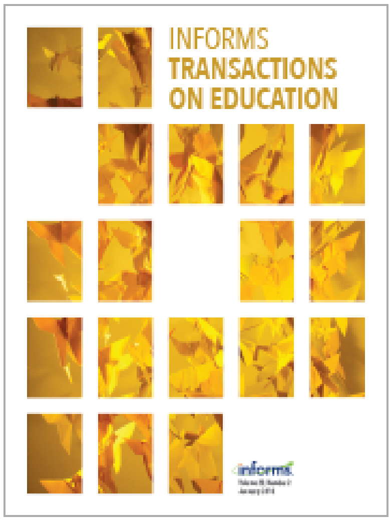

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Five Pat Hands

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The "five pat hands" proposition bet was featured in an episode of Maverick (*Rope of Cards*, Season 1, Episode 17, 1958) and a couple of episodes of Alias Smith and Jones (*The McCreedy Bust*, Season 1, Episode 2, 1971, and *The Ten Days that Shook Kid Curry*, Season 3, Episode 7, 1972). The bet consists of laying claim to the ability to arrange 25 randomly dealt playing cards into five pat poker hands.

Pat hands in draw poker are those hands that are unlikely to be improved by drawing cards. They are, in ascending order of rank, a straight, a flush, a full house, four of a kind, a straight flush and a royal flush. The odds against being dealt each of these hands are 254:1, 508:1, 694:1, 4,165:1, 72,192:1, and 649,739:1 respectively (Scarne 1961). The relative rarity of these hands may lull even a seasoned poker player into believing that assembling five of them from 25 random cards is virtually impossible. However, the number of ways of sorting n^2 cards into n hands each containing n cards is given by

$$\frac{1}{n!}\prod_{i=1}^{n}\binom{ni}{n},$$

which, for n=5, evaluates to approximately 5.19×10^{12}. This huge number of arrangements shifts the odds dramatically in favor of the proposition and renders the task of identifying such a solution, in most cases, trivial.

An informal discussion on the internet newsgroup rec.puzzles leads me to believe that the probability of success is about 98%.

One can easily construct sets of 25 cards for which no solution exists. For example, consider a set that contains the king of clubs with no other kings, no queens, no values with four of a kind and no more than three other clubs. This king therefore cannot be placed in a full house or as a kicker with a four of a kind. If there are no more than three other clubs then it cannot be part of a flush or a straight flush and, without queens, it cannot belong to a straight. The king, therefore, has no home and the five pat hands can not be completed.

In some cases, solutions exist but are strangely elusive as with the configuration in Figure 1.

An integer programming formulation is now presented to identify arrangements that fulfill the conditions of the bet.

We begin by defining sets $Cardd = 1..25$, $Cardh = 1..5$, $Hand = 1..5$, $Value = 1..13$, $Suit = 1..4$ and $Type = 1..3$, for cards dealt, cards in hand, number of hands, card values, card suits, and hand types, respectively. We need only to model for three hand types as the test for a full house and four of a kind can be done in a single stroke (see below) and straight flushes and royal flushes will be indentified as both straights and flushes.

The main variables required are:

$x_{i,j,k} = 1$ if card i is the jth card in hand k, else 0;
$vh_{j,k,m} = 1$ if the jth card in hand k has value m, else 0;
$sh_{j,k,n} = 1$ if the jth card in hand k has suit n, else 0;
$hast_{k,p} = 1$ if hand k has type p, else 0.

A number of auxiliary variables are also required and these are explained at appropriate points in the formulation. Parameters $cvals_{Cardd}$ and $csuit_{Cardd}$ will be used to store the values and suits of the 25 cards to be allocated to hands.

Constraints to enforce the conditions of the bet are as follows.

Each card dealt is in one hand only.

$$\sum_{j\in Cardh}\sum_{k\in Hand} x_{i,j,k} = 1 \quad \forall\, i\in Cardd.$$

Each hand contains five cards.

$$\sum_{i\in Cardd}\sum_{j\in Cardh} x_{i,j,k} = 5 \quad \forall\, k\in Hand.$$

Each position in each hand contains a single card.

$$\sum_{i\in Cardd} x_{i,j,k} = 1 \quad \forall\, j\in Cardh,\ k\in Hand.$$

Figure 1 **A Configuration with an Elusive Solution**

2♣	3♣	7♣	8♣	9♣	10♣	Q♣	K♣
2♦	4♦	5♦	8♦	10♦			
3♥	4♥	6♥	J♥				
3♠	4♠	6♠	8♠	10♠	J♠	Q♠	K♠

Each card in each hand is a single suit.

$$\sum_{n \in Suit} sh_{j,k,n} = 1 \quad \forall j \in Cardh,\ k \in Hand.$$

Each card in each hand is a single value.

$$\sum_{m \in Value} vh_{j,k,m} = 1 \quad \forall j \in Cardh,\ k \in Hand.$$

Ensure that x variables are consistent with vh and sh.

$$x_{i,j,k} \le vh_{j,k,Cvals_i} \quad \forall i \in Cardd,\ j \in Cardh,\ k \in Hand,$$
$$x_{i,j,k} \le sh_{j,k,Csuit_i} \quad \forall i \in Cardd,\ j \in Cardh,\ k \in Hand.$$

In order to cut down the solution search space we ensure that the cards in each hand are in ascending order by value. This will also facilitate the identification of straights.

$$\sum_{m \in Value} m \cdot vh_{j,k,m} \ge \sum_{m \in Value} m \cdot vh_{j-1,k,m} \quad \forall k \in Hand,\ j \in Cardh \mid j \ne 1.$$

To check for a straight we first compute weighted sums of consecutive card differences for each hand k using

$$\sum_{j \in 1..4} \sum_{m \in Value} 10^{j-1} m(vh_{j+1,k,m} - vh_{j,k,m}).$$

For a hand containing a straight this weighted sum will be 1,111 for an ace-low or non-ace straight, or 1,119 for an ace-high straight. In either case we set an auxiliary variable to one to record this.

Check for a flush and set an auxiliary variable to one or zero as appropriate.

$$\text{If } \sum_{j \in Cardh} sh_{j,k,n} = 5 \quad \text{then } aux = 1, \text{ else } 0 \quad \forall k \in Hand,\ n \in Suit.$$

It is possible to check both for full houses and four of a kind hands at the same time. In each case there will be only one consecutive card value difference greater than zero and if this is the case we set an appropriate auxiliary variable to one.

$$\text{If } \sum_{m \in Value} m(vh_{j+1,k,m} - vh_{j,k,m}) = 1 \quad \text{then } aux = 1, \text{ else } 0 \quad \forall j \in 1..4,\ k \in Hand.$$

As mentioned earlier, there is no need to check for a straight flush or a royal flush as these will already have been identified as both a straight and a flush.

Ensure that all hands have at least one type.

$$\sum_{p \in Type} Hast_{k,p} \ge 1 \quad \forall k \in Hand.$$

A GLPK (2010) Mathprog implementation of the above formulation is included with this article (pathands.mod). It includes the data set for the above example. This was compiled by GLPK into an MPS file and solved using SCIP (2012).

Supplementary Material

Files that accompany this paper can be found and downloaded from http://dx.doi.org/10.1287/ited.1120.0089.

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