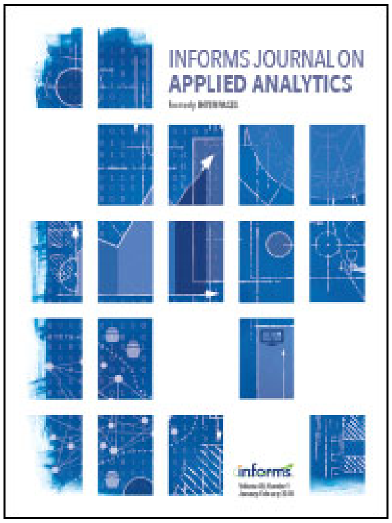

Interfaces

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Note: Further Gymnastics

Samuel Eilon, Ailsa Land,

Note: Further Gymnastics

The problem of choosing a lineup to maximize the potential team score in intercollegiate women's gymnastics is tackled by Ellis and Corn [1984], using a bivalent integer programming formulation. The authors argue that as it is not uncommon for each of the competing teams to score approximately 180 points, and as the winning margin can be less than half a point, the selection of athletes becomes a crucial decision, so that the proposed model could be an invaluable aid to the coach, who has to make the final selection.

An example is cited by the authors of a team at Utah State, given in Table 1, where the expected score in each of four events is given for 10 athletes. The coach needs to select six participants for each event, the top five scores in each event being counted in the team score, which is what the coach strives to maximize. In addition, four athletes must be "all-rounders" and participate in all four events, whereas the others can participate in up to three events. These constraints naturally led the authors to formulate a model with two classes of zero-one variables, to distinguish between the all-rounders and the others. The solution to the Utah problem was easily obtained with the commercially available MPSX package, and the authors note that with an LP formulation the solution also yields integer results, the optimal solution for the expected counted team score being 181.9.

Gymnast	Vault	Bars	Beam	Floor	Total
1	9.30	9.30	9.20	9.50	37.30
2	9.10	9.00	8.80	8.70	35.60
3	9.20	9.00	9.20	9.00	36.40
4	9.00	8.60	8.60	8.65	34.85
5	8.80	8.70	8.50	8.50	34.50
6	8.70	7.50	7.50	8.70	32.40
7	8.50	8.80	8.70	8.50	34.50
8	9.10	9.00	9.00	9.20	36.30
9	9.20	7.00	7.00	9.10	32.30
10	8.90	8.90	9.10	8.70	35.60

Table 1: The scores expected by the coach for each gymnast in each event — the Utah problem.

A follow-up by Land and Powell [1985] notes that the original formulation may be greatly simplified and shows that with four all-rounders and four events there is always an LP solution that satisfies the integer constraints. They also point out that, as there is a difference between total expected score (the sum of expected scores of athletes assigned to particular events) and the total expected counting score (where only the top five scores of the six participating athletes in each event are counted), it is possible for a problem to occur where a solution that maximizes the former is not identical with a solution that maximizes the latter. As an example they suggest Table 2 and show that the maximum expected score is 221.2 with a corresponding expected counting score of 185.6. The maximum expected counting score of 186.0 is obtained with a somewhat different solution, for which the total expected score is 220.4. Land and

0092-2102/86/1602/0069$01.25

RECREATION/SPORTS

Gymnast	Event 1	2	3	4	Total
1	9.50	9.50	9.50	9.50	38.00
2	9.50	9.50	9.50	9.50	38.00
3	9.20	9.20	8.60	8.60	35.60
4	8.60	8.60	9.20	9.20	35.60
5	9.00	9.00	9.00	9.00	36.00
6	9.10	9.10	8.60	8.60	35.40
7	8.60	8.60	9.10	9.10	35.40
8	9.20	9.20	8.60	8.60	35.60
9	8.60	8.60	9.20	9.20	35.60
10	8.90	8.90	8.90	8.90	35.60

Table 2: Land and Powell's example of scores of a group of 10 gymnasts.

Powell rightly comment that if a fraction of a point is important in these problems, then the distinction between the two criteria needs to be borne in mind, and they proceed to propose an integer programming formulation for maximizing the total expected counting score.

An interesting facet of both Ellis and Corn's original problem and that of Land and Powell, in Tables 1 and 2 respectively, is that there is no need to resort to a formal integer programming model at all in order to find the optimal solution. If, in each column of Table 2, the top five scores are selected, as shown in Table 3, then the maximum possible value for the expected team counted score is 186.0, and this bound is in fact achievable by adding an athlete to each event in such a way that four compete in all events. From Table 3, athletes 1 and 2 are automatically designated as all-rounders and any two selected from numbers 3, 4, 6, 7, 8 and 9 can also be so designated to meet the requirement for four all-rounders. The allocation of athletes to the remaining events to comply with the constraint for non-all-rounders can be completed in many ways without affecting the expected counting score. Applying this method to the original Utah problem in Table 1 readily reveals that athletes 1, 2, 3 and 8 should be selected as all-rounders, athlete 9 for events 1 and 4, and athlete 10 for events 2 and 3, to yield the optimal solution of 181.9. The remaining selection of athletes does not affect this score.

Gymnast	Event 1	2	3	4	Total
1	9.50	9.50	9.50	9.50	38.00
2	9.50	9.50	9.50	9.50	38.00
3	9.20	9.20			18.40
4			9.20	9.20	18.40
5					
6	9.10	9.10			18.20
7			9.10	9.10	18.20
8	9.20	9.20			18.40
9			9.20	9.20	18.40
10					
Total	46.50	46.50	46.50	46.50	186.00

Table 3: Solving the problem in Table 2.

The examples in Tables 1 and 2 illustrate how the application of simple logic can sometimes obviate the need for elaborate model formulation. If the authors wished to demonstrate an interesting technical point, or the novelty of their approach, they should have chosen examples that cannot be readily solved in the way described above.

References

Ellis, P. M. and Corn, R. W. 1984, "Using bivalent integer programming to select teams for intercollegiate women's gymnastics competition," *Interfaces*, Vol. 14, No. 3, pp. 41-46.

Land, A. and Powell, S. 1985, "More gymnastics," *Interfaces*, Vol. 15, No. 4, pp. 52-54.

Samuel Eilon
Imperial College
London SW7, UK

I agree with Professor Eilon ("Further gymnastics") that a "back of the envelope" approach to a real problem should always be tried first, and it certainly achieves optimum solutions on the examples in both the original paper and our subsequent note. However, we regarded it as a merit rather than a disadvantage of our numerical example that the reader could check by simple arithmetic that our model was giving the obviously correct solution.

For a slightly more subtle example, try these scores:

	Event				
Gymnast	1	2	3	4	Total
1	9.50	9.50	8.60	8.60	36.20
2	8.60	9.50	9.50	8.60	36.20
3	9.50	8.60	9.50	8.60	36.20
4	8.60	8.60	9.50	9.50	36.20
5	9.40	9.10	9.00	9.40	36.90
6	9.20	9.00	9.40	9.10	36.70
7	9.40	9.10	9.40	9.00	36.90
8	9.30	9.20	9.20	9.30	37.00
9	9.20	9.30	9.20	9.30	37.00
10	9.20	9.30	9.20	9.30	37.00

I feel Professor Eilon is missing our point that although we formally presented an ILP model, in fact there is always an optimal LP solution which is integer. We can surely by now regard running an LP as only one stage more complicated than using the back of an envelope, even if ILP can still sometimes lead us into very deep water!

Ailsa Land
The London School of Economics and Political Science
Houghton Street
London WC2 2AE
England