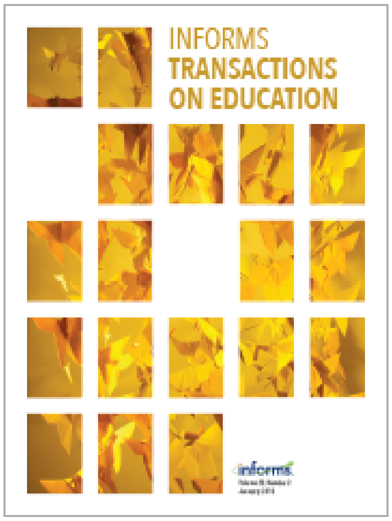


INFORMS Transactions on Education

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Game—A Carbon Emissions Game

Ian D. Frommer, Robert W. Day

informs
http://pubsonline.informs.org/journal/ited/

INFORMS TRANSACTIONS ON EDUCATION
Vol. 18, No. 1, September 2017, pp. 56–70
ISSN 1532-0545 (online)

Game

A Carbon Emissions Game

Ian D. Frommer,[a] Robert W. Day[b]

[a] Department of Mathematics, United States Coast Guard Academy, New London, Connecticut 06320; [b] Operations and Information Management, School of Business, University of Connecticut, Storrs, Connecticut 06269

Contact: ifrommer@post.harvard.edu (IDF); bob.day@business.uconn.edu (RWD)

Received: July 28, 2015
Revised: May 25, 2016; September 20, 2016; October 23, 2016
Accepted: January 10, 2017
Published Online in Articles in Advance: July 12, 2017

https://doi.org/10.1287/ited.2017.0171

Abstract. This paper describes a carbon trading game to be played in a classroom setting. The game is modeled on real-world electric power markets with a carbon dioxide (CO_2) emissions cap, often referred to as a "cap-and-trade" system. Players assume the roles of competing utilities, each with a fossil fuel plant and a renewable energy plant. Any electricity generated by fossil fuel requires offsetting carbon credits, which are available via auction. Depending on the auction price, it can be cheaper or more expensive to generate electricity with fossil fuel versus renewables. The goals of the game are: for students to become acquainted with cap-and-trade markets; to understand how regulatory policy applied to a market can induce environmental benefits; to discover how to mathematically derive strategies; to get a taste of basic game theory, auctions, and the newsvendor problem; and to meet these objectives in a fun setting. While fairly straightforward and simplified to provide symmetry among student competitors, the game can lead to surprisingly interesting results. This paper describes the game and how to facilitate it, presents strategies, analyzes games that were played, introduces novel quantitative measures to evaluate the quality of game play, and offers a number of suggestions for extending the game.

Supplemental Material: The online appendices are available at https://doi.org/10.1287/ited.2017.0171. The teaching note is available at https://www.informs.org/Publications/Subscribe/Access-Restricted-Materials.

Keywords: emissions markets • cap and trade • classroom games • CO_2 emissions • active learning

1. Introduction

Emissions of carbon dioxide from human processes have risen sharply over the past 150 years. Because these emissions are considered a significant factor in global warming and climate change, attempts have been made to reduce them. Cap-and-trade is an approach in which a fixed number of emissions credits are available to emitters (e.g., fossil fuel power plants), who must use them to offset their CO_2 emissions. The idea is to use a market-based regulatory mechanism to reduce negative environmental impact. Cap-and-trade programs have been implemented in various forms. They typically consist of an auction for credits followed by a bilateral credit trading phase. One of the first such programs to be widely implemented was the U.S. acid rain program. (Ellerman 2000) A current cap-and-trade system in place to reduce CO_2 emissions in the northeastern United States is the Regional Greenhouse Gas Initiative (RGGI). RGGI holds an annual auction for power plants to purchase credits (referred to as "allowances") to emit one ton of CO_2. Funds raised through the auction are typically directed to clean energy projects in the member states. A June 2013 auction with prices of $3.21 per allowance raised $124 million. (Singer 2013) The European Union Emissions Trading System (EU ETS) is the largest international greenhouse-gas emission allowance trading system in the world. Additional systems are in the planning stages for large-scale CO_2 emitting nations such as China and India. (Jopson et al. 2015)

To give students a feel for how such systems work and how policy can guide a market to lessen environmental impact, we designed a carbon emissions game. The game is a simplified model of an electric power market with a CO_2 emissions cap loosely based on RGGI. It is set in some imaginary realm, not unlike the northeastern United States. All numbers used are fabricated but are intended to roughly correspond to those used in practice by power plants and auctions. Players own two power plants (fossil fuel and renewable energy) and try to maximize profit and meet power demand. They participate in the primary credit auction phase, purchasing carbon credits to cover any CO_2 emissions from their fossil fuel plant. Only a limited number of credits are available each year (the "cap"), with the amount declining from year to year. For simplicity, the game does not model the secondary credit trading phase, though it could be extended to do so in the future.

The target audience for this game is undergraduate students with some minimal quantitative background (e.g., calculus is not necessary but an elementary quantitative models course would be helpful). A somewhat deeper appreciation and learning opportunity would be afforded to students who are familiar with the newsvendor model, basic game theory, and optimization. This game has several educational objectives:

1. Become acquainted with how carbon cap markets function;
2. Experience how policy can steer an otherwise free market to reduce negative environmental impacts;
3. Learn how to develop simple mathematical strategies for a competitive game;
4. Be exposed to basic auction- and game-theoretic effects as well as the newsvendor effect (NVE);
5. Meet these objectives in a fun and entertaining way.

We also perceive several benefits for the instructor:

1. Teach material in an unconventional way through a game;
2. Evaluate students in an unconventional way through the quality of their decision making (controlling for the randomness built into the game);
3. Use student game play results and possibly extensions of the game (variations on the rules, computer-simulated players, etc.) to conduct further analyses of the game and student decision making in a manner similar to the research that has emerged surrounding the Beer Game, (Sterman 1989), discussed below.

The rest of this paper is organized as follows: A literature review of classroom games for operations research constitutes the next section. The game structure, rules, and facilitation are described in Section 3. Section 4 provides an overview of strategies players should use to perform well based on auction price, coal supply, presence of shortages, and newsvendor-like considerations. Section 5 consists of general observations about games played by students followed by the introduction of metrics to rate player decision making. This is followed by a discussion of potential follow-ups and extensions to this work in Section 6 and concluding remarks in Section 7. Several appendices and a Teaching Note supplement the paper offering more in-depth analysis and game facilitation materials.

2. Literature Review: Classroom Games for Operations Research

Games have frequently been used in educational settings. Sniedovich (2002) encourages the use of games in Operations Research/Management Science (OR/MS), providing several examples. He also outlines the roles games can play in OR/MS education such as introducing mathematical modeling, simulation, algorithms (explaining/developing), heuristics, pattern recognition/analysis, and OR/MS software. An article on active learning techniques for quantitative courses (Cochran 2005) discusses games as an effective approach, and lists several examples. Wood (2007, pp. 3–4) touts the benefits of games in operations and presents a taxonomy of online games. According to his scheme, our carbon game would be a blend of an "insight" game, in which students "quickly gain a conceptual background or key insights," and an "analysis" game, in which the goal is acquisition of skills. For additional OR-related games beyond those cited here, see Griffin (2007), who also lists three reasons that an OR/MS educational game is successful, using the beer game (Sterman 1989) as an example:

1. It is relatively easy to implement;
2. It encourages interaction around OR/MS principles;
3. It generates interesting discussion about the game after it is played.

We will demonstrate that our game meets each of these criteria.

There are many examples of applying OR techniques such as integer programming to games and puzzles (Chlond 2006, 2008, 2014), (Meuffels and den Hertog 2010). In some cases, the games could be played or puzzles solved by students with the aid of the OR techniques and insights developed. Other papers directly incorporate the modeling and optimization of games into the curriculum. For example, Beliën et al. (2011) present their students with a cycling game to be solved using mixed-integer programming (MIP). Chan (2013) runs a spreadsheet version of the game show "Deal or No Deal" to teach decision making under uncertainty. Gorman (2012) uses the game "Pass the Pigs" to teach a variety of decision-making and analytics techniques.

In other cases, new games have been developed to help motivate OR-related class material using real-world applications. Our work falls into this category. Examples include games covering collaborative logistics (D'Amours and Rönnqvist 2013), health care appointment scheduling (Sauré and Puterman 2014), entrepreneurial decision making (Ben-Zvi and Carton 2007), revenue management (Talluri 2009), and supply chain sourcing (Allon and Van Mieghem 2010). Perhaps the best known and most successful of these games is the beer game (Sterman 1989). It is used to teach students about the dynamics of supply chains and has proven to be very effective at illustrating the so-called "bullwhip effect" in supply chains (Lee et al. 1997). A great deal of literature has emerged around the beer game ranging from behavioral studies of game play to mathematical analysis of the game's dynamics.

There are also works on games closer to our topic of energy emissions. Beliën et al. (2013) teach integer programming through an energy supply game. Players purchase power plants of various types, aiming to meet power demand while staying within budget and emissions limits. The educational goal is for students to formulate and solve the game as an integer program without being explicitly given the information needed. Cholette and Roeder (2012) use an interactive supply chain module in their courses in which students devise a supply chain and monitor energy use and greenhouse gas emissions. The secondary emissions trading market is explored along with other regulatory mechanisms aimed at curtailing emissions in a game described by Corrigan (2011). Our work extends the set of OR games by introducing a game pertaining to carbon emissions that not only exposes students to energy/environmental-subject matter and OR/MS techniques but also facilitates evaluation of student performance based on the quality of decision making.

3. Game Rules, Set-Up, and Facilitation

This section describes the structure and rules of the game and how it is facilitated. Note that many modifications are possible without disrupting the intent of the game. In our runs of the game, we divided students into groups of 2–4 per team; any number of teams between 3 and 6 would work well. We begin with a brief presentation about the game and the situation it models. A handout (see Appendix A) with an overview of the game, its rules, objective, an example, and pregame exercises is given to each student. Time permitting, we have done the presentation and distributed the game handout to the class before the game is played. In other cases, such as when doing the game as a "guest lecturer" we have posted the handout online for students to view ahead of time and done the presentation at the beginning of the class followed by running the game during that same class.

We will now describe how the game is played. (Keep in mind that many variations are possible.) Each team is endowed with identical assets, i.e., one coal power plant and one renewable energy power plant. Each plant has a generation capacity, generation cost, emissions profile (i.e., CO_2 credits required per unit of energy generated), and a sale price of energy. See the game handout in Appendix A for the values used in some of the first games implemented.

Note that the game assumes that the renewable energy production cost (c_r) is significantly greater than the coal energy production cost (c_c). From the perspective of short-run marginal costs, the reverse is generally true. For instance, the marginal cost of producing wind energy after a wind turbine has been built is effectively zero when the wind blows. Instead, the game takes the perspective of long-run average or "all-in" costs. Including the fixed costs of installation, inconsistent output, and lower capacity factor into the long-run costs of a renewable source, the fossil fuel source will become cheaper per unit of generation. (If not, simple self-interest would steer a changeover to renewable generation without regulatory intervention.) An important objective of the game is to illustrate how policy can steer decision makers to environmental benefit. In the simplest possible version of the game, as presented here, the players' in-game decisions are simply: How many emissions credits should we purchase? Yet the game as a whole frames a larger decision: Given the emission credit policy in place, do we build a fossil fuel plant or a wind plant? From this longer-term standpoint, it makes sense to consider the "all-in" costs of each. The relatively unfavorable long-run costs of the renewable source become favorable through policy such as capped emissions as modeled in this game, even neglecting the possibility that long-run renewable costs may decline over time through improved technologies or economies of scale.

The game is played in several simulated "years," each representing one year of power generation. Within each year, there are two stages. *Stage* 1 is an auction stage in which players submit bids for carbon credit quantities. These credits are required if the team is to generate power using its coal plant. This stage typically consists of multiple rounds, with the credit price increasing from one round to the next, ending once the aggregate player demand for credits no longer exceeds total supply. In *Stage* 2, the uncertain demand quantity for each player is realized, and each must decide on the amounts of power to generate to meet this demand, using a mixture of coal resources (covered by credits from Stage 1) and renewable capacity. Because finding the profit-maximizing mixture is simple for this stylized setting, this power dispatch step is automatically performed for each player by the game software. A team's profit for each year is then computed as follows:

$$\begin{aligned}\text{Profit} = {} & \text{Revenue from selling energy} \\ & - \text{costs to generate energy} - CO_2 \text{ credit costs}\end{aligned}$$

The team with the most money (i.e., sum of profits from each year) at the end of the last year wins. *The only decisions players make are in stage* 1 *of the game*: *How many credits to buy in the auction in each round*. This keeps the game simple enough for players to quickly learn how to play, but with just enough complexity to meet the educational objectives and produce interesting results for analysis. Future extensions incorporating greater complexity could include allowing players to purchase power plants and fuel, and given a more complex power portfolio, to determine which plants to dispatch and in what amounts.

Players will know their total generation capacity given the plant parameters supplied on the game handout. They are also given a table of recent energy demand and told that future demand is expected to be similar. (We used a simple uniform distribution between 245 and 255 Megawatt-years (MWy) to generate a table of eight annual values in games we ran (see Appendix A)). More specifics on each of the two stages are described below.

Stage 1: Auction. The players purchase credits via a multiunit ascending auction, a form of Walrasian tatonnement. (RGGI uses a different type of auction.[1]) The auction process proceeds as follows:

- There are a limited number of credits to be auctioned in each year.
- The price per credit starts at an initial level announced at the beginning of the auction.
- After any price announcement (i.e., the start of a "round"):
  - Each team bids on how many credits it would like to purchase at the current price.
  - If the total number of credits being bid for does not exceed the amount available, then each team receives its bid amount at the current auction price, and the auction ends.
  - Otherwise, a new price is announced, and the auction continues. (Note that teams are told by how much the credits bid for exceeded the available amount in the previous round.)

Stage 2: Power Dispatch and Profit Determination. After the auction, the randomized actual power demand is announced and apportioned to each player. Again, players will have access to "historical" demand data, but will not know in advance what the demand is going to be when they purchase the CO_2 credits. Fulfillment of each player's demand is determined in a profit-maximizing manner, factoring in how many CO_2 credits the player has and the plant capacities. Which plant is the more cost-effective power producer will depend on the price paid per carbon credit. We let c_c = coal power production cost, c_r = renewable power production cost, and c_{auc} = credit auction cost. Parameters c_c and c_r are fixed for the duration of the game but c_{auc} is endogenously determined and will vary from round to round. When the auction price is low, coal is cheaper (since $c_{\text{auc}} + c_c$ is less than c_r) and so is used first. When $c_{\text{auc}} + c_c = c_r$ (i.e., when $c_{\text{auc}} = c_r - c_c$) the plant costs are equal. As c_{auc} increases above the transitional value of $c_r - c_c$ (it was \$10 in the games we ran), renewables become the cheaper source (since $c_{\text{auc}} + c_c$ is greater than c_r) and so are used first. (See the example on the handout in Appendix A). Finally, profit (or loss) for each player is calculated for that year and the game continues to the next year. Demand fulfillment and profit determination can be described formulaically as follows:

Let:

l = *total load demand*,
x = *number of credits purchased*[2]
l_c = *coal power load to be generated*
l_r = *renewable power load to be generated*
S_c = *coal supply*
S_r = *renewable supply*

Then:

$$l_c = \min(x, l)$$
$$l_r = \min(l - l_c, S_r).$$

(Using the fact that $l_c \le l$ always holds so l_r will be non-negative.)
Or put another way:

$$\{l_c, l_r\} = \begin{cases} \{l, 0\} & \text{if } x \ge l \ (\textit{sufficient credits}) \\ \{x, \min(l - x, S_r)\} & \text{if } x < l \ (\textit{insufficient credits}) \end{cases}$$

Letting p = power sale price, c_c = coal power production cost, c_r = renewable power production cost, c_{auc} = credit auction cost, we can express profit = revenue − costs as:

$$\textit{Profit} = p \cdot (l_c + l_r) - (c_c l_c + c_r l_r + c_{\text{auc}} x)$$

Substituting and rearranging,

$$\textit{Profit} = \begin{cases} p \cdot l - c_c l - c_{\text{auc}} x & \text{if } x \ge l \\ p \cdot l - (c_c + c_{\text{auc}})x - c_r(l - x) & \text{if } x < l \le x + S_r \\ p(x + S_r) - (c_c + c_{\text{auc}})x - c_r S_r & \text{if } x < x + S_r < l \end{cases}$$

The three cases correspond to:

(1) having as many or more credits than total demand,

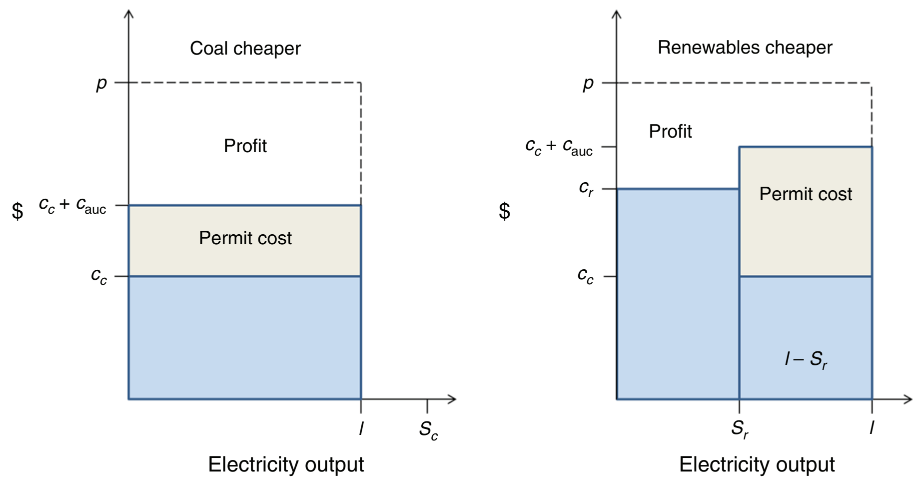
(2) total demand exceeds credits but there is enough renewable capacity to make up the difference, and

(3) demand exceeds combined total of credits and renewable capacity (i.e., missed sales).

Note that for the game to be unbiased towards any player, the variables and parameters in the profit function must be subject to the player's decision (as in the case of x), or identical for each player (as with all other parameters). In an extension of the game, players could purchase power plants with certain profiles, thus expanding the set of parameters in the profit function subject to a player's decision making. Figure 1 shows cases 1 and 2 in a depiction known in energy economics as a merit curve.

In the next section, we describe a few effective strategies. Later, in Section 5, we present examples of games played and some analysis of the performance and strategies of teams in those games.

Figure 1. Merit Curves Showing Two Cases of Sufficient Supply, in Which $l < S_c + S_r$



4. Strategies

What strategy should players use? To motivate this question, we provide students with some exercises in the carbon game handout (see Appendix A). We strongly encourage (but do not require) the students to complete the exercises before the game so that they will have a better grasp of how the game works and what strategies might be beneficial. We encourage the reader of this paper to do the same. We do not provide any strategies in the handout or otherwise, but if players perform the exercises and briefly analyze the results, a few simple strategies will emerge. The idea behind not requiring that the exercises be completed is that the game then becomes more representative of the real-world situation. To participate in a market, an entity would be required to know the rules of the market, but not to know the optimal strategy (if one exists).

The basic decision to be made by players in each round is how many credits to purchase. The information available to players consists of the specifications of their power plants (capacity, generation cost, etc.), estimated demand (based on "historical" data), available credits, and credit price. Instructor solutions to the handout (see Section 2 of the Teaching Note accompanying this article) offer guidance on strategy at an undergraduate level. The discussions of strategies in this section and in Section 1 of the Teaching Note go into more depth to allow for analysis of strategies used by players in actual games.

There are a few simple questions whose answers will determine decisions players make:

—Do we expect our competitors to play in a cooperative group-optimal manner or a selfish individually-optimal manner?

—At this point in the auction, which fuel is cheaper?

—Is there an overall shortage of available credits (i.e., if more credits were available, would we want to increase our bid quantity?)

—Are we going to factor in NVEs, in which asymmetric costs of under-supply and over-supply drive commitment decisions before a stochastic demand actualization?

We consider two strategies, i.e., one for the situation in which demand is fully known, the other for the situation in which there is uncertainty in demand. For both cases, we provide substrategies for selfish and cooperative play. The substrategies differ only when there is a credit shortage. All players can play individually-optimally without negatively impacting other players if there are enough credits to go around. But, under shortage conditions, for the market to clear, players need to reduce bid quantities from an individually-optimal amount. In this situation, auctions become analogous to a repeated, multiplayer version of the familiar "chicken" or "hawk-dove" game. One Nash equilibrium is for all players to anticipate symmetric play and bid for an equal share of the total credits available, thus ending the auction as quickly as possible and keeping the price low. Yet if one player is known to commit to continue to bid for a larger share (playing hawk to their dove), the best response of the opponents is to divide up the remaining licenses, again trying to end the auction as quickly as possible.

Therefore, with a multitude of possible pure-strategy Nash equilibria, and a difficulty in credible pre-commitment, we explore the inherent trade-off between selfless behavior (based on the symmetric or equal-shares equilibrium profile of strategies) and selfish behavior (based on other equilibrium profiles with more aggressive strategies) in the analysis of play in Section 5. This analysis gives an undergraduate-level introduction to the dynamics of repeated games with a heuristic measurement of the selfless/selfish trade-off within meaningful bounds. Though the game is simple enough for students to understand, the strategic behavior can be quite complex, and the overall exercise can open doors for students to explore deeper concepts through further reading or research. For example, instructors may introduce students to the concepts of a subgame-perfect equilibrium (in which strategies include contingency plans for non-equilibrium play by opponents) or signaling in repeated games, including how to avoid their negative effects on auctions, as described (for example) in Ausubel et al. (2006). Other avenues of further enquiry may include the use of "sniping" strategies in dynamic auctions as documented by Roth and Ockenfels (2002).

Full-Knowledge-of-Demand Situation

To describe group-optimal strategies, we first build a naïve approach that ignores the effect of cost asymmetry when facing uncertain demand (i.e., the NVE), before extending the model in the next subsection. This strategy represents selfish, non-newsvendor-aware behavior and can be easily derived as a simple piecewise function. A complete description is given in Section 1 of the Teaching Note. The essential idea is quite natural, though: When the effective price of the coal (including the auction price) is lower than that of renewables, the player should aim to satisfy all demand with coal; otherwise, coal should be used only to satisfy the excess demand that cannot be satisfied with renewables.

To account for a shortage of credits, two substrategies are possible. One is the selfish strategy just described. The second is an unselfish strategy in which the team bids for its equal share of the total credits. These substrategies are shown in Section 1 of the Teaching Note. Note that neither sub-strategy is truly "optimal" in isolation, but represents one's optimal strategy within a particular equilibrium. The group-optimal strategy forms one Nash equilibrium, i.e., if all select this strategy, then each is responding optimally to the decisions of others. Similarly, the selfish strategy is only truly optimal individually if the competitor can credibly threaten to commit to this strategy, with no fear of reprisal from opponents and no conflicting credible threats. Again, it is optimal in a particular equilibrium in which others accommodate this strategy, but other strategies may prove beneficial in response to irrational play by opponents, making subgame perfect equilibria difficult to characterize. This distinction can open a wide discussion of Nash equilibria and its properties among the instructor and students, with the simple context of the game making the concepts quite concrete and meaningful for the students through experiential learning.

Uncertainty-in-Demand Situation

In the previous description, we treated demand as deterministic, but in actual game implementations, the players are provided with a history of demand and told that the future is expected to look similar. The player would like to know the exact amount of the coming year's demand, but it is impossible to know. That said, is it preferable to err on the side of caution and overbid on credits (i.e., maybe have extras that are not used) or underbid (i.e., possibly come up short and be unable to use all coal power capacity)? This is similar to the newsvendor problem, and is an issue that can be incorporated into the discussions leading up to the game. Full development of this strategy and its attendant substrategies based on whether credit shortages are accounted for can be found in Section 1 of the Teaching Note.

5. Game Play Results and Assessment

Table 1 provides summary information on the games played by students in various courses. In this section, we discuss some general observations about the games played then provide quantitative measures to assess the quality of decision making along with the degree of cooperation (or lack thereof) exhibited by players. For a more detailed account of the games that were played, see Section 3 of the Teaching Note.

General Observations of Game Play

In each run of the game, the students were given the background information about the game including the rules and objective ahead of time. There was an optional set of exercises on a game handout that students could complete before class, and which would help them develop a strategy for play. We asked the students to take notes on their bid strategies during the game.

Each of our runs of the game began with expected demand in excess of players' coal supply, and a low auction price (e.g., \$1) so that coal started out as the cheaper fuel. After a "warm-up" first year with ample credits, the next few years exhibited a shortage of credits. In three of the five games, players' bid quantities began too high in the non-shortage first round, preventing the market from clearing. A new round would then begin with a higher auction price. The games were generally run with a few price values below $c_r - c_c$ (the transition point between cheaper fuels) and a few above (see Table 1 for the exact numbers of each). This provided the opportunity to gauge whether players understood this transition. We lowered the amount of available credits from year to year, increasing the pressure on players. In these shortage rounds, player bids were often too high, preventing the market from clearing in nine of ten years, leading to an average of four rounds in those years. Credit shortages in games played are summarized in Table 2.

We also applied a "News Flash" to the game about midway through that signaled to the players that there would be a significant reduction in expected

Table 1. Summary Information of Games Played

Game run	1	2	3	4	5
Course	Developing leaders in sustainability	Environmental security	Environmental security	Marine renewable energy seminar	Climate change seminar
Location	USCGA	USMA	USMA	USCGA	USCGA
Name	cga-1	usma-h	usma-i	cga-mre	cga-cc
Student audience	Mostly STEM majors	Environmental science majors	Environmental science majors	Various majors	Various majors
# of teams	6	6	6	5	6
Players per team	3–4	3–4	3–4	2–3	2–3
Total # of game years	5	4	5	4	3
Rounds per year					
min	2	2	1	1	1
avg	3.2	3.25	2.8	2	3
max	7	5	7	4	5
# of rounds in which:					
$c_{\text{auc}} < c_r - c_c$	6	6	6	3	3
$c_{\text{auc}} > c_r - c_c$	9	6	6	4	6

Notes. United States Coast Guard Academy (USCGA); United States Military Academy (USMA).

demand. This was also aimed at testing players' grasp of strategy.

Overall, there was a broad range in the quality of play exhibited by teams. Some teams figured out the basic strategy (not necessarily including newsvendor considerations) right away. In other cases, it took a few rounds for teams to figure it out. This was indicated in their game notes, with comments in early rounds such as "getting used to the game," and in some cases deducible from the quantification of their decisions, described more fully below. When strategy became clearer to the players, they were better able to express it in their comments. Some teams were unable to figure out a reasonable strategy to play. Among the teams that did appear to grasp the basic strategy, there was a range of behaviors along the self-interest to communal-interest spectrum. Teams frequently bid above their "share" of credits early on, leading to additional auction rounds. We quantify player behavior in the next section; a full accounting of it for the shortage case is provided in Table 15 in Section 4 of the Teaching Note. Summarizing those results in brief, we find that of the 23 total teams:

Table 2. Occurrences of Credit Shortages Within Each Game

	Year				
Game	1	2	3	4	5
cga-1	No (1,200)	Yes (800)	Yes (700)	No (700)	No (600)
usma-h	No (1,200)	Yes (800)	Yes (700)	No (700)	N/A
usma-i	No (1200)	Yes (800)	Yes (700)	No (700)	No (600)
cga-mre	No (1,000)	Yes (667)	Yes (500)	No (417)	N/A
cga-cc	No (1,200)	Yes (800)	Yes (700)	N/A	N/A

Notes. N/A indicates that the given game did not last into that year. Numbers in parentheses indicate amounts of available credits. Game 4 had five teams while the other games had six. The News flash occurs in between Year 3 and Year 4.

- Seven teams demonstrated usage of the basic strategy in each case;
- Nine teams demonstrated usage of the basic strategy when c_{auc} was less than or equal to \$10 (i.e., when coal was cheaper in total), but not when c_{auc} exceeded \$10;
- Seven teams did not demonstrate usage of the basic strategy in any case.

For teams to perform well (i.e., maximize profit), it was important that they consistently follow the full-knowledge-of-demand strategy (described in Section 4). This would ensure that the strategy would be used in the round determining payoff for that year, i.e., the last round.

Consistency of application of the strategy would also help smooth out the randomness inherent in results due to the variable demand. This is discussed in more detail later in Section 5.

Table 3 provides summary results of final profit and emissions by game. Teams making the most profit did not necessarily emit the least CO_2. Even though the game is fairly simple, we tried to mimic some truths of the energy market, one of those truths being that the most profitable energy companies are generally not the lowest CO_2 emitters. Many of the players (in real life power generation) are for-profit corporations, and their bottom line is profit. Despite the attention sustainability is getting, this fact is not likely to change significantly. So the challenge is to change the rules (in real life) somewhat, to get emissions lowered, and this is where governmental regulation comes in. Overall, the emissions are controlled, even though the players are trying to make money. The game results do show that it is possible to do well from the perspective of profit and emissions: Note that team 4 in game "cga-1," team 6 in game "usma-h," and team 1 in game "usma-i," each finish in the top two in both categories.

Table 3. Summary of Final Profit and Emissions by Game

	Profit		Emissions		
Game	Amount	Rank	Amount	Rank	Team #
cga-1	8,480	1	548	4	*5*
	8,430	2	414	1	*4*
	8,320	3	557	5	*1*
	8,080	4	470	2	*2*
	7,870	5	700	6	*3*
	6,690	6	483	3	*6*
usma-h	8,830	1	500	4	*5*
	8,510	2	445	2	*6*
	8,055	3	570	5	*4*
	8,012	4	605	6	*2*
	7,830	5	450	3	*1*
	7,190	6	230	1	*3*
usma-i	9,910	1	496	2	*1*
	9,530	2	645	4	*3*
	9,515	3	710	6	*6*
	9,337	4	601	3	*2*
	8,766	5	708	5	*4*
	5,000	6	0	1	*5*
cga-mre	8,805	1	465	5	*4*
	8,754	2	452	4	*3*
	8,736	3	448	3	*1*
	8,260	4	374	2	*5*
	5,320	5	181	1	*2*

Emissions mostly declined over the course of the games played as shown in Figure 2. As credits became scarce and their price rose, it became cheaper for teams to produce using renewables relative to coal. This is the intention of the policy. The lowered demand in the latter half of the game also contributed to this effect.

One important wrinkle in the game that could be smoothed out in future games is the issue of unmet demand. There is no requirement to meet demand, thus sometimes it went unmet. Table 4 shows the percentage of unmet demand per team. Note that this is

Figure 2. Emissions Over Time in Each Game

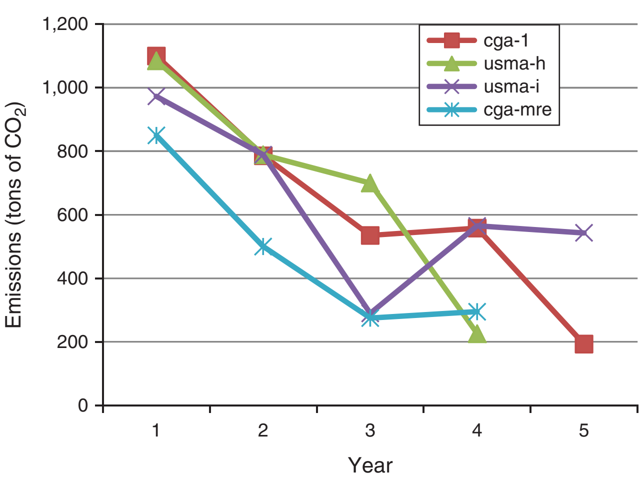


Table 4. Unmet Demand

	% of demand unmet per team by game and year				
	year (%)				
	1	2	3	4	5
cga-1	3.7	6.3	24.9	0.0	18.6
usma-h	0.5	6.9	14.6	8.3	
usma-i	10.2	10.4	41.3	2.8	5.6
cga-mre	8.4	17.9	0.8	11.7	

averaged over each team for the purpose of presenting the results concisely; in fact, some teams fully met demand in many years. It is assumed that an "outside entity" fills the demand. For instance, electricity could be imported through an interconnection with a foreign power grid. Note however that this outside power generation might create emissions in the process, an effect sometimes known as "leakage." In at least one example of game play, even had the unmet demand been met by a coal plant such as the one the teams in the game have, overall emissions would still have declined over the game (see Section 4 of the Teaching Note). Nevertheless, the game could be altered to penalize teams (perhaps emulating additional cost of "spot" markets, as in real life markets) for not meeting demand, or rewarding them if they do.

Among the teams using clearly suboptimal strategies, one stood out by consistently bidding for 0 credits. Apparently it was trying to be green and not run its coal plant at all. This is noble yet misguided, but it allowed for an interesting discussion of the pros and cons of taking such an idealistic stance. From a profit standpoint, it ended up last by a large margin. So in a competitive marketplace, barring some outside supply of cash to keep the business alive, this company would likely go out of business. In the universe of this game, going exclusively green is not a sustainable strategy. Even without considering the fate of that company, demand not met by this team could be met by an outside entity using a non-renewable fuel source, i.e., the "leakage" effect mentioned earlier. Furthermore, for a given year, the 0 bids caused the market to clear earlier than it otherwise would have. Revenue from these markets is typically directed into state clean energy projects. This would be diminished by these actions. So emissions would not be reduced, but at the same time funds for the program would be reduced, an important lesson in the complexity of interactions in regulated markets and unintended consequences.

Quantification of Decision Making Quality

In this section we provide a number of quantitative measures for assessing the quality and cooperation of players' decision making.

The measure used depends on the circumstances of the auction in the given year:

- If there is a credit shortage we use a measure denoted σ indicating how selfishly/cooperatively players played.
- If there is no credit shortage, we use a measure denoted θ indicating whether players played optimally.
- If the "newsvendor" situation applies, we attempt to measure players' awareness of it.

Selfishness Parameter, σ. We begin by focusing on game play when there is a shortage, and we initially neglect the newsvendor situation. Recall that under shortage conditions, strategies that are group-optimal and individual-optimal differ because the group-optimal player bids below or at the average number of credits available per player, while the individual optimal player may bid above.

We introduce a metric σ to denote the degree of selfishness or cooperation of a given player's decision. σ is defined as:

$$\sigma(q) = \frac{q_i - q}{q_i - q_g},$$

where q, q_i, and q_g represent the quantities of the player's bid, the individually optimal bid, and the group-optimal bid, respectively. So $\sigma = 0$ indicates a fully selfish decision, $\sigma = 1$ indicates a fully cooperative decision, and values in between represent degrees of selfishness/cooperation between the two extremes. Values of σ below zero or greater than one represent irrational bids (being irrationally selfish or overly generous, respectively).

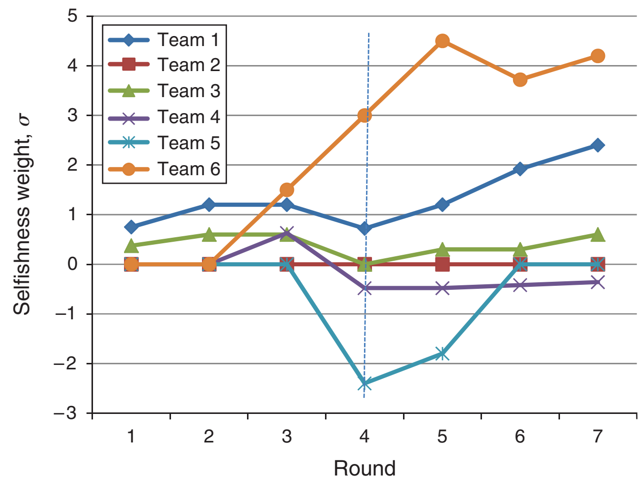
Example calculations of σ for an actual game are provided in Section 4 of the Teaching Note. We reproduce from that section a plot of σ values versus round number for a particular year in Figure 3.

The dashed vertical line in round four is drawn to indicate the changeover in cheaper fuel from coal to renewables. This plot provides quick visual insight into patterns in player strategy:

1. Teams 2 and 3 are the only teams to play rationally throughout.
2. Team 2 plays fully selfishly throughout.
3. The strategy of Team 3 ranges across the selfish to cooperative spectrum.
4. The jumps in σ for teams 5 and 6 in rounds 4 and 5 show that those teams erroneously did not alter bid strategy once the changeover in cheaper fuel occurred.

The payoff a team receives in a year depends on the team's decision in the last round of the auction (and indirectly all player decisions in prior rounds), and is also somewhat subject to chance due to the randomness of demand. Section 4 of the Teaching Note provides an example describing the linkage between decision making with respect to σ and payoff.

Figure 3. Selfishness Parameter vs. Round for "Year Two" of Game cga-1



Notes. Rational play is between 0 and 1, 0 being fully selfish, 1 being fully cooperative. Beyond is irrational.

To evaluate how well students understand the game and the strategies required to perform well in the long-run, payoff is not the right measure to use. It can incorrectly reward teams that were the beneficiary of a lucky guess in the final round, and/or of a lucky swing in the randomized demand. "Decision under uncertainty should be judged by the quality of the decision making, not by the quality of the consequences." (Hammond et al. 1999, p. 111) Put another way, judging the quality of a player using payoff based on the small number of rounds, is similar to judging the quality of a poker player after only a few hands. *In the long run, teams successfully playing the selfish rational strategy will have the highest payoff compared to less selfish competitors in the same game*.

Rather than using payoff from a particular game, reporting the percentage of times σ is between 0 and 1 (i.e., rational) for a team provides a condensed appraisal of team decision making independent of chance. For the example game we have been considering above, the results were as shown in Table 5.

Based on this measure, teams 2 and 3 appear to demonstrate a sound understanding of optimal strategy, even though, as noted above, their degrees of

Table 5. Percentage of Rational Plays by Team in Year One of Game "cga-1"

Team	% of rational plays (%)
1	33
2	100
3	100
4	50
5	67
6	33

Table 6. Percentage of Rational Plays by Team for Each Game

Game:	cga-1 (%)	cga-mre (%)	usma-h (%)	usma-i (%)
team				
1	33	75	43	90
2	100	50	71	20
3	100	100	29	50
4	56	75	57	90
5	78	50	100	0
6	44		100	20

selfishness/cooperation differ. The other teams have significant learning to do. If the game is to be used as a graded assignment, we recommend using % of rational plays as the primary measure. Table 6 gives an at-a-glance summary of the performances of all teams across all games. The 0% figure under game "usma-i" is for the "green team" that chose to avoid any purchase of credits. Full results of the selfishness parameter values by round for each game are given in Section 4 of the Teaching Note. A close inspection of those results supports the earlier claim that some teams exhibited correct strategy early on and throughout the game, others did so but only for the coal-is-cheaper situation, and a third set of teams did not play with a rationalizable strategy. There is little evidence of teams learning along the way other than some notes in some teams' strategy logs.

Note that these calculations ignore the NVE. A team might appear to be irrationally overbidding when in fact it is including a few extra credits as a buffer against the randomness of demand. This is addressed in the newsvendor section below.

Metric for the Non-Shortage Case, θ. In the absence of a credit shortage, the analysis is simpler. The group-optimal and individual-optimal bids are the same and are what players should bid. We define θ to represent the player's percent error in bidding:

$$\theta(q) = \frac{q - q_{opt}}{q_{opt}},$$

where q and q_{opt} represent the player's bid quantity and the optimal bid quantity, respectively.

Overall in the games we ran, shortages were common, so θ did not come into use often. Yet in those non-shortage cases for which σ does not apply, θ provides a simple measure to further illustrate player performance. Some sample results and discussion of θ values from actual games can be found in Section 4 of the Teaching Note.

Newsvendor Effects. The conditions under which the NVE becomes important for bidding are shown in Table 7.

Figure 4 illustrates when the newsvendor situation does and does not apply for the case in which coal is the cheaper fuel. When S_c and C/n are both less than $D_{\min}$, as in the left plot, then the bidding is capped below the area of uncertain demand, hence the newsvendor problem does not apply. On the other hand, when S_c and C/n exceed $D_{\min}$, bid amounts fall in the region of uncertain demand, and so the newsvendor problem applies.

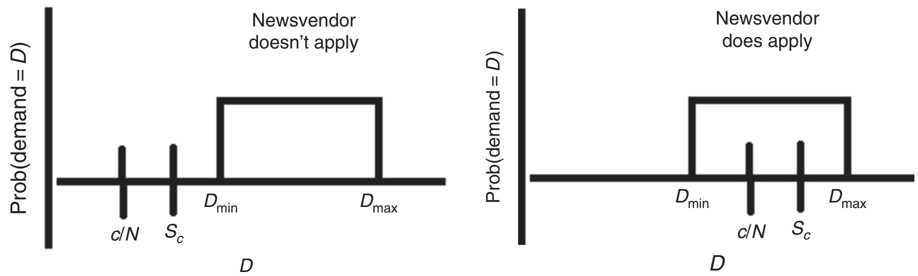
In games we ran, some teams bid in amounts above what the certain-demand strategy would indicate, suggesting they may have had awareness of the newsvendor situation. Strategy documentation tended to be fairly terse and no teams described this as part of their strategy. Yet it is conceivable that some teams had a notion about this effect and roughly implemented it by increasing bids a few credits.

Note that the σ calculations discussed above would need to be revised for any team thought to be NVE-aware. Without such a correction, those teams might appear to be irrationally over-bidding. In cases where

Table 7. Conditions for Which Newsvendor Effect Applies

Cheaper fuel	Strategy	NVE applies if:	Explanation
Coal	Group optimal	$D_{\min} < S_c$ and $D_{\min} < C/n$	Your lowest possible bid based on demand alone is less than your supply of coal and less than your share of the credits if evenly split.
	Individual optimal	$D_{\min} < S_c$	Ignoring the shortage, NVE applies when your lowest possible bid based on demand alone does not exceed your supply of coal only.
Renewables	Group optimal	$D_{\min} - S_r < S_c$ and $D_{\min} - S_r < C/n$	Your lowest possible bid based on demand alone *and adjusted by S_r since renewables are cheaper*, is less than your supply of coal and less than your share of the credits if evenly split.
	Individual optimal	$D_{\min} - S_r < S_c$	Ignoring the shortage, NVE applies when your lowest possible bid based on demand alone *and adjusted by S_r since renewables are cheaper* does not exceed your supply of coal only.

Figure 4. Display Illustrating When Newsvendor Situation Applies for the Case in Which Coal Is the Cheaper Fuel



the team's NVE estimates are quite rough, σ could actually worsen for the team with the correction. We illustrate this correction applied to an actual game in Section 4 of the Teaching Note.

6. Follow-Ups and Extensions

This section discusses some ways in which the carbon game could be modified, future directions for game analysis, additional educational programming centered around the game, and other follow-ups and extensions.

Game Modifications and Extensions

The carbon game can be altered in many ways. Some of the reasons to make changes include increasing realism, increasing the complexity of decision making and game play, and providing for greater post-game analysis opportunities by the researcher. We list several possible changes below and note that many of the changes address more than one of the reasons. Another potential source of ideas for changes is from the students who play the game; they are asked for suggestions on Question 3 of the pre-game handout. We emphasize that the game in its present form is simple with interesting behavior, something that might be lost if too many alterations are made.

Practical suggestions for game facilitation while maintaining the same overall structure include increasing the number of years per game, using more varied settings (e.g., auction prices, expected demand, credits offered), and avoiding situations with multiple equilibrium strategy profiles. Performing longer runs of the game, a two-day game with time for reflection in between, and/or using a wider variety of settings would result in more robust estimates of the player assessment metrics θ, σ, and of NVE behavior. It would also allow greater opportunity for players to learn and improve strategy over time. Multiple equilibrium profiles can arise when coal and renewables are equally priced. As currently set up, this occurs when the carbon credit auction price is 10. Without knowing what the player was thinking it is difficult to quantify and assess the quality of decision making in these cases, yet it is a simple matter to avoid such pricings.

In its present form, players have identical sets of power plants. One modification would have players starting with different numbers of plants with different types of fuel, fuel costs, and power prices. As discussed above, players are to be evaluated based on the quality of decision making, not based on the payoff they receive. Hence the evaluation should be possible given different set-ups of the players. Care would need to be taken to ensure that under the revised structure, the game still requires non-trivial decision making on the part of the players. Some of the strategy and analysis guidelines discussed earlier in the paper would need to be adjusted in light of these changes. For example, what advantages may or may not exist for players based on their resources?

Taking this concept a step further, players could start the game with cash (in equal or varied amounts) but no plants. Before the carbon market begins, there would be a sale of power plants of different types and capacities.[3] Fuel for plants requiring it (e.g., coal, oil, natural gas) could be something players need to purchase each year. In the current format, fuel costs are rolled into power production costs. In a modified game, these could be separated with the prices of the different fuels varying from year to year. Prices could reflect actual fluctuations in recent years or other realistic scenarios. Note that this could be used to demonstrate the point that renewable power costs are affected minimally or not at all by these costs. The idea of heterogeneous players is described below with respect to a simulation concept for game analysis.

Other changes aimed at increasing realism include using more realistic parameters (e.g., demand, renewable energy supply, fuel prices, auction prices, etc.) and market or auction structures, which can be readily found from sources such as the U.S. Energy Information Administration (EIA), ISO New England, RGGI and similar regional entities. Viewing renewable energy as stochastic is more realistic in light of the variability of sources such as wind and solar. Aside from fuel and carbon credit costs, players could be required to cover other costs such as labor, base-load operations, and penalty for unmet demand (discussed more fully below). Another change would allow players to trade credits between themselves, creating a "Cap-and-trade" game. The pricing structure and power supplies in the game were chosen to be simple yet led to strategies that may not be immediately obvious to the average undergraduate student. This structure could be modified to be more complex and then the results of game play and player behavior could be analyzed.

In its current form, the game has the single objective of maximizing profit. Other important objectives such as minimizing emissions[4] and meeting demand could be introduced that would complicate players' strategies. As discussed in Section 5, sometimes demand went unmet in the games that were played. What can occur in practice is the issue of "leakage" in which outside entities fill the demand without having the emissions restrictions. The game could be altered to monetarily penalize teams per MWy of unmet demand, or reward them for meeting demand. As a result, instead of simply optimizing bid strategy as described in this paper, players might have to bid for more credits than the original bid strategy suggests to meet demand.

Minimizing emissions would likely be at odds with meeting demand, or at least could drive down profit since renewables would need to replace coal power. In one sense, it is more consistent with the intent of the game to exclude the emissions objective since the point of the game is to demonstrate how the market reduces emissions by itself. Still, this is a modification that might be worth exploring.

Other Follow-Ups and Extensions

Directions for Game Analysis. Using runs of the game as presently structured, we have completed some basic analysis of game play. This type of work could be extended, being applied to the game as is, extensions of it or simulations of game play. We have in mind work that would be akin to studies surrounding the Beer Game (Sterman 1989). As to simulation, the game could be run using automated players who use various strategies across the spectrum of possibilities and/or strategies commonly used by human players.

Recently there has been much interest in studying problems in operations management in terms of the human behavior observed in the decision-making process. This has been extended to the newsvendor problem (see Schweitzer and Cachon 2000 and Bolton and Katok 2008 for examples). The carbon game provides another problem for investigation in this direction, one that is somewhat different than the traditional newsvendor problem due to its setting in the energy and emissions realm, its multiround auction structure, and its competitive multiplayer setting.

Another direction of inquiry could be from the perspective of *game theory*. Some starting points are indicated in Appendix C. For instance, a two-person game with shortage is described in which player strategies are characterized by σ values discretized along some range from 0 to 1. Because the auction may not clear depending on the bids, the payoff matrix can take on a nested structure. An extensive form game tree can be used for analysis. In addition, the presence of a Prisoner's Dilemma can be explored. The game theory connections provided by the carbon game may yield productive avenues of inquiry, but at a minimum, these connections allow students to learn about game theory through playing and analyzing the game.

Additional Educational Programming Centered Around the Game. The pre-game handout provided to students begins the learning surrounding the game. The game itself is designed to meet the learning objectives outlined in the Introduction. After the game we have typically conducted a post-game wrap-up in which we analyze player behavior, similar to the account provided in Section 5. We use the strategies documented in the paper and encourage players to discuss their experience playing the game. A more formal follow-up could be conducted. One emphasis could be on having students make greater connections between the game as played and the real world. As such, students can be asked to look at actual cap-and-trade programs, comparing situations from the game with their real-life counterparts. For example, what were the caps set at in the RGGI program? What kind of auction format was used and how did the auctions go? Who participated in the auctions? How were the funds that were raised used?

7. Conclusion

This paper presented a carbon emissions game that students can play in a classroom setting. The game fosters learning among the students in many areas. That is, students become acquainted with carbon markets, understand how policy can direct a market towards environmental benefit, discover how to mathematically derive strategies, and are exposed to basic game theory, auctions, and the newsvendor problem as they apply to the game. The game consists of multiple competitive auction rounds. As such it is engaging and fun for students. The learning opportunities for students can extend beyond what the game currently contains; we describe some possibilities in the "*Follow-ups and Extensions*" section of this paper. The benefits to teachers include the opportunity to teach material in an unconventional way through a game, to evaluate students in an unconventional way through the quality of their decision making, and the potential for mathematical and/or behavioral analysis of game play results.

Although the structure and rules are simple, the game leads to interesting results, some of which can be mathematically analyzed, others behaviorally. Players can conduct analysis to determine strategy, and the instructor can analyze games played. We have shown both in this paper, presenting strategies and analyzing games that were played. We introduced novel quantitative measures, such as the selfishness parameter σ, to evaluate the quality of game play. There are many other opportunities for analysis by student and teacher alike, several of which were described in the previous section.

We believe this game to be successful because of its meaningful subject matter and because of the balance it strikes between simplicity and complexity. As described earlier, it consists of various strategy elements that are not immediately obvious and that can be optimized using (undergraduate-level) mathematical analysis. Climate change and environmental protection are highly visible issues of concern to many. The game resembles a true carbon credit auction program. The students learn from the exercises they perform in playing the game, and that learning is relevant to their lives.

Acknowledgments
The authors thank Brian Forshaw for providing insights into the electric power sector and Marie Johnson for game data and suggestions based on runs of the game she conducted at the U.S. Military Academy at West Point. The authors also thank the area editor and anonymous referees for numerous valuable suggestions that helped improve this article.

Appendix A. Carbon Game Handout
See online Word document.

Appendix B. Game Facilitation Materials
See online Word document for player strategy sheets and profit calculation sheets.

Appendix C. Connection with Game Theory
Another direction of inquiry could be from the perspective of *game theory*. For instance, to begin simplistically, a two-person game with shortage can be considered. Player strategies can be characterized by their σ values. These values could be discretized along some range from 0 to 1. In the example below, we consider an auction round with credit price of \$3, 267 available credits, and strategies of $\sigma = 1, 0.75, 0.5, 0.25, 0$, and a clearing behavior we label $\sigma < 0$ representing a player adjusting her or his bid based on the other's to ensure the market will clear.[5] The game payoff matrix is shown in Table C.1. The payoffs are listed as row player followed by column player. The dashes indicate that the market does not clear, and so the game moves on to the next round. The row min and column max correspond to the row player. Because the max row min equals the min column max, a saddle point

Table C.2. Payoff Matrix for Round 1 of Simplified Game

	Column player's strategy	
Row player's strategy	$\sigma = 1$	Clearing ($\sigma < 0$)
$\sigma = 1$	*subgame*	3,900, 2,139
Clearing ($\sigma < 0$)	2,139, 3,900	3,270, 3,270

Table C.3. Payoff Matrix for Round 2 of Simplified Game

	Column player's strategy	
Row player's strategy	$\sigma = 1$	Clearing ($\sigma < 0$)
$\sigma = 1$	*subgame*	3,300, 1,938
Clearing ($\sigma < 0$)	1,938, 3,300	2,869, 2,869

Table C.4. Prisoner's Dilemma Payoff Matrix

	Non-cooperative	Cooperative
Non-cooperative	P, P	T, S
Cooperative	S, T	R, R

Notes. P = Punishment for both not cooperating. S = Sucker's payoff (player who is double-crossed). R = Reward for cooperation if both players cooperate. T = Temptation for double-crossing.

Table C.5. Payoff Matrix for 2-Round Game with One Decision

	Column player's strategy	
Row player's strategy	$\sigma = 1$	Clearing ($\sigma < 0$)
$\sigma = 1$	2,869, 2,869	3,900, 2,139
Clearing ($\sigma < 0$)	2,139, 3,900	3,270, 3,270

Table C.1. Game Payoff Matrix Example

	Column player's strategy						
Row player's strategy	$\sigma = 1$	$\sigma = 0.75$	$\sigma = 0.50$	$\sigma = 0.25$	$\sigma = 0$	Clearing ($\sigma < 0$)	row min
$\sigma = 1$	—	—	—	—	—	3,900, 2,139	0
$\sigma = 0.75$	—	—	—	—	—	3,784, 2,422	0
$\sigma = 0.50$	—	—	—	—	—	3,667, 2,704	0
$\sigma = 0.25$	—	—	—	—	—	3,551, 2,987	0
$\sigma = 0$	—	—	—	—	3,270, 3,270	3,270, 3,270	0
Clearing ($\sigma < 0$)	2,139, 3,900	2,422, 3784	2,704, 3667	2,987, 3,551	3,270, 3270	3,270, 3,270	2,139
Col. max	2,139	2,422	2,704	2,987	3,270	3,900	

Figure C.1. Extensive Form Game Tree

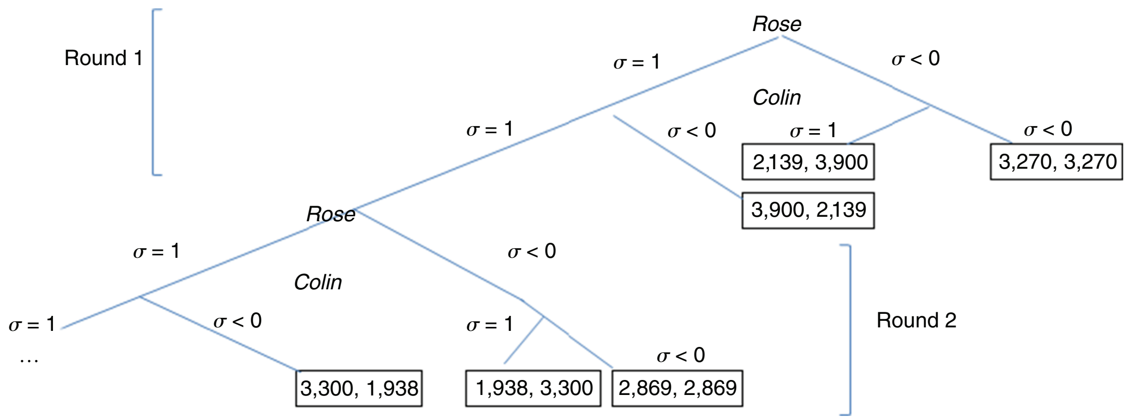


Table C.6. Payoff Matrix Corresponding to Game Tree

<table>
<tr><th colspan="2"></th><th colspan="4">Colin's 2-round strategies</th></tr>
<tr><th colspan="2">Rose's 2-round strategies</th><th>$\sigma = 1, \sigma = 1$</th><th>$\sigma = 1, \sigma < 0$</th><th>$\sigma < 0, \sigma = 1$</th><th>$\sigma < 0, \sigma < 0$</th></tr>
<tr><td>$\sigma = 1$</td><td>$\sigma = 1$</td><td>—</td><td>3,300, 1,938</td><td colspan="2" rowspan="2">3,900, 2,139</td></tr>
<tr><td>$\sigma = 1$</td><td>$\sigma < 0$</td><td>1,938, 3,300</td><td>2,869, 2,869</td></tr>
<tr><td>$\sigma < 0$</td><td>$\sigma = 1$</td><td colspan="2" rowspan="2">2,139, 3,900</td><td colspan="2" rowspan="2">3,270, 3,270</td></tr>
<tr><td>$\sigma < 0$</td><td>$\sigma < 0$</td></tr>
</table>

exists in this game as long as potential future rounds are ignored. Note that the game is neither zero-sum nor even constant-sum.

Consider the above game simplified by removing all but the extreme strategies: $\sigma = 1$ and $\sigma < 0$. For a first round with $c_{auc} = 3$, the payoffs are shown in Table C.2.

The upper left entry occurs when the market does not clear. The payoffs are deferred to the next round (denoted as "subgame" in the table) when they will take on one of the values 1,938, 2,869, 3,300 or be deferred yet again in case of a stalemate. In the next round (for which we set c_{auc} at 6), the payoffs are shown in Table C.3.

We can think of this payoff matrix as being nested in the upper left entry of the one from round one. We will come back to this point shortly. As structured here, the question of whether the game amounts to a Prisoner's Dilemma can be raised. For that to be the case, following the notation from Winston (2003), the requirement is that $T > R > P > S$ (see Table C.4).

Each of these holds except that the P value is not determined. If we assume that players will play cooperatively in the second round (e.g., out of frustration that the market did not clear), then the overall payoff matrix for this game is as shown in Table C.5. Since $P > S$ (i.e., the payoff for both playing non-cooperatively, 2,869, exceeds the payoff the double-crossed party receives, 2,139, when one plays cooperatively and the other does not) the game is now a Prisoner's Dilemma. Again, this version presupposed a fixed behavior in round 2, which is not necessarily a good assumption.

Because each year of the game can consist of a sequence of decisions (one per round), analysis using an extensive form game tree can be undertaken. We provide a rough sketch (see Figure C.1) in which "Rose" is the row player and "Colin" is the column player, and a payoff matrix (see Table C.6) from which the nested behavior of the game can be seen. The original round decisions form the outer blocks around the nested games in the upper left corner. As long as the market does not clear, additional subgames would be generated as the rounds progress, with payoffs nested in the upper left corner of the original payoff matrix.

In the games we ran, we observed varying degrees of concession when the market did not clear. Of the players who understood the game, many slowly lowered their bids to help facilitate market clearing, though in many cases, they did not reduce their bids enough for this to happen right away. Other plays yielded little or no ground. All payoffs diminish when bid quantities are not lowered enough to end the auction. While we did not conduct formal analyses of this situation for this paper, we did perform some back-of-the-envelope calculations for conditions common in the beginning rounds. These showed that for the selfish player, not only does payoff diminish as c_{auc} increases through stalemated rounds, but the selfish player's share of total profit diminishes as well. This is a factor to be weighed against the benefits accrued by selfish play if the market does clear. More fully exploring this trade-off could be another research direction.

The application of game theory to the carbon game may prove to be a fruitful topic for future research. It can also be a worthwhile exercise for students to learn about several fundamental elements of game theory including zero-sum games, the prisoner's dilemma, game trees, and nested games.

Endnotes

[1] RGGI uses the sealed-bid, uniform price auction. Bidders bid prices and quantities (possibly in multiple pairs). Credits are served in

descending order of bid price in amounts requested until credits are exhausted. Players are charged a uniform clearing price, which is typically the price of the lowest bid to win credits, though this price determination is slightly more complicated in the case of RGGI. Because the auction type we chose consists of multiple iterations, rather than the one round used in the uniform price auction, it leads to a more exciting, suspenseful, and educational game, and players experience the dynamics of demand revelation in iterative auctions, an added educational benefit.

[2] Players' credit bid quantities may not exceed their coal plant capacities. This capacity limit forbids certain spiteful objectives to keep superfluous (unusable) licenses away from competitors. This could be relaxed in a more complex version of the game permitting carbon allowance trading.

[3] A game with this type of set-up that was played at an office holiday party is described in Dikeman (2008).

[4] Question 4 on the handout provided to students before the game challenges the students to devise a method that would incorporate a reward for emissions minimization.

[5] In practice, this behavior would be impossible since the bids are sealed but this behavior is close to one in which a player tries to force the market to clear as quickly as possible by anticipating the selfish behavior of another player.

References

Allon G, Van Mieghem JA (2010) The Mexico-China sourcing game: Teaching global dual sourcing. *INFORMS Trans. Education* 10(3):105–112.

Ausubel L, Cramton P, Milgrom P (2006) The clock-proxy auction: A practical combinatorial auction design. Cramton P, Shoham Y, Steinberg R, eds. *Combinatorial Auctions* (MIT Press, Cambridge), 115–138.

Beliën J, Goossens D, Van Reeth D, De Boeck L (2011) Using mixed integer programming to win a cycling game. *INFORMS Trans. Education* 11(3):93–99.

Beliën J, Colpaert J, De Boeck L, Eyckmans J, Leirens W (2013) Teaching integer programming starting from an energy supply game. *INFORMS Trans. Education* 13(3):129–137.

Ben-Zvi T, Carton TC (2007) From rhetoric to reality: Business games as educational tools. *INFORMS Trans. Education* 8(1):10–18.

Bolton GE, Katok E (2008) learning by doing in the newsvendor problem: A laboratory investigation of the role of experience and feedback. *Manufacturing Service Oper. Management* 10(3):519–538.

Chan T (2013) Deal or no deal: A spreadsheet game to introduce decision making under uncertainty. *INFORMS Trans. Education* 14(1):53–60.

Chlond M (2006) An alien IP. *INFORMS Trans. Education* 7(1): 149–152.

Chlond M (2008) A tank attack puzzle. *INFORMS Trans. Education* 8(3):149–152.

Chlond M (2014) Three set pieces. *INFORMS Trans. Education* 14(2): 105–107.

Cholette S, Roeder T (2012) Embedding a sustainability module into quantitative business courses. *INFORMS Trans. Education* 13(1):44–56.

Cochran JJ (2005) Active learning for quantitative courses. Smith JC, Greenberg HJ, eds. *Tutorials in Operations Research* (INFORMS, Hanover, MD), 237–256.

Corrigan J (2011) The pollution game: A classroom game demonstrating the relative effectiveness of emissions taxes and tradable permits. *J. Econom. Education* 42(1):70–78.

D'Amours S, Rönnqvist M (2013) An educational game in collaborative logistics. *INFORMS Trans. Education* 13(2):102–113.

Dikeman N (2008) Carbon trading, the game. *Cleantech Blog* (December 26). Accessed October 19, 2014, http://www.cleantechblog.com/blog/2008/12/carbon-trading-game.html.

Ellerman AD (2000) *Markets for Clean Air: The U.S. Acid Rain Program* (Cambridge University Press, Cambridge, UK).

Gorman M (2012) Analytics, pedagogy and the pass the pigs game. *INFORMS Trans. Education* 13(1):57–64.

Griffin P (2007) The use of classroom games in management science and operations research. *INFORMS Trans. Education* 8(1):1–2.

Hammond JS, Keeney RL, Raiffa H (1999) *Smart Choices* (Harvard Business School Press, Boston).

Jopson B, Hornby L, Clark P (2015) China to announce carbon trading plans. *Financial Times* (September 25).

Lee LH, Padmanabhan V, Whang S (1997) The bullwhip effect in supply chains. *Sloan Management Rev.* (April 15), 93–102.

Meuffels WJM, den Hertog D (2010) Solving the battleship puzzle as an integer programming problem. *INFORMS Trans. Education* 10(3):156–162.

Roth AE, Ockenfels A (2002) Last minute bidding and the rules for ending second price auctions: Evidence from eBay and Amazon auctions on the internet. *Amer. Econom. Rev.* 92(4):1093–1103.

Sauré A, Puterman M (2014) The appointment scheduling game. *INFORMS Trans. Education* 14(2):73–85.

Schweitzer ME, Cachon GP (2000) Decision bias in the newsvendor problem with a known demand distribution: Experimental evidence. *Management Sci.* 46(3):404–420.

Singer S (2013) Northeast states tighten carbon emission rules for power plants. *The New London Day* (July 19).

Sniedovich M (2002) OR/MS games: A neglected educational resource. *INFORMS Trans. Education* 2(3):86–95.

Sterman JD (1989) Modeling managerial behavior: Misperception of feedback in a dynamic decision-making experiment. *Management Sci.* 35(3):321–339.

Talluri K (2009) The customer valuations game as a basis for teaching revenue management. *INFORMS Trans. Education* 9(3): 117–123.

Winston WL (2003) *Introduction to Probability Models: Operations Research*, Vol. II (Duxbury Press, Pacific Grove, CA).

Wood SC (2007) Online games to teach operations. *INFORMS Trans. Education* 8(1):3–9.