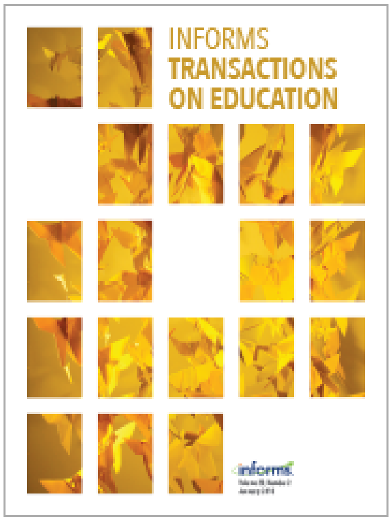

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The Effectiveness of Using a Web-Based Applet to Teach Concepts of Linear Programming: An Experiment in Active Learning

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The graphical solution method to a two-variable linear program (LP) provides valuable insights about the general nature of multivariable linear programming models. As a result, introductory operations research/management science textbooks typically present a graphical solution method to a two-variable LP as a prelude to the presentation of more complex problems. Construction of a two-dimensional feasible region combined with iso-profit (or iso-cost) lines on a blackboard or overhead projector can be tedious, at best. Even with tools such as PowerPoint, that include drawing tools, it is difficult to show students what happens in a graphical LP as constraint lines and iso-profit lines shift around on a graph. To overcome this hurdle, this paper presents a Web-based Java script applet that was used by both instructors and students to graphically illustrate/learn fundamental concepts of LP models. It then describes the results of a study that compares student performance on exams of those who did use the applet versus those who did not. Results show that the students who used the applet to learn about LP concepts performed significantly better than those who did not. Implications for using such active learning techniques and models in the classroom are discussed.

Key words: linear programming; active learning; sensitivity analysis; Web-based applet; linear program graphical solution
History: Received: June 2010; accepted: September 2010.

Introduction

The graphical method of solving linear programming problems is not a realistic applied solution approach because the method is limited to two-variable problems. However, the graphical technique is notably valuable from an instructional standpoint because the technique pictorially illustrates numerous aspects of the more complex, algebra-based, solution algorithm. Therefore, numerous introductory operations research/management science textbooks illustrate a graphical solution to a two-variable problem (Anderson et al. 2008, Render et al. 2009, Stevenson 2009, and Taylor 2010). Specifically, a graphical solution to a linear program provides a student with intuitive visual aids to facilitate their understanding of such concepts as a basic feasible solution, the feasible region, an unbounded solution, degeneracy, slack, surplus, binding constraints, nonbinding constraints, right-hand side ranges, and objective function of coefficient ranges.

Construction of a two-dimensional feasible region combined with iso-profit (or iso-cost) lines on a blackboard or overhead projector can be tedious, at best. Graphically guiding students through specific concepts (i.e., basic feasible solutions, or right-hand side ranges) borders on the impossible when using such traditional media. One possibility is to show a graphical representation on an overhead transparency and physically move lines on the graph. For this, the instructor needs a unique slide for each constraint line, where the constraint line in question is shown and manipulated manually. Within programs such as PowerPoint or Excel, there are drawing tools available, but even with these tools, it is very difficult to capture the dynamic nature of a linear program on PowerPoint or Excel slides. To overcome these hurdles, this paper presents a Web-based Java applet that can be used by both instructors to graphically illustrate fundamental concepts and by students to practice determining the effect of changes to constraint lines on the optimal solution. The applet has the advantage of allowing students to actively move constraints and the objective function line within a standard problem (by clicking and dragging) to facilitate their understanding of numerous LP concepts and to experiment with changes within a graphical representation of a linear program.

Other Web-Based Applets

Several other Web-based applets exist related to the graphical solution of a linear program. Visual Lin Prog (Vanderbei 2010) is a tool that allows students to solve their own general linear program (LP) problems. Once formulated, the tool allows the student to watch the solution process step by step, which is very useful for illustrating the simplex algorithm. However, the student cannot move constraint lines around on the graph to see the effect of changes in the constraints or objective function. To view the effect of changes, new constraints would have to be entered.

The Intermediate College Algebra tool (Exploring Linear Programming) (Green 2010), the Graphical Simplex Algorithm (2D) (Zhang 2010), LP Explorer 1.0 (Hall and Baird 2002), and The McGill LP tool (cgm) (Shepard 2010) are other existing Web-based applets that allow students to learn about linear programs by providing some type of solution, and in some cases, a graph. However each one has one or more limitations regarding student experimentation with model parameters. None allow students to manipulate the right-hand sides of the constraints in order to see the effect of changes to these values on the feasible region or the solution. None allow students to change the slopes of the objective function coefficients to find upper and lower bounds on the objective function coefficients.

Animated Linear Programming Applet (Wright 2010) is a Java applet that appears to be the most closely related to the applet described in this paper. This applet does allow the user to "click and drag" constraints across the feasible region for two-variable problems. However, the Animated Linear Programming Applet appears to present complete results for just one previously defined standard problem. The applet does not allow for a user-defined problem to be solved. When the authors attempted to input a different problem, errors occurred in the input table of the Animated Linear Programming Applet.

A Web-Based Applet as an Active Learning Technique

Active learning has been used in several different disciplines to engage students in the learning process (Gardner 2008, Umble and Umble 2004, Prince 2004, Cook and Hazelwood 2002). It is basically defined as any instructional method that asks students to complete activities and to think about the activities as they are being done (Bonwell and Eison 1991). Thus there is an active component to the learning. Proponents of the use of active learning in the classroom suggest that active learning techniques get students more involved than they would normally be simply watching an instructor solve a problem or listening to a lecture (Auster and Wylie 2006, Whiting 2006). In the case of learning about LP, it is extremely important for students to interact with the problem itself and with the graphical representation of the problem in order to fully understand how the different components of an LP work together. Students can formulate LP problems and then solve them using any number of LP computer packages, such as LINDO (LINDO Systems 2010). However these programs only show them how to structure the inputs of the model and then allows them to look at the solution printed out in computer format. So it helps them set up the inputs and see the outputs, but it does not enhance understanding of what happens in the middle. One way to help students truly understand the intricacies of an LP solution is to allow the student to be able to interact with the graphical representation of the problem.

The applet described here was designed to allow students to interact with a graphical representation of the problem. It lets the student input the parameters of an LP problem and then to manipulate the graphical representation/solution of that problem. By actually having the ability to slide constraint and objective function lines around on the graph, the student is able to see the immediate effect on the solution of having more or fewer resources available (by moving the constraint lines) or of changing the prices or profits of the products (by changing the slope of the objective function line). It is an interactive graph that engages the student by demonstrating how changes to the inputs change the output. The applet presented here is a good example of an active learning technical tool that engages students and provides them with an effective learning environment.

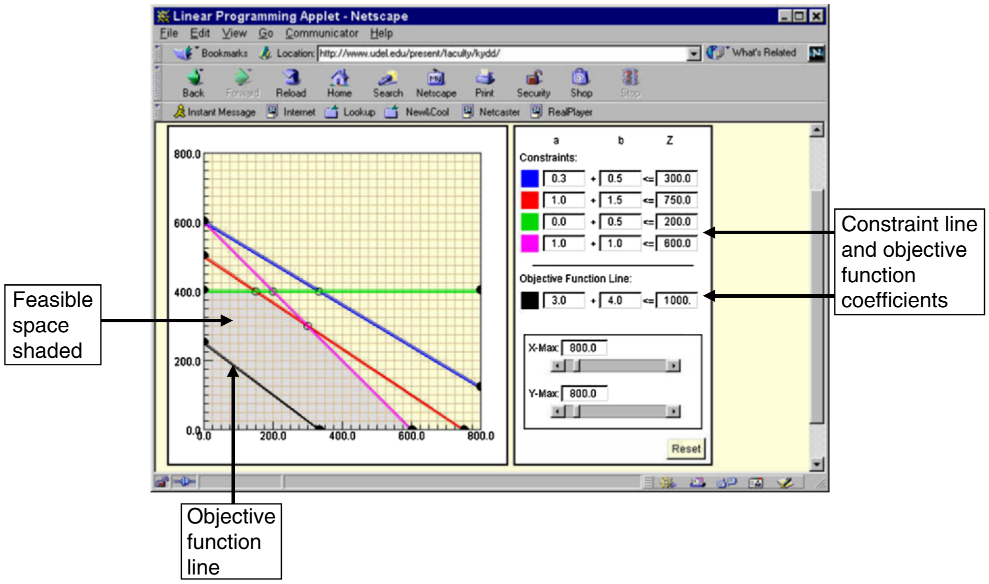
Description of the LP Java Applet

The applet presented here (Kydd 2010) was originally developed for the purpose of teaching sensitivity analysis on a graphical solution to a linear program. It is intuitive and instructive to show students what happens on a graph when finding bounds on resources so that they can have a picture in their minds of what these bounds are all about. However once the applet was completed, it was obvious that it could also be used for showing initial feasible solutions, finding the optimal solution and also doing the sensitivity analysis, as originally conceptualized. Thus, each of these categories is discussed briefly below.

Finding the Feasible Region and Optimal Solution

Students can first see the graphical representation of a linear program using this applet. The constraint line coefficients and right-hand side (RHS) values,

Figure 1 **Graphical Representation of a Linear Program**



along with the objective function coefficients can all be entered into the spaces on the right side of the applet. Then the lines are shown on the graph, and also the feasible space is shaded in gray (see Figure 1). This lets the students see immediately what their problem looks like in graphical form.

They can then click and drag the objective function line away from the origin until it touches the last feasible point, thereby showing them which intersection point is the optimal solution (for a standard maximization problem) (see Figure 2).

In this applet, a very helpful feature is that every intersection point on the graph is circled, and as the lines move, so do the circles, thus highlighting at all times the intersection of the two lines in question. This feature helps the students to see these intersection points, as well as helps the instructor to draw attention to the intersection points. As we know, it is at these intersection points where virtually all the action is in LP! So when the objective function line hits the intersection point of the red and pink lines, the students have found the optimum. The value of Z at this point is shown at the right (see Figure 3), so they know immediately what the objective function is equal to at the optimal point.

Figure 2 **Graphical Representation of a Linear Program Showing Optimal Solution Point**

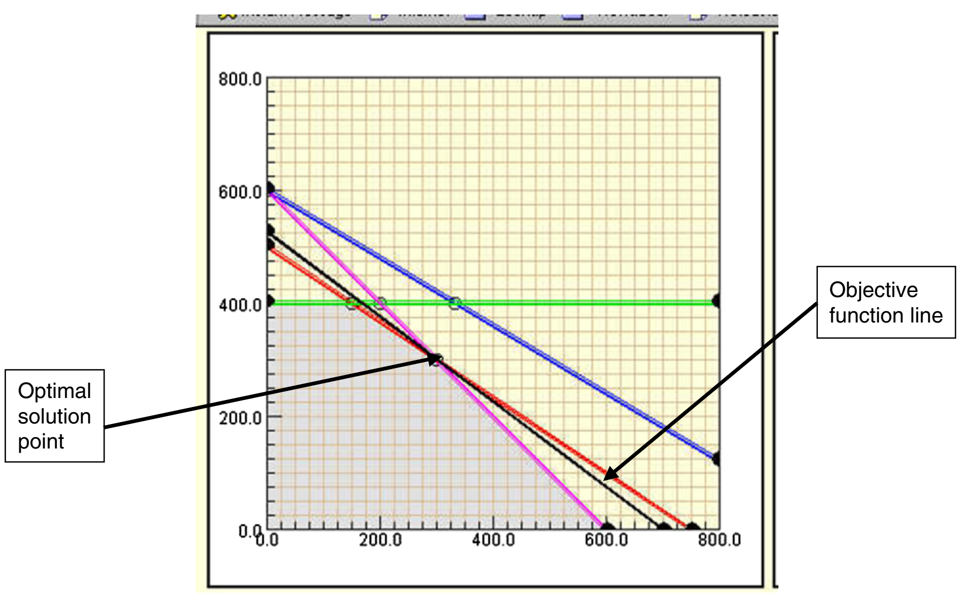


Figure 3 Value of Objective Function at Optimum

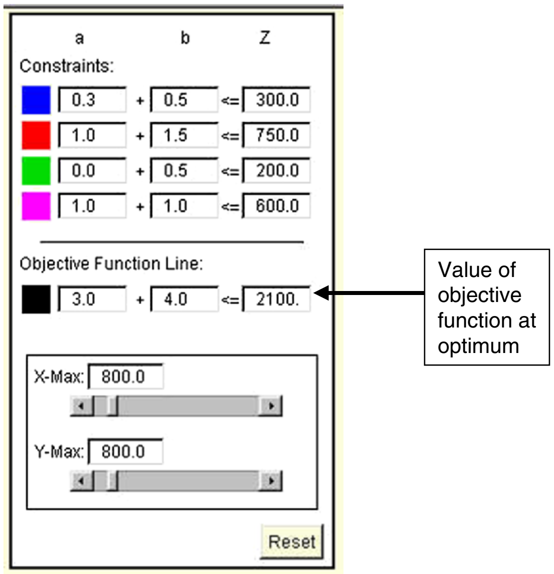
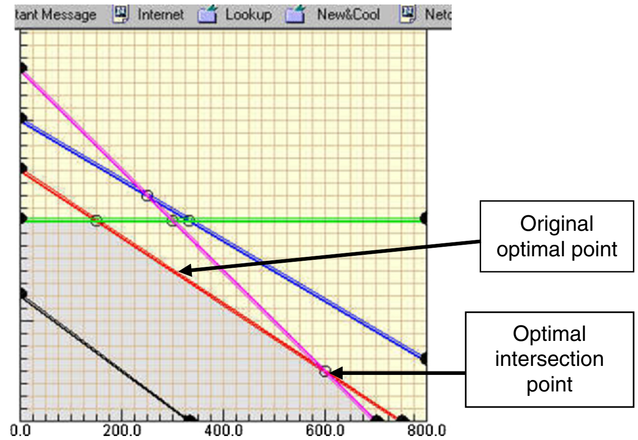


Figure 4 Graphical Representation of Bounds on Constraint Lines



Finding the Upper and Lower Bounds on the Resources (Constraints)

Once the optimal solution has been identified on the graph, students can then be shown how to complete sensitivity analysis for each resource or constraint. Here again, it is very helpful to have each intersection point highlighted by a circle. For each constraint line, the students can slide the line both away from the origin and toward the origin to determine where the upper and lower bounds are for that line (or resource). When sliding the line away from the origin, the intersection point will itself slide either up or down the other line that it intersects with to give the optimal solution. For example, in this problem the optimal solution occurs at the intersection of the red and pink lines. To find the upper bound for pink, students need to slide the pink line out and to the right. While doing this, the circle at the intersection between the red and pink lines slides down the red line (see Figure 4).

When it finally hits the next intersection point at the x-axis, the upper bound point for pink has been determined. If the pink line is moved further in that direction, the intersection of red and pink is no longer optimal because the point has moved out of the feasible space. Because the student is sliding the line and can watch where the circle, or intersection point, moves to, he or she can easily see where the upper bound occurs. At the same time, the numerical value of the upper bound can be seen at the right as the new RHS for that constraint equation. Thus, students can determine where the bound occurs and what the value is. They are also able to determine the allowable increase for that resource. In this case, the upper bound for the pink constraint line is 750, which can be seen at the right (see Figure 5).

The same thing applies for finding the lower bound but in the opposite direction. Each constraint line can be moved in turn to determine upper and lower bounds.

Seeing the Effect of Changing the Slope of the Objective Function

The final part of sensitivity analysis that can be shown easily through use of this applet is the effect of changing the slope of the objective function on the optimal solution. In this case, the student can grab one end of the objective function line and change the slope by dragging the endpoint to a new location (see Figure 6). Then the new optimal solution can be determined by again sliding the objective function line away from the origin to the last feasible intersection point (see Figure 7).

Bounds on the objective function coefficients can be determined by changing the slope of the objective function line until it is the same as each of the two constraint lines in turn that originally intersected to give the optimum. When the objective function line is coincident with one or the other of these two lines, then one of the objective function coefficients will show as having changed in the right side window of the applet. This new value will be the upper or lower bound on the objective function coefficient value (see Figure 8). For example, if the objective function line is coincident with the red line, then the coefficients are shown as 2.67 and 4. With these coefficients, the objective function line now has the same slope as the red line (slope $= -1/1.5 = -2/3$).

Method

A study was completed regarding the LP applet described here using students in six sections of an undergraduate operations management course in one

Figure 5 **Upper Bound Value for Constraint Line**

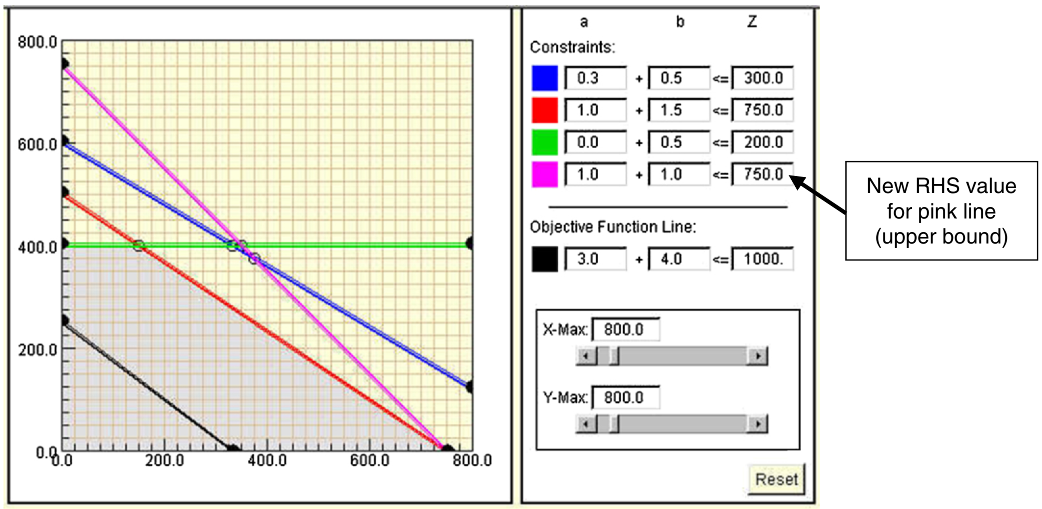


Figure 6 **New Slope of Objective Function**

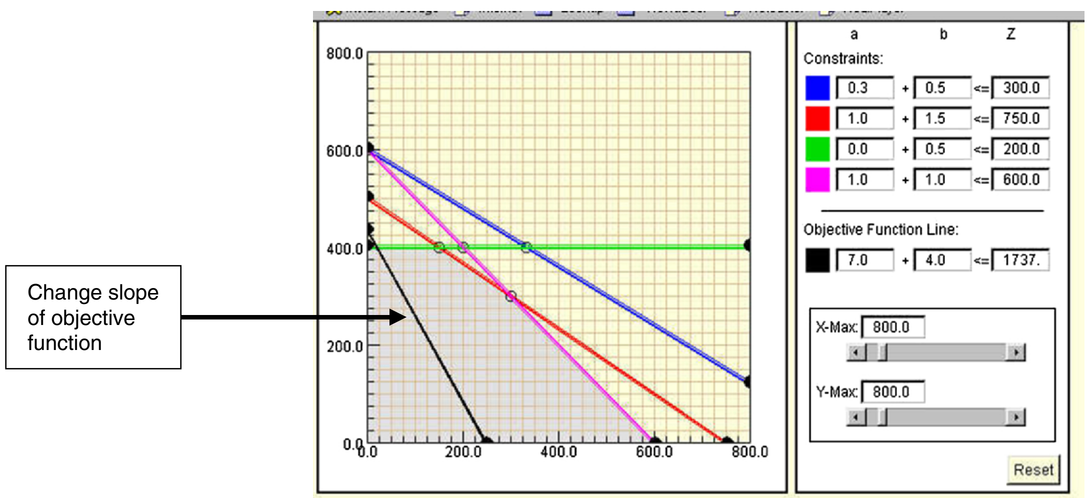


Figure 7 **Finding New Optimal Solution Based on Modified OF Slope**

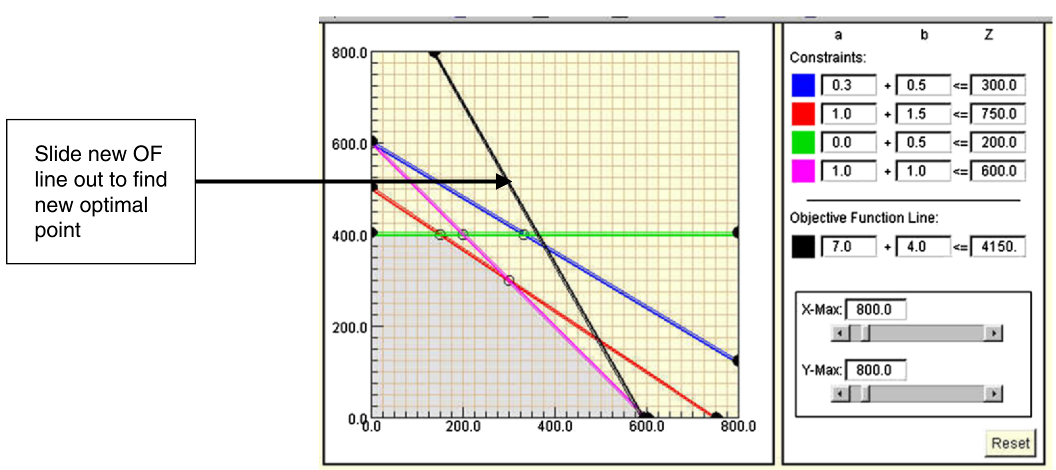
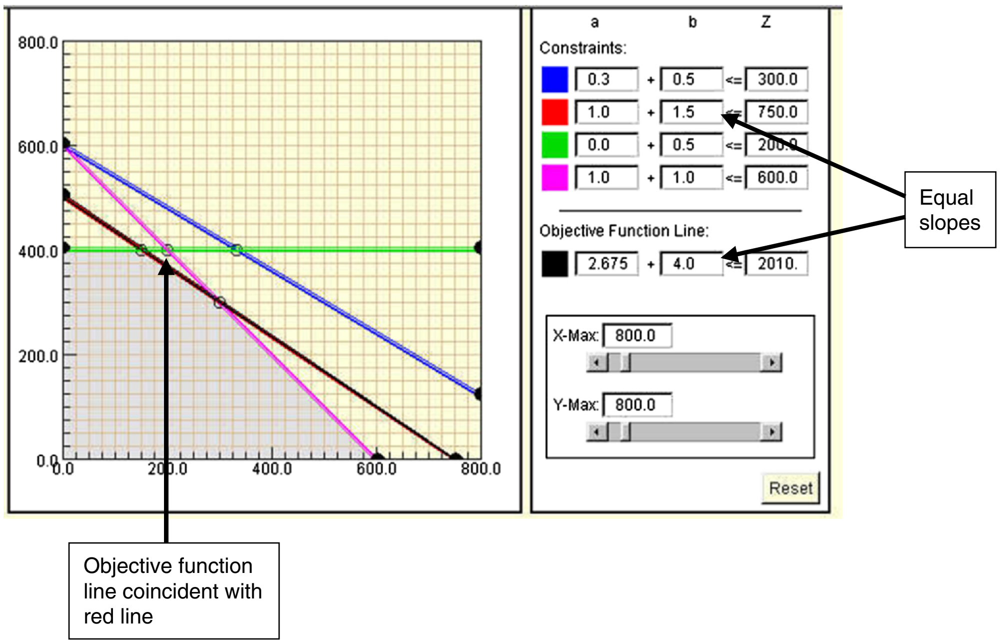


Figure 8 **Finding Bounds on OF Coefficients**



semester at a large university in the eastern United States. In two of the sections, chosen because the author was the instructor, the above-described LP applet was used both during the class lectures on LP and outside of class for student practice. Specifically, the applet was used during class lectures to demonstrate feasible solutions and the feasible region, the optimal solution, and ranges on the right-hand side values. The ranges were shown with the applet by sliding each constraint line away from the origin and then toward the origin in turn, until the line moved outside of the feasible region. The applet was also made available to the students in these same two sections to use outside of class. The students were provided with the Web address of the applet and instructed to practice finding the bounds on their own time. No data was collected as to how many students actually used the applet outside of class; however a practice problem for use with the applet was suggested to these students and several asked questions about this problem during office hours when the students were preparing for the exam. In the other four sections, the use of overhead transparencies and manually sliding "pencil" constraint lines around on the graph illustrated the same concepts. The same practice problem was given to these students to practice on with paper and pencil in preparation for the exam.

There were between 45 and 55 students in each of the six sections of the course. Although students could not be randomly assigned to sections, the students in each section were very similar in terms of their basic characteristics. All of the students in this study were either juniors or seniors in college, and all of them were pursuing a major in business administration. Ages ranged from 19 to 22 across all sections. There were no notable differences among the six sections in terms of gender. This course (Introduction to Operations Management) was a required course for all of the students in this study. All of the students in this study had the same prerequisites for the operations management course. LP is not taught in any other class, so the seniors did not have any unfair advantage here.

Although all sections of the course were not taught by the same instructor, the specific material involving LP and sensitivity analysis (two 75-minute lectures) was taught by the same instructor in all six sections. Thus, the way in which material was presented across sections, with the exception of using the applet, was basically the same.

An exam was administered in each of the six sections, which consisted of the same set of questions on LP and sensitivity analysis. For purposes of this study, the questions examined the student's knowledge about LP in general, and specifically about graphical sensitivity analysis, i.e., finding upper and lower bounds on constraint lines, or ranging. The specific exam questions can be seen in the appendix; part (e) is the ranging question. The questions center on an LP problem and solution that was given to the students in the exam. Computer output and a graphical representation of the problem were also provided to the students.

Results

To examine whether student performance on an exam was impacted by use of the LP applet described in this paper, student scores were recorded for (1) the full exam, (2) the complete LP question on the exam, and (3) the score on just the LP ranging part of the LP question on the exam. To standardize the scores across the control group versus the group exposed to the applet, percentages of each problem correct were used as the main dependent variable. Other analyses were also conducted on points earned on each part of the exam in order to verify the findings in this study.

The data was grouped by control versus experimental group; thus there were 203 students in the control group (four sections) and 108 students in the experimental group (two sections). The two sets of data were tested as to whether the variances on mean percentage correct were equal or not. For the entire LP question, the variances were found to be significantly different ($F = 0.6845$; $p = 0.0109$) for the two groups (thus, a t-test on the difference between means assuming unequal variances was used here). For the LP ranging part of the LP question, the variances were not found to be significantly different ($F = 0.802$; $p = 0.091$) for the two groups (thus, a t-test on the difference between means assuming equal variances was used here).

The major test completed was a comparison of means between the two groups to determine whether students in the experimental group performed better than those in the control group on the LP ranging question and also on the entire LP question. For the LP ranging question, those students who were in the experimental group were found to perform significantly better than those in the control group (see Table 1(a)). The experimental mean was 0.518 whereas it was only 0.315 in the control group showing that use of the LP applet in preparation for this particular question (on ranging) appeared to have a positive impact on percentage of the question correct. For the entire LP question, there was no significant difference in mean scores between those students who were in the experimental group versus those in the control group (see Table 1(b)). The experimental mean percentage was 0.526 and the control group mean was 0.535. So although interaction with the LP applet appeared to help students understand ranging better than their counterparts who had not been exposed to the applet, it did not seem to help them understand other aspects of LP problems better. Because this result might seem to indicate that the experimental group performed worse than the control group on the nonranging part of the LP question, the mean scores on only the nonranging part of the LP question were compared. Although the average number of points scored was slightly higher for the control group, there was no significant difference between the means (see Table 1(c)).

Table 1(a)　Mean Percent Score for Ranging Part of LP Question

	Control	Experimental	t-value	Significance
Mean	0.315	0.518	−4.07	0.00003
Variance	0.161	0.201		
Sample size	203	108		

Table 1(b)　Mean Percent Score for Full LP Question

	Control	Experimental	t-value	Significance
Mean	0.535	0.526	0.224	n.s.
Variance	0.092	0.134		
Sample size	203	108		

Table 1(c)　Mean Score for LP Question Minus Ranging Part of LP Question

	Control	Experimental	t-value	Significance
Mean	17.192	14.962	1.974	n.s.
Variance	82.918	159.068		
Sample size	203	108		

To further explore the data collected in this study, additional analyses were run to determine the effect of student performance on other parts of the exam on performance on the ranging question. The first regression model was specified as $Y = a + b1 * X1 + b2 * X2 +$ 6 additional terms, where Y was the score on the LP ranging question, $X1$ represented the student's grade on the LP question excluding the LP ranging question, $X2$ represented the student's grade on the exam excluding the LP question, and each of the six class sections was represented by a 0-1 dummy variable. The results (Table 2) showed that the regression was significant overall at the $p < 0.001$ level ($R^2 = 0.33$). The two variables, $X1$ and $X2$, were both significant at the $p < 0.001$ level whereas none of the dummy

Table 2　Ranging Score vs. (LP-Ranging) Score and (Exam-LP) Score

ANOVA	df	SS	MS	F	Significance F
Regression	8	3,070.281	383.785	23.801	3.1014E−28
Residual	303	5,583.759	18.428		
Total	311	8,654.039			

	Coefficients	Standard error	t stat	P-value
Intercept	5.435	1.507	3.607	0.0004
LP − ranging	0.109	0.012	8.924	4.407E−17
Exam − LP	0.162	0.0202	8.020	2.326E−14
Sect 10	1.143	0.954	1.198	0.232
Sect 11	1.449	0.914	1.586	0.114
Sect 12	2.063	0.932	2.213	0.0276
Sect 13	0.025	0.925	0.0265	0.979
Sect 14	0.767	0.963	0.796	0.426
Sect 15	0.720	0.834	0.864	0.388

variables representing section was significant. Thus, although none of the individual sections had any impact on how the students performed on the ranging question of the broader LP question, both the score on the LP question minus the LP ranging question and the score on the exam minus the LP question were significant in accounting for the variance in the dependent variable (performance on LP ranging question alone). What this tells us is that both performance on the LP question minus the LP Ranging question and performance on the exam minus the LP question helped to explain performance on the ranging question. So performance on various other parts of the exam is linked to performance on the ranging question, which makes sense because students should perform consistently across all parts of the exam.

The second regression model was similar to the one above but without $X1$ and including a variable $X3$ that represented a dummy variable for control versus experimental group. The model was $Y = a + b1 * X2 + b2 * X3$, where $X2$ again represented the student's grade on the LP question excluding the LP ranging question, and Y again represented the score on the LP ranging question. The results (Table 3) showed that the regression was significant overall at the $p < 0.001$ level ($R^2 = 0.25$). The variable $X2$ was significant at the $p < 0.01$ level. The variable $X3$ was significant at just over the 0.05 level ($p < 0.058$). This suggests that the score on the LP ranging question could partially be attributed to score on the LP question minus the LP Ranging question and also partially to the experimental treatment. This makes sense in that if a student understands LP in general, then that might help the student score higher on the ranging part of the problem. Also it shows that the control versus experimental dummy variable was mildly significant in predicting performance on the ranging question. This result supports the hypothesis regarding how the use of the LP applet led to higher percentages correct on the ranging question. This result is also supported by the t-test on the difference between the mean percent scores for the ranging question as reported earlier (difference between the means was significant at the $p < 0.001$ level).

Table 3 **Ranging Score vs. (LP-Ranging) Score and Condition (Control vs. Experimental)**

ANOVA	df	SS	MS	F	Significance F
Regression	2	1,291.778	645.889	49.634	2.7838E−19
Residual	292	3,799.804	13.0130		
Total	294	5,091.582			

	Coefficients	Standard error	t stat	P-value
Intercept	0.141	0.405	0.349	0.72704
LP − ranging	0.175	0.0184	9.498	8.053E−19
C vs. E	−0.980	0.516	1.899	0.0589

The student's grade on the overall exam should not matter here because there were other topics covered on other parts of the exam that had nothing to do with linear programming or the LP applet described in this paper. To check this, the values for $X1$ (exam score excluding LP question score) were first normalized. An analysis of variance (ANOVA) was then carried out on the normalized mean scores for the exam score minus the LP question for all sections. There were no significant differences across sections. The six classes of students tested approximately the same on the parts of the exam that were not on LP. Thus the students in the experimental group sections were not different (i.e., smarter) in terms of performance on other parts of the exam than the control group sections.

Finally to test for differences within treatment groups (control sections versus experimental sections) on percentage correct on the ranging part of the LP question, the entire LP question and the entire exam, several ANOVAs were run. There were no significant differences among the four sections that were in the control group for any of these variables. Thus, these four sections were similar on all three variables. Similarly for the two sections that were in the experimental group, there were no significant differences on any of these same three variables.

Analysis of variance tests were run on the control group sections (four sections) and separately on the two experimental sections for the normalized values for $X1$ (exam score − LP question). There were no significant differences among the four sections that were in the control group for this variable. There were also no significant differences across the two sections in the experimental group. This demonstrates that within the control versus experimental groups, there were no differences on performance on the parts of the exam that were not on linear programming across sections in each group.

Discussion

The LP applet used in this study was found to be helpful for student understanding of certain aspects of linear programs. Results of this study showed that students who are high performing will do well on all parts of an exam, and vice versa (performance on the ranging question was strongly related to performance on other parts of the exam, and on performance on other parts of the LP problem). However the students also performed better on the ranging part of the LP exam question when they had been shown the concept using the applet compared to a pencil-and-paper demonstration. These results show the value of utilizing a computer-based tool as an active learning technique. Because the students first watched and then used the LP applet to complete ranging tasks, it appears that their interaction with the LP applet aided learning. However, it is interesting that use of the LP applet did not improve learning overall for the topic of linear programming. Both the control group and the experimental group performed equally well overall on the LP problem. One would think that if a student gained a greater understanding of ranging in LP (one of the more difficult aspects to understand regarding an LP solution) based on using the LP applet developed here, then the student would also understand LP solutions better in general, because all of the different concepts are related to each other, particularly within a linear programming graphical solution.

There are a few possible explanations for these results. The first is that the focus of the LP applet in this setting was to teach students about ranging in particular. It is true that the applet was used to demonstrate other concepts as well, such as an initial feasible solution, corner points, and redundant constraints. However, the primary focus was on the student's ability to determine upper and lower bounds on resources (i.e., ranging) for a specific problem solution. One particular problem was suggested to the students for practice on ranging, and so if they used the LP applet primarily for this problem, then their use of the applet would have been primarily (and perhaps only) to practice ranging. Data on frequency of use of the applet and also what students used it to practice were not collected, because students had access to the applet outside of class. In future studies such data should be collected and also perceptions by the students as to whether the applet helped illustrate basic LP concepts to them or not.

The second possible reason why use of the LP applet did not appear to improve general understanding of linear programming problems is that the instructor who taught linear programming to all sections of the class included in this study has been teaching these concepts for over 20 years. Perhaps the instructor's experience in presenting graphical solutions to LP problems outweighs the value of the LP applet. Explanations and examples used to illustrate all parts of an LP solution were identical in both the control and experimental groups. The only difference was that in the control group, the instructor used slides and an overhead projector to show the graphical solution technique instead of the LP applet. Thus future studies of the effect of using interactive tools on learning and understanding should probably be completed across instructors with different levels of experience to see if the LP applet could potentially help instructors with less experience in presenting difficult topics. Also it is possible that the novelty of having a guest lecturer in the control group classes caused the students to pay closer attention than they might have. Additional focus could have led to a greater understanding of LP problems in general in this group.

Future research on the use of computer-based teaching tools should focus on a few factors. First it needs to be determined whether it is the tool itself, the interaction between the tool and student (the active learning part of the process), or the way in which the instructor uses the tool that impacts student learning. There have been numerous anecdotal stories regarding teaching tools that have been very successful under the hands of a particular instructor, but when others try to use it, the effect is not as positive. Also the applet itself can be improved and extended so that students can use it to practice all aspects of solving an LP problem. Extensions to this applet include building in additional functions to make it more user friendly, and expanding the number of constraints it can handle.

This applet represents one more technical, Web-based teaching tool that could be very useful in getting difficult mathematical concepts across to students, particularly to those who are not planning to go into a technical field but who have to learn the basics of an applied field such as operations management. By getting the students engaged with a tool such as this, it appears that "active learning" methods or tools can help students learn. Benefits to using this applet include not only illustrating important points in the classroom setting, but also allowing students to work with the graphical representation on their own outside of class. To date, it appears that this applet can have very positive effects on student understanding and learning.

Appendix. LP Exam Questions

The Ohio Creek Ice Cream Company is planning its production output for next week. Demand for Ohio Creek premium and light ice cream continues to outpace the company's production capacities. Ohio Creek earns a profit of \$100 per hundred gallons of premium and \$100 per hundred gallons of light ice cream. Three resources used in ice cream production are in short supply for next week: the capacity of the mixing machine, the amount of high-grade milk, and the quantity of sweetener available. After accounting for required maintenance time, the mixing machine will be available for 140 hours next week. One hundred gallons of premium ice cream requires 0.3 hours of mixing, 90 gallons of milk, and 10 cups of sweetener. One hundred gallons of light ice cream requires 0.5 hours of mixing, 70 gallons of milk and five cups of sweetener. Only 28,000 gallons of high-grade milk and only 3,000 cups of sweetener will be available for next week.

A linear programming formulation for this problem is given:

$$\text{Max} \quad Z = 100P + 100L$$

$$\text{subject to} \quad 0.3P + 0.5L \leq 140 \text{ hours}$$

$$90P + 70L \leq 28{,}000 \text{ gallons}$$

$$10P + 5L \leq 3{,}000 \text{ cups}$$

$$P, L \geq 0.$$

A computer-generated solution to this problem is provided.

Please answer the following questions regarding this problem.

(a) How many hundreds of gallons of premium and light ice cream should the Ohio Creek Co. produce next week? How much profit will they make?

(b) How much of each of the resources is left over at the optimal solution? Verify algebraically the amount of high-grade milk that is left over.

(c) Suppose that there is a sudden jump in the demand for the light ice cream, and the profit margin for Light doubles. Using the graph, determine what the optimal solution would now be. Explain.

(d) Suppose that Ohio Creek finds out that it can lease additional mixing time next week, but it will cost $75 per hour. Should the firm lease additional mixing hours or not? If so, how many hours should it purchase? If not, explain why not.

(e) From the *graph only*, determine the upper and lower bounds for the mixing machine hours within which the optimal intersection point remains the same. Show the points that you use to find these bounds.

(f) If Ohio Creek can obtain an additional 2,000 gallons of milk at no cost, how much additional profit can it make? Explain your answer.

(g) Ohio Creek is considering introducing a new, super-diet type of ice cream that uses 0.6 hours of mixing time, 75 gallons of milk and no sweetener per hundred gallons of ice cream. It will bring in a profit of $100, the same as the other two brands. Should Ohio Creek produce the new diet ice cream? Why or why not? Show all work and explain completely.

(h) Show the complete LP formulation for this problem that includes the new super-diet ice cream along with the premium and light.

Computer-generated solution to the LP problem

Objective function value
(1) 35,000.00

Variable	Value	Reduced cost
P	175.000000	0.000000
L	175.000000	0.000000

Row	Slack or surplus	Dual prices
(2)	0.000000	83.333336
(3)	0.000000	0.833333
(4)	375.000000	0.000000

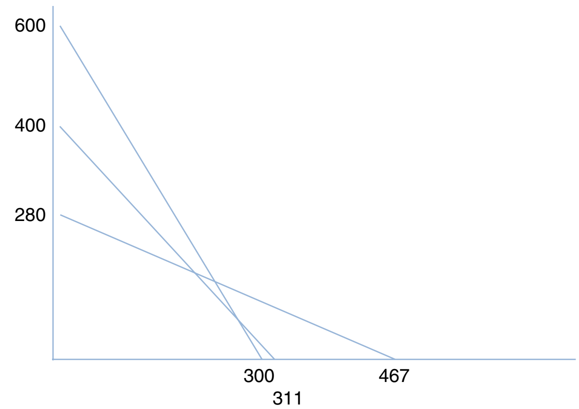
No. Iterations = 2

Ranges in which the basis is unchanged:
OBJ coefficient ranges

Variable	Current Coef	Allowable Increase	Allowable Decrease
P	100.000000	28.571428	39.999996
L	100.000000	66.666664	22.222223

Right-hand side ranges

Row	Current RHS	Allowable Increase	Allowable Decrease
2	140.000000	60.000000	36.000000
3	28,000.000000	2,571.428711	8,400.000000
4	3,000.000000	Infinity	375.000000



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