yet another problem in facility location, that of finding a 1-center. The standard 1-center problem is as follows: For n point sites in the plane, find the location of a point facility that minimizes the maximum of the distances from the point to the other sites. This problem has been studied thoroughly, and a linear-time method is known. In their paper, the authors pursue the generalization of the 1-center problem to that of finding a *line segment* of fixed length that minimizes the maximum of the distances to the point sites. Using the prune-and-search method, they give an $O(n)$ algorithm for the case when the segment has a fixed orientation. Without the assumption of a fixed orientation, they give an $O(n^4 \log n)$ algorithm.

The eighth paper, *On the Optimal Bisection of a Polygon*, by Elias Koutsoupias, Christos H. Papadimitriou, and Martha Sideri, analyzes the complexity of the following "pie-cutting" problem: Cut a given polygonal "pie" into two (not necessarily connected) sets of equal area by using a minimum-length "knife action," i.e., such that the sum of the perimeters of the two regions is minimized. The authors show that the problem is NP-hard and that, in fact, for any $\epsilon > 0$ it is NP-hard to obtain an area bisection with a cut whose length is within a factor $(1 + \epsilon)$ of the optimum. On the positive side, they give an algorithm that is polynomial in n and $1/\epsilon$ and cuts an n-gon into two regions of *nearly equal* area (i.e., their ratio differs from 1 by at most ϵ), using a perimeter no more than that of the optimal area-bisecting cutting.

The ninth paper, *Monotone Pieces of Chains*, by Vijay Chandru, V. T. Rajan, and R. Swaminathan, considers the problem of decomposing the boundary of a simple polygon into a minimum number of monotone chains. (A polygonal chain is monotone in a direction d if any line orthogonal to d intersects the chain in a point or an interval). One can think of this problem as another kind of optimal cutting problem, in which one wants to cut out a given shape, using the fewest possible resets, for a machine that is constrained always to cut in the $+y$ direction, while the material moves back and forth in x.

The tenth paper, *"Lion and Man": Upper and Lower Bounds*, by Laurent Alonso, Arthur S. Goldstein, and Edward M. Reingold, examines a continuous pursuit-evasion problem in which a lion pursues a man within a circular region of radius r, in an attempt to get within distance c of the man, at which point the man is eaten. The authors prove upper and lower bounds (roughly on the order of r/s) on the length of time it takes the lion to get to a point where he can eat the man, assuming that each contestant is limited to the same maximum speed, s.

We are grateful to the many people that assisted in making this issue a reality. Numerous referees were involved in the process of reviewing submissions, with many of them under tight time constraints. The issue was conceived and took shape under the supervision of Harvey J. Greenberg, Founding Editor of the *ORSA Journal on Computing*. We are especially grateful for the diligent efforts of the Editorial Assistant, Tanya McMurtry. We thank the new Editor, Bruce L. Golden, who has been instrumental in getting this issue to press. And, finally, we thank all of those authors who submitted and contributed to the success of the issue.

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