Vol. 16, No. 1, September 2015, pp. 1–5
ISSN 1532-0545 (online)
INFORMS Transactions on Education
http://dx.doi.org/10.1287/ited.2015.0143

Numerous Sigma Level Tables Need Correction: Commonly Used Tables Ignore Permitted "Left-Side" Defects

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Many popular textbooks in the fields of operations management and quality contain process sigma level tables that are slightly inaccurate and inconsistent among each other. These discrepancies annoy and are confusing to fastidious educators and students and could lead to minor estimation errors in practice. We document instances of the error and provide explanation, calculations, and an accurate sigma level table.

Keywords: Six Sigma; DPMO; quality; teaching operations management; teaching supply chain management
History: Received: October 2014; accepted: April 15.

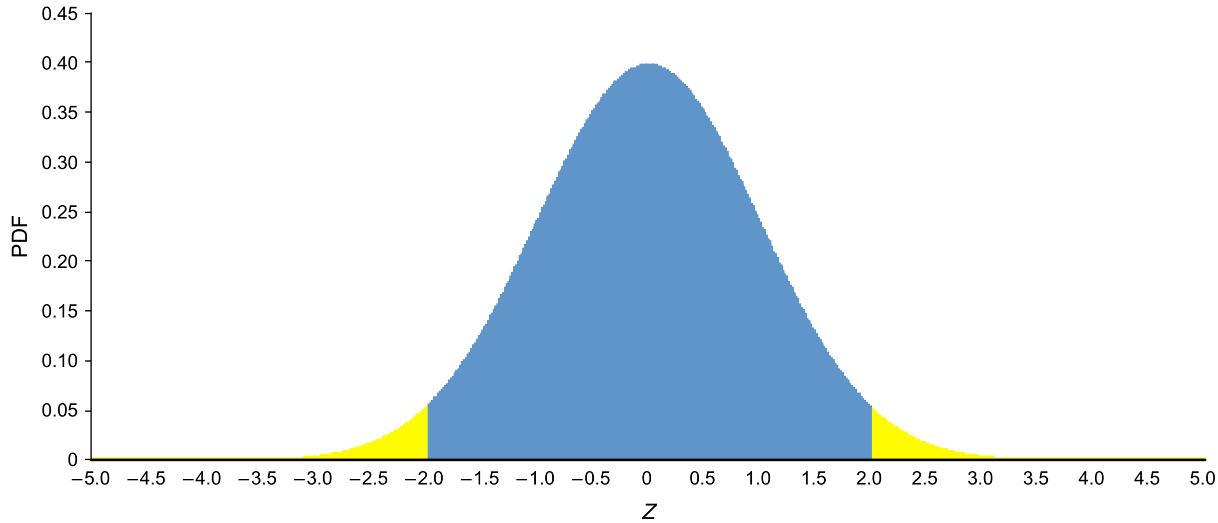
1. Background

A Six Sigma process is commonly known as one that produces no more than 3.4 DPMOs (defects per million opportunities) (Hoerl 1998). Organizations use methodologies such as DMAIC (define, measure, analyze, improve, control) and DMADV (define, measure, analyze, design, verify) to spur process improvements and to build new processes in attempts to achieve this goal of near perfection. For interested readers, more detail regarding these methodologies can be found in Hahn et al. (2000) or Subramaniyam et al. (2011), for example. Six Sigma pioneering organizations Motorola, General Electric, and Allied Signal are widely praised for implementation and development of six-sigma methodologies, saving their organizations billions of dollars. Thus, many organizations benchmark current practices to determine levels of performance and identify projects for potential improvement.

When Bill Smith and a team of Motorola managers developed the six-sigma concept, they assumed that a process would be allowed to shift or drift over time by as much as 1.5 standard deviations and still be considered a six-sigma process (Gupta 2006, Harry and Lawson 1992). We incorporate this now widely accepted mean process shift in the present work.

A sigma level is a measure of process performance (Meredith and Schafer 2010). Often, processes that perform at the six-sigma level or higher are referred to as world class or best of breed, three to five sigma level processes are considered average, and two and below sigma level processes indicate a need for improvement, as they are deemed noncompetitive (Harry 1998). A sigma level of process performance can also be communicated as a percentage of defects a process is expected to produce. This percentage is commonly multiplied by one million to obtain DPMOs. Many books and articles include tables that provide this translation. Our classroom experience in teaching Six Sigma includes undergraduate and graduate business courses in supply chain and operations management, and in project management. Six Sigma, and the DPMO and process sigma level calculations, are an important portion of a module in quality management, which consumes three hours of our normal semester class. In class assignments, students commonly use a textbook or online table (http://www.isixsigma.com/new-to-six-sigma/sigma-level/sigma-performance-levels-one-six-sigma/) to look up sigma levels of a business, service, or operations process and corresponding DPMOs. We have found numerous examples where this translation is inaccurate, and especially notable at lower sigma performance levels. As we show, a two-sigma level process produces 308,770 DPMOs, not 308,537 DPMOs, as is widely and incorrectly specified. With the incorrect DPMO specification, permitted "left-side" defects have been ignored. In our class discussions, we explain to students that defects may occur above (right side in a plotted distribution) or below (left side) the acceptable range of values, yet many commonly used tables

Figure 1 **Two-Sigma Level Process with No Shift**



ignore the left side permitted defects. This omission is an error with consequences. For example, a container that is underfilled or overfilled is a problem. A plane that arrives either early (a gate might not be available) or late (passengers are inconvenienced) is a problem. A burger patty may either be too large or too small, undercooked or overcooked, etc. It makes little sense to consider only right-side defects in the technical calculations. This article documents some of the numerous instances of this mistake. We strive to increase awareness of the inaccuracy and offer analysis that supports use of a correct table.

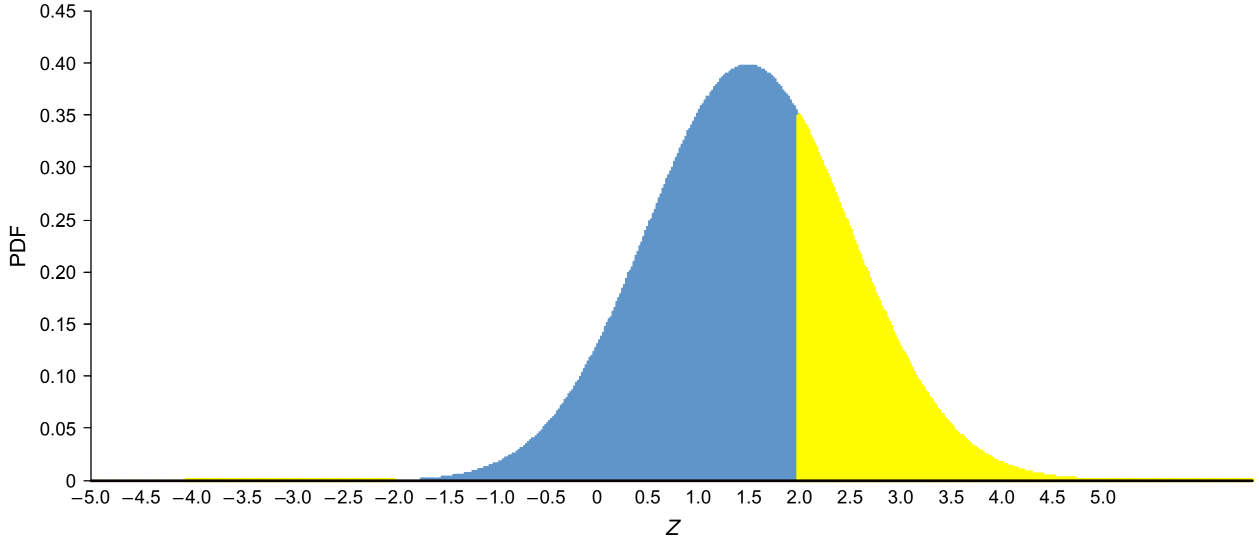
2. A Two-Sigma Process Example

Since the problem is most obvious at lower sigma levels, we will focus on a two-sigma level process to illustrate the inaccuracy. Figure 1 depicts a straightforward two-sigma level process with no allowance for a process shift or drift. In this scenario, a process would be expected to generate 45,500 defects per million opportunities, half of which could be greater than two standard deviations from its process mean and half of which could be smaller than its process mean as shown.

Adjusting for the 1.5 standard deviation shift allowance, Figure 2 shows the same process shifted 1.5 standard deviations to the right.

A "shifted" two-sigma process allows defects up to 30.87702% of the time, 30.85375% defects on one side of the process mean, and 0.02326% defects on the other side of the process mean or a total of 308,770 defects per million opportunities. In the numerous instances of incorrect process sigma level tables, the smaller side percentages of allowable defects are either overlooked or omitted. It is easy to see why this is the case, as it is difficult to observe the small

Figure 2 **Two-Sigma Level Process with 1.5 Standard Deviation Shift to the Right**



area in the left-hand side of the tail three and half standard deviations away from the shifted process mean. A process that has a two-sigma level of performance allows for defects above 0.5 standard deviations above its process mean (in this case the process mean is 1.5) and 3.5 standard deviations below its process mean. In a standard normal distribution this area is 0.30854 and 0.00023, respectively. These percentages of defects allowed translate to 308,537.54 right-side defects per million and 232.63 left-side defects per million, for a total of 308,770.17 defects per million. Many tables do not include the 233 left-side defects in this total.

As an example, according to McDonald's website (http://www.mcdonalds.com) about 70,000,000 customers are served per day. Using the incorrect calculation for a two-sigma level process ($0.30854 \times 70{,}000{,}000$), 21,597,800 customers would receive an order with some kind of defect (e.g., too many or too few french fries). Using the correct calculation of a two-sigma level process ($0.30854 \times 70{,}000{,}000 + 0.00023 \times 70{,}000{,}000$), an additional 16,100 customers could have had their order filled improperly, and McDonald's would still have a two-sigma level order fulfillment process. Using the correct calculations, organizations are able to have a few more defects per million at a specified sigma level than they might have realized. Conversely, organizations that use the incorrect calculation might underestimate the number of DPMOs they are delivering for a given sigma level. This error would lead them to underestimate the true costs of failure associated with a given level of quality.

The following formulas and Excel functions illustrate the calculations required to determine the permitted DPMOs for a stated sigma level:

σ: the sigma level of the process
Z_L: the standardized lower specification limit
Z_R: the standardized upper specification limit
W: the specification width, $W = Z_R - Z_L$ and $\sigma = Z_R + 1.5$, $W = 2\sigma$
A_L: area under the normal curve to the left of the lower specification limit and $A_L = \text{NORM.S.DIST}(Z_L, \text{TRUE})$
A_R: area under the normal curve to the right of the lower specification limit and $A_R = 1 - \text{NORM.S.DIST}(Z_L, \text{TRUE})$
D_L: total left-side defects per million, $D_L = 1{,}000{,}000 \times A_L$
D_R: total right-side defects per million, $D_R = 1{,}000{,}000 \times A_R$

The calculations below show both the incorrect and correct way to obtain the DPMOs for a two-sigma level process.

Incorrect Calculation, Two-Sigma Level Performance with Shift (*left-side permitted defects ignored*):

$Z_R = 0.5$
$A_R = 1 - \text{NORM.S.DIST}(0.5, \text{TRUE}) = 0.30854$
$D_R = 308{,}537.54$
Total DPMOs $= 308{,}537.54$

Correct Calculation, Two-Sigma Level Performance with Shift (*left-side permitted defects included as shown in Figure 2*)

$Z_R = 0.5$
$A_R = 1 - \text{NORM.S.DIST}(0.5, \text{TRUE}) = 0.30854$
$D_R = 308{,}537.54$
$Z_L = -3.5$
$A_L = \text{NORM.S.DIST}(-3.5, \text{TRUE}) = 0.00023$
$D_L = 232.63$
Total DPMO's $= D_R + D_L = 308{,}770.17$

When implemented using our prior correct calculation example with $Z_L = -3.50$, $Z_R = 0.50$, $A_L = 0.00023$, $A_R = 0.30854$, $D_L = 232.63$, $D_R = 308{,}537.54$, $W = 4.00$, and the specified sigma level, $\sigma = 2.00$ (*user input*) the following results are obtained:

	Z	A	D
Left side	−3.50	0.00023263	232.63
Right side	0.50	0.30853754	308,537.54
Total DPMOs			308,770.17
Sigma level			**2.00**
W			4.00

Table 1 shows the commonly used (incorrect or omitted) right-side defects per million and the accurate defects per million for various sigma levels.

The commonly used right-side defects column ignores allowed left-side errors. This inaccuracy is most evident at low sigma levels as we described for a two-sigma level process performance. At the one-sigma level, commonly used tables fail to account for 6,210 defects, or about 0.9 % of the total defects. Table 1 correctly specifies allowed errors on each side of a process mean.

Table 1 Correct and Commonly Used Incorrect or Omitted DPMOs

Sigma	Left-side defects	Right-side defects	Correct DPMOs
1.0	6,209.67	691,462.46	697,672.13
1.5	1,349.90	500,000.00	501,349.90
2.0	232.63	308,537.54	308,770.17
2.5	31.67	158,655.25	158,686.93
3.0	3.40	66,807.20	66,810.60
3.5	0.29	22,750.13	22,750.42
4.0	0.02	6,209.67	6,209.68
4.5	0.00	1,349.90	1,349.90
5.0	0.00	232.63	232.63
5.5	0.00	31.67	31.67
6.0	0.00	3.40	3.40

3. Scope and Limitations

Use of tables that incorrectly depict only single-sided errors is widespread. Using our university library and our supply chain department faculty's own bookshelves, we reviewed over 100 popular books covering the area of quality; we found 26 containing six-sigma DPMO tables. As shown in Table 2, only seven were found that used a table with correct calculations. Of the seven correct tables, there were slight differences in DPMOs at the one- and two-sigma levels. Seventy-three percent, 19 of the 26 sources, used an incorrect, right-hand only version table. It should be somewhat disturbing, that in our review, tables that should display exactly the same information had occurrences of at least three different sets of numbers.

Many published sigma/DPMO tables give a slightly incorrect DPMO value, ignoring or omitting permitted errors below (or on one side only of) a process's mean. The current Wikipedia entry (https://en.wikipedia.org/wiki/Six_Sigma) regarding this topic publishes a sigma/DPMO table using right-side only defects but notes "defects beyond the far specification limit are

Table 2 **Publications**

	Source	Title	ISBN	Page	Sigma/DPMO table correct?
1	Wisner and Tan (2011)	Principles of supply chain management: a balanced approach	0324657919	274	No
2	Meredith and Schafer (2010)	Operations Management for MBAs	0470485767	148	Yes
3	Chase and Jacobs (2006)	Operations Management for Competitive Advantage	0073121517	354	No
4	Swink et al. (2011)	Managing operations across the supply chain	0073403318	170	No
5	Foster (2010)	Managing quality: integrating the supply chain	0136088503	399	No
6	Burt and Starling (2003)	World class supply management: the key to supply chain management	0072831561	130	No
7	Burton and Sams (2005)	Six sigma for small and mid-sized organizations success through scaleable deployment	1932159215	19	No
8	Bertels (2003)	Rath and Strong's Six Sigma Leadership Handbook	0471251240	493	No
9	Besterfield (2003)	Total Quality Management	0130993069	148	Yes
10	Shaffie and Shahbazi (2012)	Lean Six Sigma	0071743855	125	No
11	Omachonu and Ross (2004)	Principles of Total Quality	1574443267	443	Yes
12	Tennant (2001)	Six Sigma: SPC and TQM in Manufacturing and Services	0566083744	Appendix	No
13	Tennant (2002)	Design for Six Sigma: Launching New Products and Services Without Failure	566084341	182	No
14	Breyfogle (2008)	Integrated enterprise excellence: business deployment: a leaders' guide for going beyond lean six sigma and the balanced scorecard	193445415X	424	Yes
15	McCarty (2005)	The Six Sigma black belt handbook	0071443290	383	No
16	Eckes (2003)	Six sigma for everyone	0471281565	24	No
17	Pande and Holpp (2002)	What Is Six Sigma?	0071381856	Appendix	No
18	Pande and Neuman (2002)	The Six Sigma Way Team Fieldbook: An Implementation Guide for Process Improvement Teams	0071373144	179	No
19	Nash and Poling (2006)	Using Lean for Faster Six Sigma Results: A Synchronized Approach	1563273438	39	No
20	Hoyle D (2007)	Quality Management Essentials	0750667869	50	Yes
21	Gitlow and Levine (2006)	Design for six sigma for green belts and champions: applications for service operations-foundations, tools, DMADV, cases, and certification	0131855247	41	No
22	Kumar et al. (2006)	Reliability and six sigma	0387302560	8	Yes
23	Jones (2014)	Quality management for organizations using lean six sigma techniques	1439897829	64	Yes
24	Gulati (2009)	Maintenance and reliability best practices	0831133115	383	No
25	Truscott and Truscott (2003)	Six sigma continual improvement for business: a practical guide	0750657655	4	No
26	Bowie (2011)	Lean acres: a tale of strategic innovation and improvement in a farm-iliar setting	0750657655	148	No

not included in the percentages." The author(s) of this entry perhaps assume that this omission is a convention, but such a statement is not included in the entry.

In practice, using a popular textbook table as a benchmark, a process would achieve a slightly lower sigma specification than it merits, more notable at low levels of performance. Further, in practice, a number of other issues could lead to sigma level estimation errors that are more significant than the one discussed in this manuscript. These issues include but are not limited to model error and parameter uncertainty. For example, the results of a manufacturing or service process may not follow a normal distribution and the actual drift may be different from 1.5 standard deviations. Process parameters (the true mean and standard deviation) are estimated from data. If the sample data are not representative of the process, the sigma level calculation will be inaccurate. The errors introduced by these factors may be compounded or offset by the omission of permitted left-side defects in many published six-sigma tables, and probably do not present a major quality benchmarking issue for practitioners. However, inconsistencies in these sources can be confusing to students as well as practitioners. As educators we should strive to publish correct, two-sided table values.

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