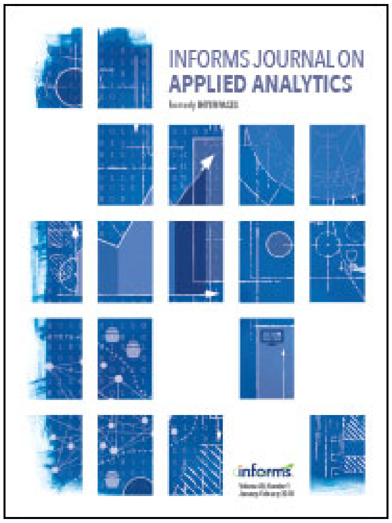

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A Tutorial on Building Policy Models as Mixed-Complementarity Problems

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After 50 years of development, building and solving mixed-complementarity problems (MCPs) have become commonplace for policy models that analyze markets. These models are used to develop policies that reshape markets, or introduce markets that replace other organizational forms. In this tutorial, we give some background on building economic equilibrium models, starting with the use of linear programming, and show how MCPs can be used to answer policy questions that require manipulation of the solutions to linear programs of economic sectors. We illustrate the use of MCPs using examples from King Abdullah Petroleum Studies and Research Center projects, including a model of domestic energy markets in Saudi Arabia, which is in the process of changing some of its pricing policies and market regulations.

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Mixed-complementarity problems (MCPs) have been part of the operations research toolkit since the 1960s. Early developments include Lemke and Howson (1964) on bimatrix games; Lemke (1965) on bimatrix equilibria, which include the solution conditions for quadratic programs; and Cottle and Dantzig (1968) on complementary pivot theory. Because of the development of good software (Dirkse and Ferris 1995), MCPs, which remained largely theoretical for the first several decades of their development, are now prominent in the building of policy models for microeconomic sectors, with energy modeling capturing a large share of the modeling activity. These models focus on the outcomes of the actions of economic agents, where economic agents can either be individuals (as producers and consumers) or organizations that deliver goods or services.

This paper addresses members of the community who do not necessarily focus on microeconomic modeling in the models they build; however, they want to know more about current methods for representing sectors using economic equilibria. We describe in nonmathematical terms the advantages of using MCPs over other representations of markets, and discuss how to determine when they are preferable to a more traditional (and possibly simpler) optimization approach (e.g., linear programming). We provide short technical appendices to give some mathematical structure to the material. Our focus is on partial equilibrium models—models of single or multiple economic sectors, but not of the whole economy. Examples of these models include the TIMES model housed in the International Energy Agency (Loulou and Labriet 2008, Loulou 2008), the U.S. Energy Information Administration's National Energy Modeling System (NEMS) (Energy Information Administration 2013), and the King Abdullah Petroleum Studies and Research Center (KAPSARC) energy model (Matar et al. 2013, 2015a), a multisector equilibrium model of the Saudi energy economy (KEM-SA). The experience of building a model that captures government interventions in energy markets within Saudi Arabia provides insights into the circumstances under which adopting a

MCP formulation is preferable to using optimization methods.

The most fundamental decision in building a microeconomic model is whether to model the outcomes of decisions or the actual decisions the agents make. The standard answer is that agents' actual decisions cannot be modeled under the following conditions.

- The agents are heterogeneous in resource endowments and cost structures as producers of goods or services; or
- as consumers, the agents place a multiplicity of values on products and services.

In such situations, econometric methods can be used to model the outcomes of decisions.

This involves gathering data on the outcomes and statistically estimating production functions and demand curves. For example, using econometric techniques for estimating end-use demand is usually the best strategy because consumers are heterogeneous, exist in large numbers, have a range of incomes, and value goods and services differently. The consumer response to gasoline price changes illustrates this. The outcome of gasoline consumption is a consequence of millions of decisions made by individual households, each of which has its special circumstances. Similarly, an estimate of a production function on the manufacturing sector is based on inputs of capital, labor, energy, and materials. An aggregation over all of manufacturing represents a multitude of processes and circumstances, and the estimated production function captures shifts in the aggregate use of labor; for example, as the costs of capital, materials, or energy increase.

By contrast, when a sector in a market has relatively few agents and the production and decision processes are understood, the market outcomes can be estimated by modeling the decisions, technologies, and cost structures of the agents. Some models use a mixed strategy, such as estimating demands for energy services econometrically and then modeling the choice of technologies to meet those services based on cost. Thus, the methodologies can be mixed together.

When modeling decisions, one enumerates the choices that agents can make and develops data on the inputs, outputs, costs, and revenues associated with each choice. One then forecasts the outcomes that result from the agents making profit-maximizing or cost-minimizing choices. As an example, to build a model of the oil refining industry in a region, one would assemble data on the capital and operating costs, capacities, inputs, and outputs of all of the refinery units in the region and build a linear programming model of the region's capacity as if all the refineries operated as one refinery. Models of this type go by several names. They were originally called process models because production processes are explicitly modeled. Other names include bottom-up models because a sector is modeled starting at the bottom and is built up; and technicoeconomic models because of their explicit representation of technologies. We use process model because Alan Manne, the inventor of this approach (Manne 1958), used the term.

This paper introduces the MCP approach with a simple example of a competitive market. It presents a framework for determining when to build a MCP model instead of using a standard optimization approach. We show the features of an optimization model of a competitive economy, explain the limitations of the optimization approach, and demonstrate how MCP models address the weaknesses of optimization models. We mention some of the many MCP models that have been built and point the reader to additional MCP models. We conclude with a description of how to build a MCP of an energy system, drawing on KAPSARC's work in modeling Saudi Arabia's energy economy. We then mention two other models developed at KAPSARC.

Complementarity: A Natural Way to Describe Market Equilibria

The Classical Law of Supply and Demand

Consider an idealized economy consisting of a single good for which demand and supply curves are given. Demand and supply are functions of prices, but economists usually depict them in inverted form: the demand curve shows the price at which consumers are willing to buy a given quantity of the good; the supply curve shows the price at which producers are willing to sell a given quantity of the same good.

The demand curve slopes downward because consumers consume less when the price is higher. The demand curve is defined by the marginal value of consumption (i.e., the satisfaction expressed in monetary terms) that the consumers get from consuming an additional unit of the good. For a given market price, consumers increase their consumption until their marginal value of consumption equals the price. The supply curve slopes upward because the producers produce more if the price is higher. Under the assumption of perfect competition, the supply curve is defined by the marginal cost: producers increase production until the cost of producing the last unit of a good equals the market price. The two curves intersect at a point called the equilibrium point, a price P_e and quantity Q_e such that the quantities demanded and offered match at the given price. Figure 1 illustrates this classical law of supply and demand.

This classical representation assumes that the supply curve is smooth and that the good is produced at the equilibrium. The reality, however, might be more complex; at the equilibrium, the good may not be produced or the price may include an economic margin (or rent) if production capacities have limits. A full representation of the market then leads to the complementarity conditions, as we illustrate next.

MCP Formulation of a Market Equilibrium

To explain the complementarity conditions, we modify the supply curve in Figure 1 to simplify the discussion. We replace the upward-sloping supply curve with a single supply step, where all the producers use the same technology and have the same operating cost C, and the total available production capacity is M. We could add more steps to the supply curve to reflect the different costs of multiple producers; however, that would complicate the discussion without adding any insight. In Figure 2, we show the three possible equilibria that can occur. In Figure 2(a), the demand curve lies above the production cost for any quantity at or below capacity M. All the available supply is used and the market clears at a price equal to the marginal value of capacity for consumers. This price is greater than the operating cost $P_e > C$; that is, P_e includes an economic margin (also called scarcity rent): $R = P_e - C$. In Figure 2(b), at cost C capacity is greater than demand, the market price is C, and producers have no margin. Finally, in Figure 2(c), the cost C of operating the capacity is always above the demand curve, and no production or margin is present.

As Figure 2 shows, either the unused capacity (case 2a) or the margin (case 2b) is 0. This is the complementarity condition, and it represents the equilibrium version of the complementary slackness property in optimization. Table 1 summarizes the complementarity conditions defining these equilibrium outcomes; therefore, it describes a MCP. The term mixed refers to the condition in which the quantity produced lies between a lower bound (zero) and an upper bound (the available capacity M). MCPs also include equality constraints that are complemented with variables that

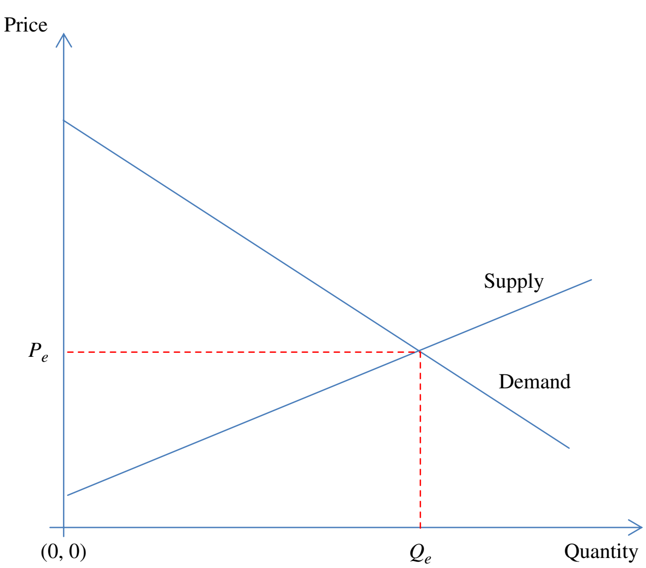

Figure 1: (Color online) In the standard picture of a competitive market equilibrium, the supply and demand curves intersect at the market equilibrium.

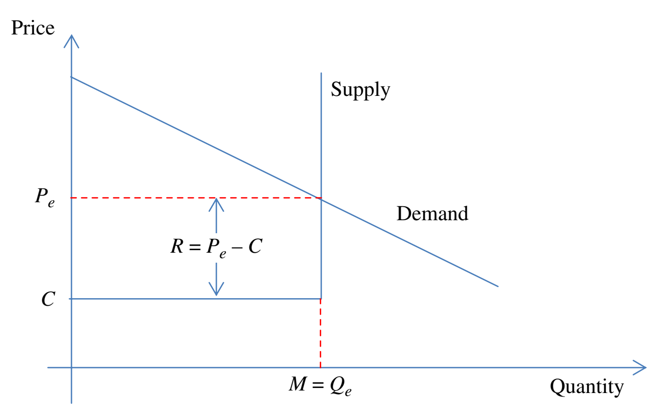

Figure 2(a): (Color online) When the demand curve is above the production cost, producers receive an economic rent of *R*.

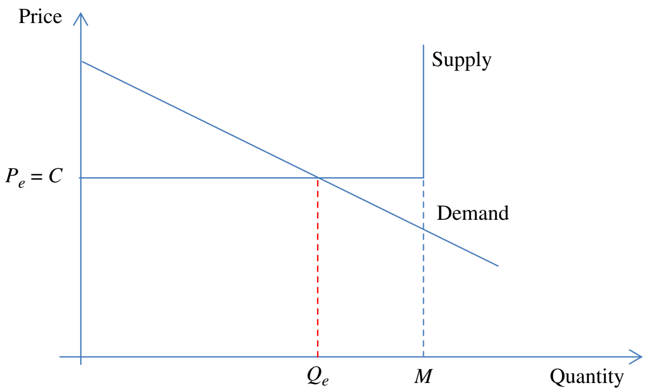

Figure 2(b): (Color online) When the demand curve intersects the production cost, producers do not earn economic rents.

are unconstrained in sign to maintain a square matrix. Being feasible in the equality guarantees the complementarity condition is met automatically. Appendix A provides a formal representation of this MCP. If an algorithm finds the solution to the complementarity conditions in a model, then it has also found the economic equilibrium.

The limited capacity depicted in Figure 2 sometimes refers to a resource; in such a case, the margin made by the owner of that resource (beyond a market rate of return on investment) is commonly referred to as a rent. Rents permeate energy economies, as the following three examples illustrate.

- Coal mines with very thick seams have resource rents because they are easier and cheaper to mine than mines with thin seams, and the market price is determined by the production cost with thinner seams.
- Existing electrical generation equipment has capacity rents when demand grows faster than anticipated because of time lags in building new capacity or because favorable sites are no longer available.
- Producers who are closer to their customers have location rents. For example, the mine-mouth prices of Appalachian coals, even after adjusting for heat content, are higher than the prices for coal produced in Wyoming because these coals are nearer to customers in the east.

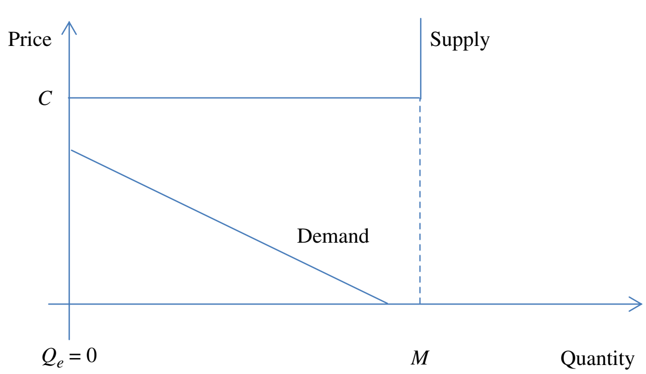

Figure 2(c): (Color online) With a production cost above the demand curve, producers produce nothing at the equilibrium.

Efficiency of the Market Equilibria

Assume that a benevolent planner determines the quantity sold in a market by considering both supply and demand information. Consider the situation that Figure 1 describes. By being benevolent, this planner chooses the quantity Q, which maximizes the area between the supply and demand functions. This is the difference between the consumers' accumulated willingness to pay and the producers' cumulative cost, and is called the economic surplus. This area consists of the revenues above total costs to the producer plus the total value of the product above the price for consumers. Maximizing the economic surplus leads to choosing a quantity that is equal to the equilibrium quantity Q_e. In Figure 1, the triangle made by the supply and demand curves and the price axis covers this area between the demand and the supply curves for Q between 0 and Q_e. These concepts also apply to Figure 2 with the equilibrium values Q_e outlined in Table 1.

Thus, optimizing the economic surplus is equivalent to finding competitive market equilibria. Furthermore, an economy is efficient when it provides the greatest economic surplus, which is at the intersection of the

Case	Quantity produced	Margin beyond cost
Production cost below demand curve	$Q_e = M$	$R = P_e - C \geq 0$
Production cost intersects demand curve	$0 \leq Q_e \leq M$	$R = P_e - C = 0$
Production cost above demand curve	$Q_e = 0$	$R = 0 \geq P_e - C$

Table 1: This table summarizes the three cases.
Note. **Different relationships between supply and demand curves lead to different equilibrium outcomes.**

supply and demand curves. This relationship between the surplus and the equilibrium is the formal expression of Adam Smith's (1776) "invisible hand" where the self-interests of the individual agents lead to the maximum social surplus. This maximum aggregate social welfare is realized by achieving competitive market equilibria.

Having illustrated why adopting a complementarity framework represents an intuitive way to describe economic equilibria, we now explain how using linear programming to find an efficient solution is the same as determining market equilibria. This is essential to understanding the economic foundations and limitations of using optimization in building energy models.

Linear Programming: A Standard Approach for Equilibrium Models

Linear programing, which was developed as a planning tool, has found its greatest use in optimizing the production of goods and services in large firms. The benefits come from centrally coordinating organization-wide production processes. From an historical perspective, Kantorovitch (1958) invented linear programing in the Soviet Union to optimize the plans of the central planning agency and Dantzig (1951, 2002) independently invented linear programming to optimize manpower planning in the U.S. military. One of the first commercial uses of linear programming was to model an oil refinery (Manne 1958). Because the savings generated were so large, oil companies funded the improvements in linear programming software for the next few decades. During this early period, the power industry began using it to plan investments (Massé and Gibrat 1957), and capacity-expansion models became a standard application of linear programming in utilities. The original uses of linear programming led planners to frame their thinking around centrally coordinating complicated organizations.

Linear programming became an important tool for understanding markets after Samuelson (1952) and Enke (1951) recognized that the algorithm developed to solve linear programs—the simplex algorithm (Dantzig 1951)—was a method for finding equilibrium in a competitive market, a market in which prices equal the cost of the marginal unit of production and no firms exercise market power. A linear program can be viewed in several ways. The most common is to use standard algebra to think of the model as a set of equations that express how the solution to the equations must meet conditions that represent the coordination of activities in an organization. Another way to frame the model is in terms of agents and activities in which each activity (i.e., each variable in the model) is treated as a Leontief production function that converts inputs into outputs in fixed ratios. We use this latter view, which Dantzig (1963) emphasized.

Think of an activity as an agent that can decide its production level, depending on whether it is profitable to produce. Each step in the simplex algorithm can be considered a bidding process in which each agent bids its reduced cost to enter the solution; the agent with the highest reduced cost wins and enters the solution, reaching a level at which it drives some other agent out of the solution.

We now show how the activity view leads to abstracting economic sectors as process models. Let the supply functions be of the type described in Figure 2 with a constant marginal cost, possibly up to some maximal capacity M, and multiple steps in a step function with increasing costs on the steps. Substitution effects (i.e., variations in the proportions of inputs and outputs) are captured by having multiple activities with different proportions, which can vary the mix of inputs and outputs in the solution, as in the process-selection model. Demand functions in a linear program are either fixed quantities that do not vary with price, unlimited demand at fixed prices, or a step function with multiple steps, which have lower prices with higher quantities. Step functions can approximate arbitrarily closely the functions in Figure 1, making linear programming a general tool for modeling more complicated supply and demand functions.

Constraints express limits on resources, demand requirements, balances of inputs and outputs, product or input characteristics, or policy or technical restrictions. The levels of the activities in the optimal solution depend on the choice of the objective function. The objective functions most commonly used in linear programming-based models of economies maximize economic surplus. If demand is fixed, minimizing cost produces the equilibrium, and if market prices are fixed, maximizing profits leads to the equilibrium.

Each activity of a linear program uses a combination of goods and labor and (or) capital services to produce one or more outputs. If a production activity uses one or more inputs from sectors not included in the partial equilibrium model, those inputs have assumed prices that serve to determine the expenditures on those inputs per unit of production. Those expenditures become the cost coefficient of the activity. Each good and (or) service used and produced by the activity is typically constrained and these constraints have dual variables, also known as shadow prices (i.e., the additional value that would be captured if the constraints were relaxed by one unit), in the terminology of linear programming. If the activity's outputs are sold to consumers (and not to other activities), the dual variable on the constraint that links the activity's supply with demand equals the marginal value of the output for consumers and is the market-clearing price. If the outputs are sold to other activities, then the same dual variable represents the marginal value of the intermediate good from the perspective of the purchasing activities. The marginal value is also the economic gain that would result from adding an additional unit of output in the model at no cost.

At the optimal solution, the marginal value of an output is its implicit price in the model. Shadow prices have an interpretation of rent when they apply to a constraint that represents a limited resource. A rent associated with a limitation on the availability of a resource represents the value of an additional unit of the resource for the model. The rent accrues to the owner of that resource; it can be nonzero only if this constraint is binding. In addition, the activity can have direct costs in the objective function. The marginal cost of operating an activity is the sum of the direct costs and the rents and (or) prices paid for inputs. This defines a nonsmooth supply curve (e.g., the curve in Figure 2(a)).

For an activity that operates in the optimal solution, the costs of operating the activity (i.e., direct costs plus rents and (or) prices paid) equal the marginal value(s) of the activity's output(s). That is, marginal cost equals marginal value, which equals price. This is equivalent to the classical equilibrium mentioned above for a perfectly competitive economy. If the marginal cost of an activity exceeds the marginal value of the activity's output, the activity does not operate, as Figure 2(c) shows.

Dantzig's (1951, 1963) original method for solving linear programs, the simplex algorithm, uses a pricing mechanism to determine whether an activity should operate. Here, the five steps in the simplified version of the algorithm give a story for how a market reaches an equilibrium.

1. Start with a set of trial prices with one price per constraint.

2. For each activity, determine if it is profitable to produce by taking the trial prices of the constraints and the production coefficients, as in the reduced-cost calculation.

3. Increase the production of the most profitable activity (i.e., the activity with the highest reduced cost) until some other activity is driven out of business.

4. Operating this activity at its maximum economic level changes the trial prices. The effect of the changes in the trial prices is that the marginal profit of the introduced activity is reduced to zero.

5. At the optimal solution, the activities not in the solution cannot enter profitably, and using the dual variables of the constraints as prices leads to zero profits for all positive activities, as would be the case in a competitive economy.

The properties of the linear programming solution that we have outlined are the same properties of a competitive equilibrium. For each activity, the following applies.

- Supply equals demand at a set of market prices (this results from the surplus, profit, or cost objective function).
- Marginal cost equals marginal value, which equals price.
- No one can profitably enter the market using the available technologies, because the profit from adding one unit of production is negative or zero for any activity not already in production.

The assumption that prices equal marginal costs or marginal values or both is central to energy models based on linear programming (e.g., TIMES). An energy system is composed of multiple interacting firms (or sectors), each of which maximizes its profits or minimizes its costs. Assuming perfect competition or that all the firms are run efficiently by a single planner is similar to assuming that all prices of commodities traded among firms are marginal costs or marginal values.

When sectors are combined into a single model, as in TIMES, and one sector supplies another, these sectors are connected through material balance constraints or transportation activities, where the supply delivered equals the amount consumed. The purchase cost in the consuming sector comes from the dual variable of the material balance constraint. This dual (the implicit price as we explain earlier) is also known as the transfer price.

When a multisector model is built, it is often easier to build models of individual sectors and then combine them. Noting that the objective function of a sector's model includes the cost of all inputs from outside the sector, the objective function of the combined model adds together the objective functions of the sectors after removing costs or prices are that determined internally within the multisector model, but are external to the individual sectors. For example, a model of only the electricity sector includes fuel costs. Once fuel models are combined with the electricity model, the costs of fuels are dropped from the portion of the objective function devoted to the electricity model, and the fuel costs affect electricity-generation activities through the dual variables, or transfer prices, on the material balances that connect the fuel models with the electricity model. The resulting combined objective function can be optimized subject to the set of constraints faced by the sectors. That is, the key assumption made implicitly in standard process models is that transfer prices of commodities between firms or sectors are marginal costs or marginal values or both. This allows the modelers to optimize a unique objective function that represents the whole energy system.

To summarize, activities represent individual agents and the surplus objective leads to the market clearing. The different objectives of the firms or sectors can be combined into a single objective because each activity in the model can be considered a price-taking agent that takes a positive value only if it is profitable, given the shadow prices on the constraints, independent of which firm has that activity in its production process. The solution is a competitive equilibrium because marginal cost or marginal value (or both) equals price.

Limitations of Optimization in Representing Economic Sectors

Optimization methods cannot always be used to represent markets. Finding the equilibrium by maximizing the economic surplus requires computing the difference between the area under the demand curve(s) and the area under the supply curve(s). These areas are integrals in calculus terminology. Although the areas exist when the model represents a single product or the demand curves have the property that no change in a product price affects the demand for another product, the areas might not exist in commonly occurring situations in which demand and supply must be represented by a system of equations that does not satisfy these properties. When the area below the demand curves does not exist, demand is said to be nonintegrable.

A measure of the response of the demand for a product to prices is known as the price elasticity—the percentage change in the product's demand as a function of the percentage change in the product's price and the prices of competing products. In models of markets with multiple products, the value of the elasticity of demand for product 1, given a change in the price of product 2, is normally different from the value of the elasticity of demand for product 2, given a change in the price of product 1. As a property of integral calculus, these elasticities must be identical for the integral to exist and demand to be integrable. That is, the Jacobian matrix of partial derivatives of the demand curves must be the Hessian matrix of the function that measures the area under the demand curves. Price elasticities can also be introduced for supply curves involving several products, with similar properties needed to guarantee that supply is integrable.

The strength of optimization in representing competitive markets is also its weakness. In many markets, the price does not equal the cost of the marginal units of supply. Although optimization can also represent markets with monopolists, other markets without perfect competition need to be modeled. Examples are listed as follows.

- When a few firms dominate a sector, they can exercise market power and set marginal revenue, rather than price, equal to marginal cost as oligopolists. In this situation, each of the large firms has its own profit-maximizing objective function, and the objective functions cannot be added together into a single appropriate objective function in an optimization model (unlike a monopoly model in which the objective function of the model changes from maximizing surplus to maximizing profit for the monopolist).

- An industry can operate under particular pricing rules; examples include price indexation clauses that link liquefied natural gas and crude oil prices in long-term contracts or price-escalation clauses that link the cost of a project to price indices.
- Governments can impose regulations that alter prices. They can impose caps on the prices of some essential commodities, grant subsidies, or impose pricing rules. Examples include administering the transfer prices of fuels among industrial sectors in Saudi Arabia and charging consumers the average cost of electricity under traditional rate-of-return regulation.
- Governments can impose sales or value-added taxes. Linear or nonlinear programs can represent taxes that are a fixed amount (i.e., excise taxes); however, governments usually impose taxes that are a percentage of the price (e.g., value-added taxes). Linear or nonlinear programs cannot represent such taxes because the tax level must be known in advance when determining the coefficients in the objective function. Yet, a sales tax is a percentage of the price, the dual variable of the demand constraint.

Before the development of good algorithms to solve MCPs, a collection of algorithms was developed that involved solving linear programs, adjusting the solutions, and resolving the linear programs until the market equilibrium was found. Murphy et al. (1982) showed how to use this approach to find the equilibrium in markets in which oligopolistic producers can exercise market power, Hogan (1975) addressed the problem of asymmetric cross-price elasticities, and Greenberg and Murphy (1985) presented algorithms that deal with the tax problem and government price controls. All these algorithms require computational skills and modeling efforts that MCP tools do not require. For example, to deal with asymmetric cross-price elasticities in a linear programming model, start from a trial solution and use the following steps.

1. Construct an approximation of the demand curve that runs through the trial solution and has cross-price elasticities set to 0.
2. After finding trial equilibrium, use the new trial prices in the demand curve with the cross-elasticities to provide new trial quantities.
3. Construct a new approximate demand curve through this new price-quantity pair and repeat the cycle until successive trial solutions are within a tolerance.

In a specific example of trying to capture the effects of average-cost prices for electricity under traditional utility regulation, the difference between the marginal and average cost of electricity in a trial solution is subtracted from the transportation cost of the transportation activity that links the generation regions to the demand regions. This difference is updated at the same time the demand approximation is updated. These adjustments are in the category of Gauss-Seidel algorithms.

An important advantage of a MCP is that the model is completely separate from the solution algorithm. With optimization-based approaches, the model and algorithm must be mixed together, making model management harder.

Modeling Using the MCP Framework

In formulating an optimization program, one writes the variables and equations that represent the physical flows and the costs of each step in making and delivering products. The economic rents and prices are calculated as part of determining the levels of the activities undertaken by the agents, as we discuss above. The linear program with supply, production, and demand activities and constraints (e.g., on resources and demands) is known as the primal problem. MCPs represent the decisions of individual agents on whether to produce in the same way as primal linear programs. In addition, MCPs can contain supply and demand functions that use functional forms that cannot be included in optimization models. A MCP, therefore, provides a more general description of a competitive market than the linear or nonlinear models that maximize economic surplus. MCPs can also accommodate representations other than purely competitive markets; therefore, they enable a higher degree of generality. They include an explicit statement of price relationships and complementarity conditions that differ from the competitive-market price relationships in these markets

(Table 1), because the rents and prices appear directly as variables.

As an example of price relationships, when firms exercise market power, the pricing formulas do not set marginal cost equal to price. They set marginal cost equal to marginal revenue, taking into account that a product price is lower at a higher level of demand. That is, let P_0 be the intercept on the price axis. With an inverse demand curve of $P = P_0 - bQ$, the supplier revenue is $Q(P_0 - bQ)$ and the marginal revenue for a monopolist is $P_0 - 2bQ$, which cuts the supply curve at a lower quantity than the demand curve. Note that an equilibrium with a monopolist can be modeled as an optimization. However, in an oligopoly the marginal revenue curve for player i is $P_0 - \hat{Q}_{-i} - 2bQ_i$, where $\hat{Q}_{-i}$ is the production of all firms other than i and Q_i is the production by i. Because multiple players are optimizing separate objective functions, an oligopoly cannot be represented in an optimization model when the players' cost structures differ. In MCPs, when a sector such as electric power is subject to rate-of-return regulation, the pricing formula calculates the average cost of electricity. With value-added taxes, the delivered prices are the supply prices multiplied by one plus the tax rate.

Every optimization with continuous variables can be written as a MCP. The MCP of a linear program consists of the constraints from both the primal and dual linear programs and the complementarity conditions, which require that a dual variable can be positive only if the associated constraint is binding; a primal variable can be positive only if the corresponding dual constraint is binding. However, many MCPs cannot be written as optimization models. Appendix B provides a simple example of such a MCP.

Optimization is still preferred when the market structure and government policies meet the right conditions, because building a linear programming model takes less time than building a MCP; the prices are found in the solution to the primal linear program, which means that formulating the dual linear program and specifying the complementarity conditions are therefore unnecessary. An additional advantage is that the solution algorithms for linear programs are faster than MCP solvers and can handle larger models.

Building MCP Models

KAPSARC has developed a formal procedure for building MCPs. This is valuable because multisector equilibrium models are large, complicated models that must be assembled from smaller models. MCPs require that the pricing relationships be written in addition to the production activities; therefore, we first build linear programs, convert them into MCPs, and then introduce the features that differ from those in competitive markets. Next, we list the steps for building a model or sector from scratch.

1. To substitute for the subsequent connections to other sectors when building individual model components, estimate rough initial prices for inputs and quantities for outputs. This keeps the components feasible and bounded.

2. For each sector, build a primal linear program separate from the full model, debugging the competitive model.

3. Using the same coefficients as in the primal program, write the dual linear program.

4. Run both primal and dual programs, checking that the values of their objective functions are equal (the strong duality property of linear programs) and that the solution values for prices and quantities in the primal and dual match (beyond degeneracy issues). This helps catch the inevitable errors that occur when rewriting the primal as a dual.

5. Take the constraints from the primal and dual models and write the complementarity conditions to form the initial MCP.

6. Modify the pricing rules and add regulatory conditions.

7. Build transportation linkages or material balances to connect the sectors and run the full model.

The sectors are developed separately so that any modeling errors can be found more readily; in addition, the run times are fast, allowing quicker debugging cycles. This underlies the decomposed structure of NEMS; however, we do not face any of the convergence issues that can occur with NEMS (Murphy and Mudrageda 1998).

KAPSARC has a project underway to develop software that automatically generates the dual equations in GAMS, the modeling language that we use. This will eliminate a time-consuming step. An experimental program that is available through the GAMS Development

Corporation (GAMS 2014) produces an MCP from an optimization program after the full model has been generated from the GAMS equations. This approach does not support a decentralized model building in that the mnemonics that tell the modeler where an activity or constraint fits in the general model structure are lost, and the model cannot be altered to include regulatory conditions.

We are developing a model management system that includes a database of model components in the form of GAMS equations that are separate from the data. For example, we can take the general model statement for the electricity submodel for Saudi Arabia (which we describe in section *KEM-SA: A MCP Example*) and reuse it for other countries, modifying the model structure as necessary, adding the new model structure to our collections of models, and producing new data for the other countries. As part of this model management system, we are developing databases that contain the input data to the model and an archiving system to allow the retrieval of models and runs used in our studies.

KEM-SA: A MCP Example

The assumption that prices equal marginal costs or marginal values does not hold in the case of Saudi Arabia. Transfer prices of fuels among energy (or energy-intensive) sectors are administered to lessen the losses for electric and water utilities and to favor the development of specific industrial activities, such as petrochemical production. These sectors are dominated by large organizations, suggesting price regulation as an appropriate approach to control market power.

Because of the price controls, until December 2015, the power and water (desalination) utilities bought crude oil from Saudi Aramco at a price lower than US$5 per barrel, whereas the marginal value of crude oil consumed domestically is the export price (US$107.8 per barrel on average for Arab light crude in 2011, the model's solution year). The actual marginal value of a barrel of oil may be lower than the export price because Saudi Arabia is a major oil-exporting country with significant spare production capacity; however, it is certainly above US$5 per barrel. Table 2 gives the prices of fuels for power and water production and feedstocks for the local petrochemical industry

	Product	Price
Natural gas	Methane	0.75 USD/mmBtu
	Ethane	0.75 USD/mmBtu
Crude oil	Arab light	4.24 USD/barrel
	Arab heavy	2.67 USD/barrel
Petroleum products	Diesel	3.60 USD/barrel
	HFO 360cst	2.08 USD/barrel

Table 2: The local prices for the domestic power, water, and petrochemicals sectors in 2011 were set below world prices.
Notes. **mmBtu is one million British thermal units, HFO 360cst is a type of heavy fuel oil.**
Source. **National Commercial Bank (2012).**

in 2011. In addition, the administered prices can differ by consuming sector. For example, cement companies bought heavy crude at US$6 per barrel and power utilities bought it at US$2.67 per barrel.

Because of the administered prices, the current Saudi energy system cannot be represented as a straightforward linear program with a unique objective function; however, it can be represented as a regulated competitive equilibrium. The fuels demanded by the consuming sectors are supplied at the administered prices by the upstream oil-and-gas sector or the refining sector.

KAPSARC developed a MCP formulation that simulates an economic equilibrium over the Saudi energy system with intersector administered prices. KEM-SA is solved as a single MCP, with a workflow designed to facilitate decentralized component development, as we describe above. KEM-SA is the first MCP formulation of a portion of the Saudi Arabian economy. The MCP structure allows the model to make decisions on domestic sales of oil using the administered prices, while selling oil internationally at world prices. Natural gas presents interesting challenges because the low administered price leads to a shortfall in supply relative to potential demand. This results in rationing, requiring an allocation mechanism in the model.

Our formulation allows us to analyze policy options for redesigning the pricing structure in the energy sector in a way that the Saudi population would perceive as fair, while inducing conservation and optimal technology choices. It provides a framework to test potential policies that lead to more efficient equilibria, while retaining either current or alternative administered prices. Matar et al. (2013) provide a detailed

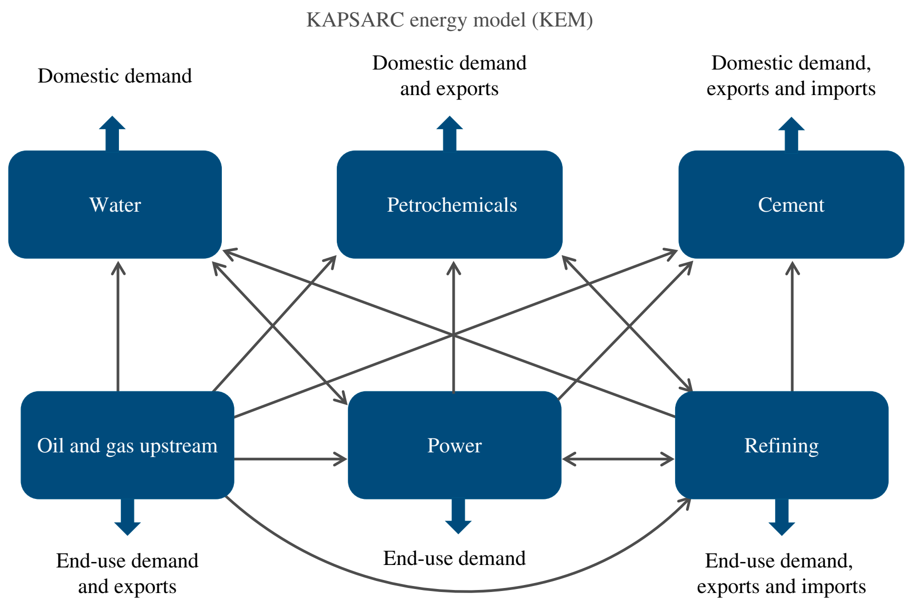

Figure 3: (Color online) The KAPSARC energy model represents six sectors and the major flows among them.
Note. **We define end-use demand as all energy consumption not captured in the submodels.**

description of the model and its calibration. Next, we summarize the key features.

KEM-SA is a partial-equilibrium model that integrates into one model six sectors, each captured in a submodel. The full model is obtained by concatenating all primal and dual linear programs of the sectors, which were initially described separately, and then altering the dual to represent prices that are not marginal costs, following the steps in the *Building MCP Models* section. The model can be run as a single-period, long-term static equilibrium model or as a multiple-period model with a user-specified planning horizon. The equilibrium is found using GAMS and the PATH solver. Figure 3 shows the sectors and the flows among them. Each submodel can also be run separately, taking exogenous prices for inputs used in the sector and exogenous quantities for outputs demanded by other sectors. The power and water sectors, which meet only local demand in Saudi Arabia, are modeled as cost minimizers. The refining, cement, and petrochemical sectors, which also export and import, are modeled as profit maximizers that face fixed local demand and capped export amounts. Oil and gas production is taken exogenously from Saudi Aramco's production data. The oil and gas upstream sector is, therefore, represented with available supplies at administered prices and a pipeline structure that minimizes the cost of meeting regional demands for gas and crude oil.

Using a long-term static version of the model calibrated to 2011 data, we analyzed various scenarios in comparison to a current-policy baseline. One of these scenarios, termed the price-deregulation scenario, measures the impacts of pricing fuels at their marginal values. The model set the marginal values of crude oil and oil products consumed domestically at the 2011 export prices. This particular scenario estimates the upper limit for economic gain resulting from changing transfer prices and could have been formulated as a standard linear program. As Matar et al. (2015a) show, current fuel consumption could be reduced substantially, without altering consumer prices, while preventing the annualized net cash flows of the utility sectors from falling below zero. The policies examined could potentially have generated economic benefits exceeding US$23 billion in 2011, or about four percent of Saudi Arabia's GDP. Although the benefits are lower with the 2015 oil prices, they remain positive. This economic benefit comes mainly from intersector fuel-pricing policies that provide incentives for shifting the mix in technologies used in generating electricity and producing water toward more efficient technologies.

Although senior officials in countries that manage prices understand the benefits of deregulation, they are concerned about the transition and the direct and indirect effects of inflation on the income statements of organizations and prices charged to consumers. Therefore, alternatives that garner most of the benefits of deregulation with less impact on prices are of interest to policymakers either as targets or as part of a transition to deregulated markets. We examined a scenario in which we introduced investment credits on energy-efficient technologies and simultaneously raised fuel prices. We minimized the total cost to the government, while meeting the equilibrium conditions defined by the MCP with the added constraint that the national utility stays within a budget constraint and does not incur losses greater than under the current pricing structure. This policy comes very close to reproducing the energy savings of deregulation without bankrupting the affected firms. Because policies that radically change pricing can shock the system, we also examine policies that ease the transition in Matar et al. (2015b).

A model with an objective function and constraints that are equilibrium conditions is known as a mathematical program subject to equilibrium constraints (MPEC). The conceptual difference between an MPEC and a standard optimization model is that the organization that runs the latter can command the organization to implement the solution (e.g., a refinery owner operates all units in the refinery based the model solution). An MPEC represents a longer chain of events and actions to produce the desired result. The optimizing organization maximizes its objective function through setting incentives that lead to behaviors by market participants that shape the equilibrium solution.

KAPSARC used the results we describe to initiate a dialogue with senior decision makers in Saudi Arabia.

Other KAPSARC Energy Models

We mention two other KAPSARC models that look at different energy markets and illustrate the interactions of policy questions and modeling choices. KAPSARC is developing an energy model of China to better understand the country's energy markets and their effects on world commodity markets. It is developing the model by addressing a sequence of questions about the Chinese energy economy. The first version of the model is a linear program of coal supply and transportation with fixed demands. The result is a study that examines the effects of expanding rail capacity in China on world coal markets (Rioux et al. 2015). The study shows that a major contributor to the collapse in the prices of internationally traded coal is that the rail expansions give domestic coal producers in the interior of the country greater access to the coastal regions, replacing imports that have to find different markets.

The first version of the model is a linear program, because the model contains fixed, historical demands for coal and does not include pricing issues that require a MCP. To examine alternative environmental policies for coal and electricity in general, the model has been expanded to incorporate electricity generation. The current modeling activity includes adding in the pricing rules of the national government for the various types of electricity generators. This requires a MCP because the pricing rules do not take into account capacity utilization or the value of electricity at different times of the day. Furthermore, although all electricity has the same value in the market, the government prices it differently depending on the source. We will then be able to compare the current pricing scheme versus one that is more closely aligned with market pricing and the interaction of pricing schemes and environmental regulations.

Questions about oil markets include whether large suppliers (such as Gulf Producers) can profit from allocating oil among regions and whether large consumers in Asia can make strategic purchases to lower their total costs of oil. Alkathiri et al. (2015) examine the potential to capture location rents by large suppliers and large consumers. In this study, the central core of the model is a transportation model that has added structure to represent the qualities of different crudes and their product yields. Samuelson (1952) uses the transportation model when showing that an optimization model is an equilibrium model. In this model, minimum allocations and strategic purchases are included by imposing lower bounds on flows between regions and/or countries, providing a spatial equilibrium, given the regional allocations. The study examines three scenarios, two Stackelberg games and a Stackelberg-Nash game, where both the producer and consumer coalitions act as Nash players. In the first Stackelberg game, a major supplier or coalition of suppliers acts as a leader that maximizes revenue; that is, the major supplier, acting as a leader, maximizes revenue knowing how the followers shift their actions in response to the leader's decisions. In the second Stackelberg game, a major consumer or coalition of consumers in the Far East acts as a strategic buyer to minimize cost. In the Stackelberg-Nash game, both the supplier and consumer coalitions are Nash players, while taking into account the actions of the competitive fringe. The first two scenarios are MPECs that lead to the Stackelberg equilibrium, and the third is known as an equilibrium problem subject to equilibrium constraints (EPEC). In the first two scenarios it is profitable to act as a Stackelberg leader; however, there is no equilibrium in the third scenario, raising issues of the strategic interactions of the large players.

Energy Issues and MCP Models

Although the oil price is a natural focus in energy markets, economic equilibrium models that are policy models of energy sectors have little to say about near-term oil prices, because the financial markets absorb all currently available information, akin to markets for stocks. Equilibrium models are useful for longer-run projections, at least beyond the period covered by financial markets for energy commodities, where the demographic and economic fundamentals (e.g., population growth, resources, costs) and economic growth shape the outcome. Given the uncertainties and surprises, such as recessions, and the rapid productivity improvements in the development of shale oil, these longer-run forecasts are treated as baselines for exploring alternative futures and measuring the consequences of changes in policies rather than a prediction.

Unlike operational models, which include a direct link between the model and actual decisions, policy models inform discussions in ways that can be either direct, as with models such as NEMS, or can inform discussions in the broader energy community to influence policy outcomes, as with KEM-SA. Consequently, the models we mention in this section do not necessarily link directly to decisions. Nevertheless, the modelers are part of the social network that connects academics and the policy community.

A good starting point is the article by Hogan (2002), which looks back at the role of modeling in energy policy formulation and implementation; this modeling led to the restructuring of energy markets during the Carter administration. Greenberg and Murphy (1980) cover the specifics of modeling Carter's national energy plan. Building and solving MCPs was not feasible at that time, although the basic model formulations were established then.

Gabriel et al. (2012) extensively cover the MCP literature on energy. Ruiz et al. (2014) draw from Gabriel et al., covering MCPs in greater mathematical detail than we do in this paper, and also include an extensive bibliography of MCPs in energy models. To narrow our focus, we mention a few models that examine two policy areas, market power in natural gas and market design in electricity. In the United States, market power by gas suppliers is not an important issue and pipelines are monitored closely to restrict their ability to exercise market power. Europe faces a very different situation in that Russia is a dominant supplier (Mock 2014, Troianovski 2015). Furthermore, Norway and the Netherlands are large producers. Thus, MCP models of European gas markets focus on market power. Abada et al. (2013), Gabriel and Smeers (2006), Holz et al. (2008), and Huntington (2009) are examples of European gas models.

The number and scale of models of electricity markets is far greater than for natural gas. From an engineering perspective, this sector is more complicated than the natural gas sector, and the questions to be addressed include market design and the implications for reliability and market power. Carlson et al. (2012) discuss how one independent system operator runs a market in the United States after a change from traditional regulation. Other papers on this topic include Boucher and Smeers (2001), Hobbs (2001), Garcés et al. (2009), and Ehrenmann and Neuhoff (2009). Again, the modeling work and literature extend well beyond the few models we mention in this paper.

Conclusions

This nontechnical overview of the modeling choices and their economic implications is intended to broaden the understanding of MCPs in the operations research community. Our goal is to assist modelers in making

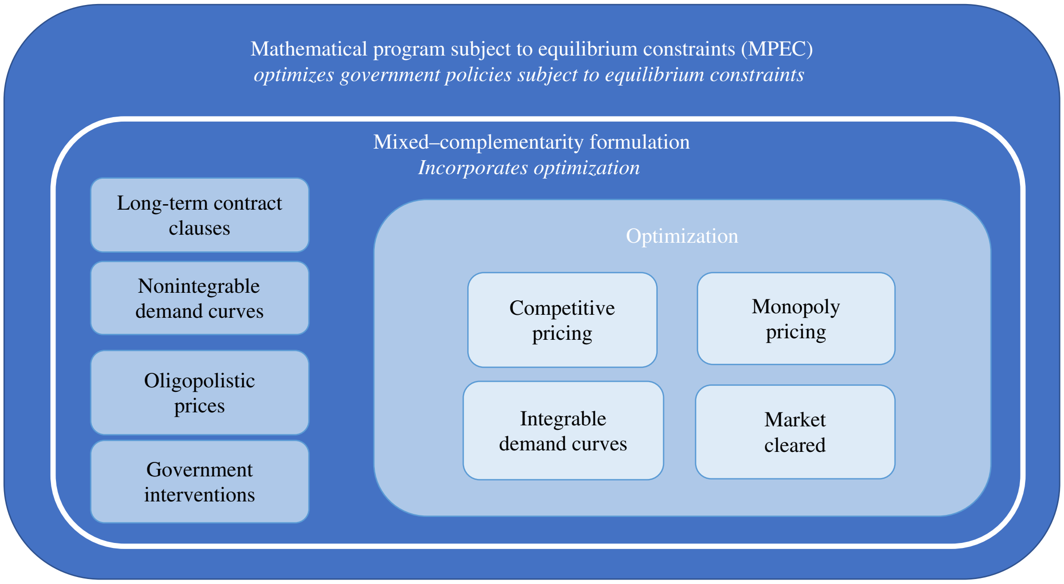

Figure 4: (Color online) The layers of modeling capabilities reflect the complexity of the markets and the policies analyzed.

modeling choices between MCPs or optimization models. Building MCPs is more complex than building optimization models; therefore, we illustrate the types of deviations from market pricing that make MCPs especially interesting and worth the extra effort when developing alternative pricing policies and assessing the market power of large players. Figure 4 illustrates the nested layers of modeling capabilities we discuss in this paper. Nagurney (1993) and Gabriel et al. (2012) provide more detailed discussions of MCP modeling and solution methods.

Appendix A. Formal Statement of the Complementarity Problem

Let $P = mv(Q)$ be the (inverted) demand function, where mv stands for marginal value. Finding the economic equilibrium described in Figure 2 is equivalent to finding the Q and R that solve one of the three complementarity conditions:

$$mv(Q) - C - R = 0 \quad \text{and} \quad Q = M \text{ and } R > 0 \qquad \text{(Figure 2(a))},$$

or

$$mv(Q) - C = 0 \quad \text{and} \quad 0 \le Q \le M \text{ and } R = 0 \qquad \text{(Figure 2(b))},$$

or

$$mv(Q) - C < 0 \quad \text{and} \quad Q = 0 \text{ and } R = 0 \text{ (Figure 2(c))}.$$

Using the $\perp$ symbol as a mathematical shorthand for expressing complementarity, the above statement is equivalent to finding the Q and R that solve the following system of inequalities:

(1) $0 \le M - Q \perp R \ge 0$,

(2) $0 \le C + R - mv(Q) \perp Q \ge 0$,

where:

(1) is equivalent to: $M - Q \ge 0$ and $R \ge 0$ and $R(M - Q) = 0$. This states that there is an economic rent if and only if the limit on production is reached.

(2) is equivalent to: $C + R - mv(Q) \ge 0$ and $Q \ge 0$ and $Q(C + R - mv(Q)) = 0$. This states that the good is not produced if the marginal cost exceeds the marginal value, and that the marginal value is equal to the marginal cost plus an economic rent if the good is produced.

Appendix B. A Simple MCP with Government Intervention

In the example of the economy with a single good, we assume that, as a social goal, the government does not want the price of the good to exceed the value P^{up}. To ensure that production always meets consumer demand, the government pays a subsidy per unit of the good sold if the marginal cost of meeting demand at the capped price P^{up} exceeds P^{up}. This subsidy S is defined as the amount of the marginal cost of production that is subsidized.

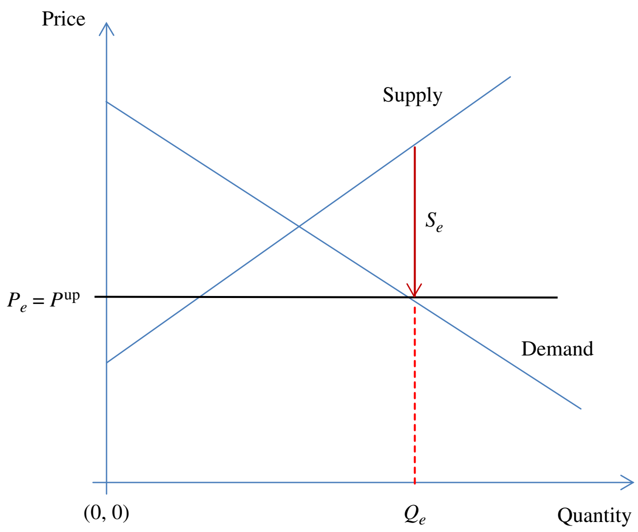

Figure A.1: (Color online) Price is subsidized by the amount $S_e > 0$.

Let $P = mc(Q)$ and $P = mv(Q)$ be the inverse supply and demand functions; mc stands for marginal cost and mv stands for marginal value. The price the consumer sees after the subsidy is $P - S$. The possible market equilibria resulting from the government intervention can be described by the following MCP:

(1) $0 \le P^{\text{up}} - mv(Q) \perp S \ge 0$,
(2) $0 \le mc(Q) - S - mv(Q) \perp Q \ge 0$,

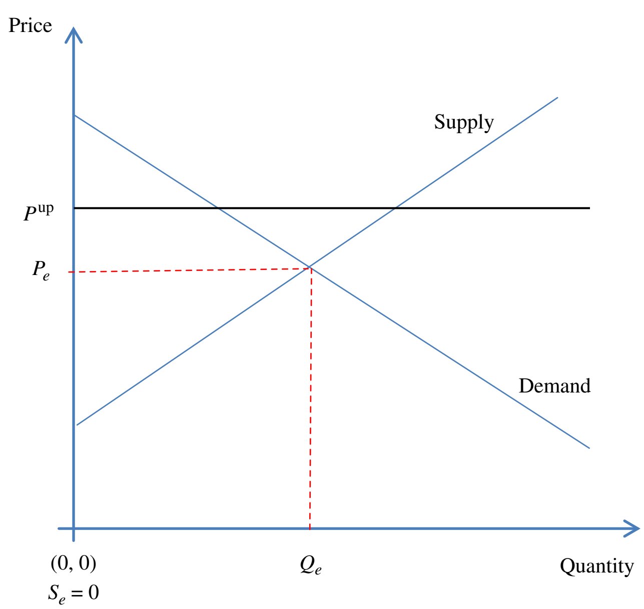

Figure A.2: (Color online) Market clears below the price P^{up}.

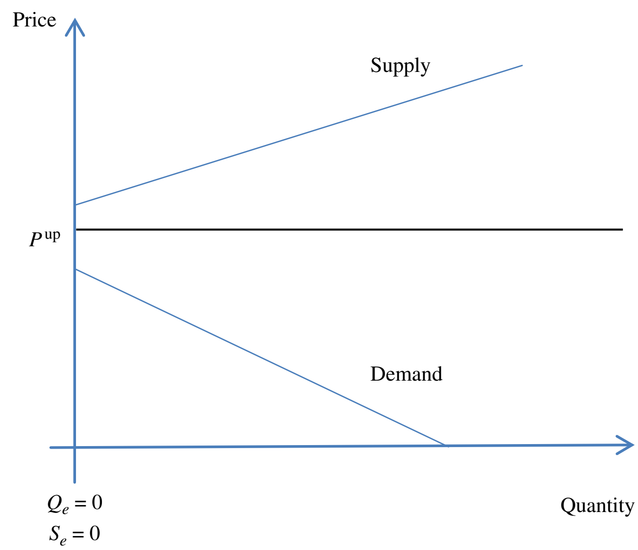

Figure A.3: (Color online) Demand is 0.

where:

(1) states that there is a subsidy if and only if the cap on price is reached,

(2) states that the good is not produced if the marginal cost less the subsidy exceeds the marginal value, and that the marginal value is equal to the marginal cost less the subsidy if the good is produced.

Figures A.1–A.3 describe three possible equilibria, where equilibrium quantity and subsidy are Q_e and S_e, respectively.

The equilibrium conditions of this MCP do not correspond to the optimality conditions of any optimization model.

Appendix C. Building a Linear Complementarity Problem

The formal statement of a linear program using matrices and vectors is as follows:

$$\max v \cdot Q,$$
$$A \cdot Q \le b,$$
$$Q \ge 0.$$

The corresponding dual linear program is as follows:

$$\min R \cdot b,$$
$$R \cdot A \ge v,$$
$$R \ge 0,$$

where R is the vector of dual variables (rents) associated with the constraints in the primal program.

Let Q^* and R^* be the solutions to the primal and dual programs. We have

$$v \cdot Q^* = R^* \cdot b,$$

where Q^* and R^* are the values of Q and R that satisfy the following MCP:

$$0 \le b - A \cdot Q \perp R \ge 0;$$

$$0 \le R \cdot A - v \perp Q \ge 0.$$

This MCP is similar to that given in Appendix A. It states that a dual variable can be positive only if the associated constraint is binding, a nonbinding constraint has a dual variable equal to zero, an activity can be operated only if its marginal cost equals its marginal value, and an activity that is not in the solution has a marginal cost greater than or equal to its marginal value. These complementarity relationships are the Karush-Kuhn-Tucker optimality conditions of the primal linear program.

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