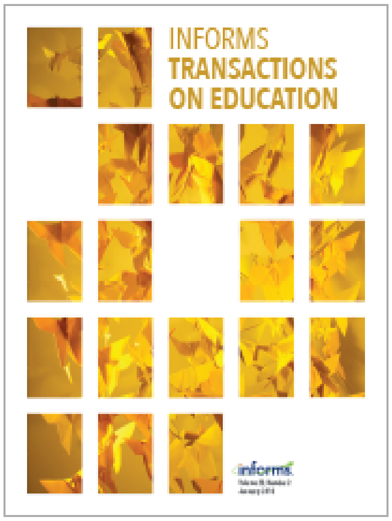

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Game

Introduction to Probability: Bison Hold'em Poker Game

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Abstract. Probability literacy is the foundation of business analytics. Although traditional textbook probability exercises may be convenient, students might have insufficient life experience to contextualize them. Aiming to stimulate students' interest in learning probability, this article presents an interactive pedagogical game based on Texas hold'em poker. In order to compete with some robots in the game, the students must earn in-game currency by answering a set of poker-related probability questions covering general conditional probability and Bayes' rule. Using this game and framework, we achieve the goal of introducing the basics of probability to students in a fun and immersive way. Student feedback has shown that this interactive pedagogical activity effectively and efficiently advances business analytics education. The activity is designed for undergraduate students in business analytics courses. The game is developed using Visual Basic for Applications for Microsoft Excel and can be easily implemented in class. Regardless of student's prior exposure to Texas hold'em, they can easily play along and enjoy the game while learning.

Supplemental Material: Supplemental materials are available at https://doi.org/10.1287/ited.2019.0222.

Keywords: spreadsheet game • experiential learning • probability

1. Introduction

A core component of analytics is the transformation of data into insights for better-informed decision making. Given the importance of analytics, organizations are increasingly committed to data-driven decision making, and educators are continuously innovating the teaching of analytics to advance students' understanding (Davenport and Patil 2012). The well-established topics for teaching analytics are, in the ascending order of competitive advantage, descriptive, predictive, and prescriptive analytics. A common thread across the topics is probability (Gorman and Klimberg 2014). Applications of probability are ubiquitous in business analytics; the more advanced the topic, the higher the probability literacy that is expected from students. Therefore, analytics education must build on students' probability knowledge, and the teaching of analytics is inseparable from the teaching of probability. However, students' willingness to learn and engage with the subject can be impacted by the motivating examples and learning activities. To this end, we propose an innovative activity for increasing students' interest in learning probability concepts and offering opportunities for them to explore the notion of probability in stimulating ways.

We aim to leverage some casino game in which students can easily play as part of their learning process to increase students' interest in learning probability. Of all the casino games, Texas hold'em poker has been ranked as one of the most popular forms of casino poker and is the game of choice within the annual World Series of Poker (Billings et al. 2003). The origin of the game can be dated back to the early 1900s (Texas State Legislature 2007). This popular game is played around the world (PokerStars.com 2019) and has inspired a wide variety of game-play strategy studies (Bowling et al. 2015, Brown and Sandholm 2019). Essentially, Texas hold'em is about computing outcome likelihoods based on given information. The rich probability content within Texas hold'em has made it a popular subject for studying or teaching decision-making uncertainty (Chen and Ankenman 2006, Smith et al. 2009, Barr and Dowie 2011, Sagan 2015, Ma 2016). As such, we choose to model our game closely after classic Texas hold'em poker.

Texas hold'em deals with drawing cards from a single deck of cards without replacement; hence, the events taking place during the game are dependent. Given the cards that were revealed on the table, players must mentally construct possible outcomes with probability and make (betting) decisions accordingly. That is, the whole game is about assessing conditional probability distributions of possible outcomes at all times. Furthermore, as new information becomes available during a game (e.g., revealed community cards, opponents' betting decisions or facial expressions), players can update their prior belief on their opponents' potential hands. Experienced players are able to pick up new information and continually update the (prior) probability estimates for possible outcomes, forming the basis of Bayes' rule. Therefore, we focus on leveraging the framework of Texas hold'em to teach conditional probability and Bayes' rule in our bison hold'em poker game.

The learning activity proposed in this paper is designed for undergraduate students with an entry-level probability understanding who tend to have little experience quantifying uncertain events. In our experience, immersing the students in an uncertainty-laden game with virtual money for betting provides a more engaging learning atmosphere than simply working on textbook questions. Besides the intended audience, instructors of graduate students or students with higher probability literacy can certainly benefit from the versatility of the game. Relative to conventional teaching approaches (e.g., lectures) on the same topics, the primary advantages of using our game are the engaging learning process (game play), relatable examples of probability theory (winning hands), and props-free demonstration (spreadsheet environment). Spending as little as 10 minutes of class time, instructors can use this flexible game to create some interesting learning experience for their students.

Teaching probability with the activity introduced in this paper has many benefits versus using real playing cards in class. For example, instructors using real cards must prepare several decks of cards and put students into groups, which makes managing the game play of all groups while keeping their attention challenging. Moreover, groups with students unfamiliar with the rules of Texas hold'em may require more guidance than is possible by a single instructor. Our spreadsheet-based game requires less preparation time and material expenses than its off-line counterparts. Additionally, our computerized game enforces the game rules to facilitate the game play. Although there are some games available on the internet providing similar functions, most online Texas hold'em games require payment, social media accounts, or agreements on end-user licenses before playing. Also, most online games connect real people as players in real time, so instructors have little control over the gameplay for teaching purposes. In comparison, our game is self-contained, granting instructors total control, and completely free. To the best of our knowledge, this is the first pedagogical tool allowing students to play a Texas hold'em poker game in a spreadsheet environment with rule-based robots and virtual betting. Texas hold'em games of other versions that operate in the spreadsheet environment tend to be for demonstration instead of simulating actual game play (Rochelle 2019).

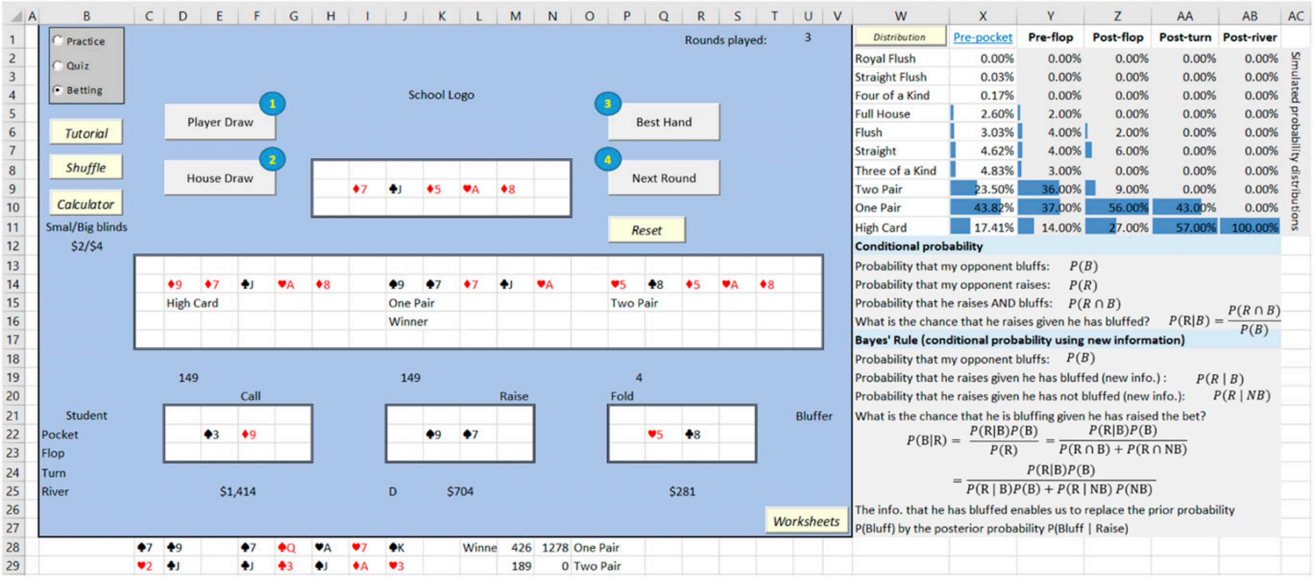
The remainder of the paper proceeds as follows. In Section 2, we review the relevant literature and differentiate our spreadsheet-based bison hold'em poker game from other spreadsheet-based games teaching probability. In Section 3, we explain the interface of our game. In Section 4, we recommend ways to implement the game. In Section 5, we report the student evaluations and assess the game effectiveness. In Section 6, we conclude the paper.

2. Literature Review

In the literature related to methods for activity-based learning for probability, there exist many creative approaches. For example, Bailey and Fry (2010) used an intriguing news story to motivate students to structurally analyze unstructured events, reinforcing the students' cognitive capability for assessing and decomposing probabilities of uncertain events. McLaren and McLaren (2014) posed rather philosophical questions to the students to highlight how the same verbal probabilistic expressions can be conceived subjectively, especially when the students come from different cultures. This learning exercise exposed the students to the concept of probability without the typical formalism of probability theory. These studies validated the importance of using real-life examples for facilitating students' learning.

Using spreadsheets for teaching management knowledge for undergraduate business students has been common in recent years (Bell 2000, Teich et al. 2005, Jaureguiberry and Tappata 2015). Bickel (2004) used baseball to not only teach probability, but also to introduce Markov processes to undergraduates and graduates in technical and nontechnical disciplines. The author understood that baseball might not catch everyone's attention, especially for students from countries where baseball is not a significant sport. Nonetheless, years of teaching with the real-life example proved the approach to be successful. Chan (2013) used a spreadsheet-based version of the popular game show *Deal or No Deal* to motivate the learning and discussion of the topics of expected value and utility function. Chen and Bailey (2017) developed an interactive game reducing a reverse auction to a spreadsheet model, allowing students to experientially learn about reverse auctions and how they work in practice. The spreadsheet game was shown

Figure 1. Game Interface Screenshot



to actively keep students engaged. We draw on the advantages of these teaching approaches in creating our unique spreadsheet game for activity-based learning.

With respect to research on poker games, Reiley et al. (2008) proposed simplified Texas hold'em poker as an instructional classroom game to introduce player behaviors, game theory, and Bayes' rule. Although similar in spirit to our game and approach, the authors used only four kings and four queens for the game, and their activity could only engage one student at a time. Additionally, the authors bet real money in the classroom, which could create a host of concerns. In contrast, our game presents several unique features. First, the Visual Basic for Applications (VBA) code ensures our game can proceed as expected even if the users/students are not familiar with the game rules. Second, the rule-based robots in our game are capable of strategically playing against student players, creating a more interactive environment. Third, the students may choose to play against one or two robots, significantly increasing the uncertainty of the outcome. Fourth, the students would be able to bet and win reward points, eliminating any unintended consequences of using real money. Fifth, all the game results would be noted for the instructors or students to review later on. Finally, we incorporate probability quizzes into the game play, seamlessly combining playing with learning.

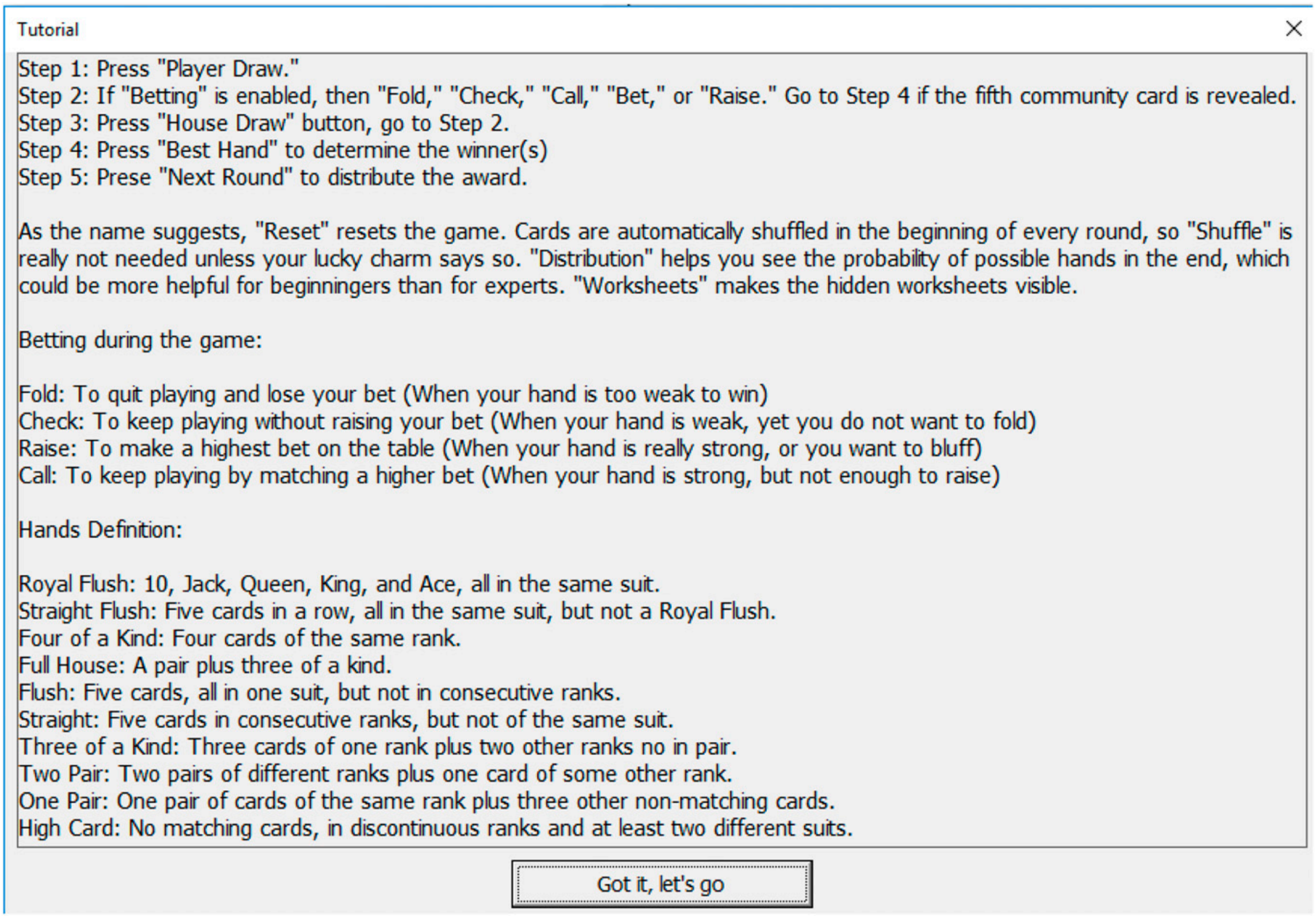
3. Game Interface

Our bison hold'em poker game can operate in recent versions of Excel (e.g., 2007 or later). At the beginning of every game, each player gets two random pocket cards that shall not be exposed to the opponents. Five community cards are successively revealed in three stages: the flop, turn, and river. The flop refers to the first three community cards; the turn refers to the fourth community card; the river refers to the last community card. Betting takes place prior to the revealing of the flop, turn, or river. The betting sequence proceeds clockwise (viewing the table from above). Before the first three community cards, that is, flop, are revealed, players can check, call, raise, or fold, depending on where the player sits relative to the player holding the dealer button. The player holding the dealer button is always the last player to take actions and, hence, can collect the most information from the opponents. At the conclusion of each game, the dealer button moves clockwise to the next player. Note that the purpose of the dealer button is to ensure that every player has a fair chance to be the last one betting. Figure 1 shows the main game interface.

3.1. Virtual Table

Range B1:V27 is the virtual table on which the game takes place. The buttons in range B1:B10 are for changing game modes or providing more information at any time during the game. In particular, the three radio buttons enable one of the three modes of the game: *practice* (default), *quiz*, or *betting*. We recommend using practice mode in the beginning. Under practice mode, none of the quiz questions or the betting options appear, minimizing the distractions for students who want to familiarize themselves with the poker rules. Students should enter quiz mode after acquiring the necessary knowledge to answer quiz questions, which is explained in Section 4.4. Betting mode allows students to compete with the robots and is recommended to be used toward the end of class time. Students can freely switch between modes at any time. Relevant in betting mode, the stake placed for betting is displayed

Figure 2. Game Tutorial Screenshot



in D19, and the student's cumulative reward point is displayed in E25.

The light yellow buttons with italic text on the virtual table serve the following purposes. The *tutorial* button displays a window as shown in Figure 2 explaining the main game-play operations, actions to take for betting, and definition of winning hands in Texas hold'em. The *shuffle* button reshuffles the cards. In fact, the cards are coded to be automatically reshuffled prior to every new game. Thus, it is not necessary to press this button. Nonetheless, we give the students this option in case they feel compelled to do so. The *calculator* button activates the Windows calculator to facilitate probability calculations for the quiz questions, yet students can always use their own devices or another spreadsheet for the purpose. The *reset* button (P11: Q11) resets the game to its default setting. The *worksheet* button (W28:W29) allows students to hide/unhide the supporting worksheets housing necessary game operations.

The other four light gray buttons tagged with circled numbers are key to the game play. To get the game going, students have to click them in the following order:

① *Player draw* (D5:F6): This is the button to receive two (pocket) cards. This fail-safe button functions once. The players' pocket cards are displayed in E22: F22, and the robots' pocket cards are displayed in K22:L22 (and Q22:R22 if two robots).

② *House draw* (D8:F9): This is the button to prompt the computer dealer to sequentially reveal the community cards displayed in H8:N10. Students need to click this button three times to reveal all five community cards. This fail-safe button functions only after the player draw button was clicked.

③ *Best hand* (P5:R6): This is the button to reveal every player's best hands to determine the winner(s). This fail-safe button functions only after all community cards were revealed. The computer determines the best five-card combinations out of the seven cards (pocket and community cards), which are then displayed in D14:T14. The robots' cards remain hidden until this button is clicked.

④ *Next round* (P8:R9): This is the button to proceed to the next round. Besides distributing the reward points, the button notes the game-play records in row 27 onward for later analysis if needed. This fail-safe button functions only once.

3.2. Assisting Panels

Range W1:AB11 is the chart that displays some simulated probability distributions. Especially useful for those who are unfamiliar with Texas hold'em, this chart visualizes how likely the students are to get certain hands at the end of the game. The unconditional "prepocket" probabilities in range X2:X11 are calculated by enumerating all possible five-card combinations out of 52-choose-7 card combinations. A hyperlink in X1 provides some reference on the calculation. To plot each column of the chart in range Y2:AB11, students click the *distribution* button that triggers a simulation of 100 sets of random community cards given whatever pocket and community cards are revealed so far. This button functions before or after each time the house draw button is clicked. As more cards are revealed, students may find it interesting to observe how the distributions converge to one of the hands. The reason that we use the simulation for the chart is that computing the conditional probabilities is an NP-complete problem. Although the standard errors of the simulated probabilities are less than 0.0005, the chart should be for reference only.

Range W12:AB27 displays the formulas of the conditional probability and Bayes' rule for students to learn how to translate words to mathematical symbols as well as how the formulas can be applied with the probabilities. The more frequently the students have to look up this information when solving the quiz questions to earn reward points, the more familiar they will be with the basics of conditional probability and Bayes' rule.

4. Activity Implementation

We recommend the following ways of implementing the bison hold'em poker game in class. A computer laboratory may be unnecessary as long as students have access to Excel (e.g., on their own devices).

4.1. Rules

Because not all students played or even heard about Texas hold'em poker, we recommend starting the session with a brief overview of the rules of poker. Also, we recommend covering some basics of probability, such as complement events, union of events, intersection of events, or probability concepts related to playing cards for the students prior to the session to prepare them for the activity. The goal of this session is to stimulate the students' interest in playing the game. The instructor should expect to spend about 10 to 15 minutes in this session.

4.2. Tutorial

After the introduction, the instructor may demonstrate actual game play. Under practice mode, students can play several rounds of the game to familiarize themselves with the game interface and operations. In addition to the four main buttons labeled with circled numbers on screen, that is, "play draw," "house draw," "best hand," and "next round," the instructor should remind the students to use the "distribution" and "tutorial" buttons. Also, the instructor should clarify that the conditional probability distributions displayed at the top right corner of the screen are based on a simulation of 100 sets of random community cards. The goal of this session is to ensure the students know how to play the game and have a chance to ask questions they might have. The instructor should expect to spend about 5 to 10 minutes in this session.

4.3. Lecture

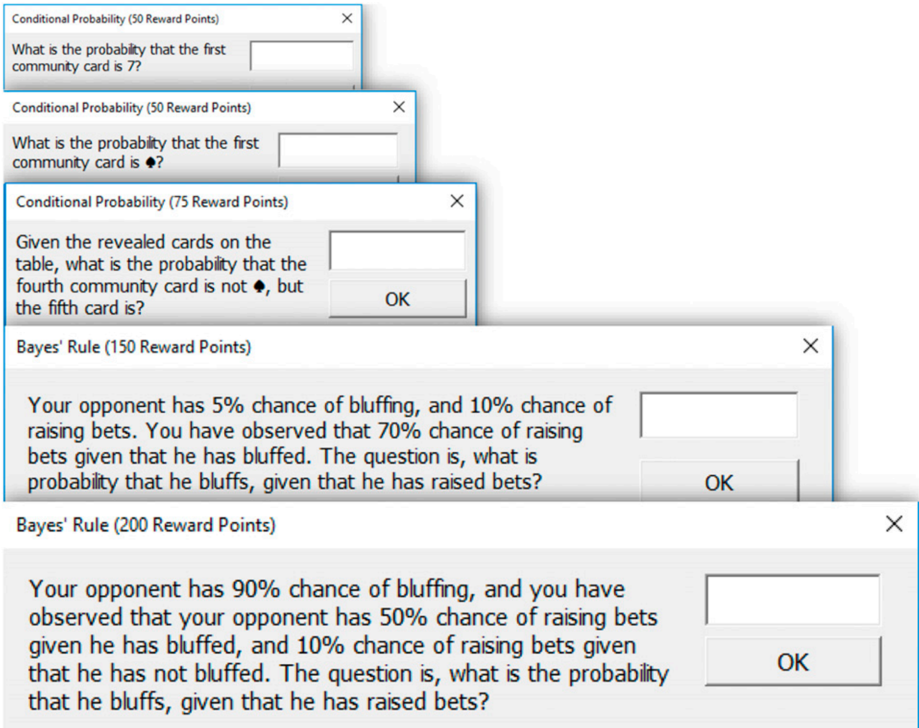
The instructor should explain that students must answer quiz questions to earn *reward points* before playing the betting mode of the game. We recommend using lecture materials, such as slides or handouts, to go over numerical examples related to conditional probability. To increase students' engagement, the instructor may announce that a prize will be awarded to students with the highest reward points. The goal of this session is to ensure the students understand the basic probability concepts and formulas needed to answer the quiz questions correctly to earn reward points. Covering Bayes' rule along with all the other information in one class could be challenging, so the instructor should use the instructor's judgment on whether to include Bayes' rule, especially if the students' probability literacy is low. The instructor should expect to spend between 15 (excluding Bayes' rule) and 30 minutes (including Bayes' rule) in this session. At the end of this session, the ideal outcome is that students would be interested in learning probability and applying the formula in order to earn reward points so that the instructor does not need to sell the concepts of probability to the students.

4.4. Game Play

After the lecture, the students should be ready to enter quiz mode of the game. Pedagogically, quiz mode is the most valuable and constructive aspect of this Excel game for it is instrumental to advance or assess students' probability knowledge. Once the "quiz" radio button is selected, quiz questions keep popping up as the game progresses. Figure 3 illustrates some of the questions that the students may see on the screen.

Templates of six conditional probability questions and two Bayes' rule questions are written in the VBA codes. The templates automatically generate quiz questions of different difficulty levels. As students proceed in the game, the quiz questions become increasingly challenging, but students accumulate more reward points by answering challenging questions than by answering easy questions. To prevent the

Figure 3. Screenshots of Several Quiz Questions, Which Pop up One at a Time



students from simply memorizing the answers, the templates use random numbers. Even if a student gives an incorrect answer, the students still get some reward points for trying. In our experience, many students raised their hands and asked for help in solving the quiz questions, which proves that the students have an incentive to learn. The students' objective in this session is to gain as many reward points as possible via answering quiz questions.

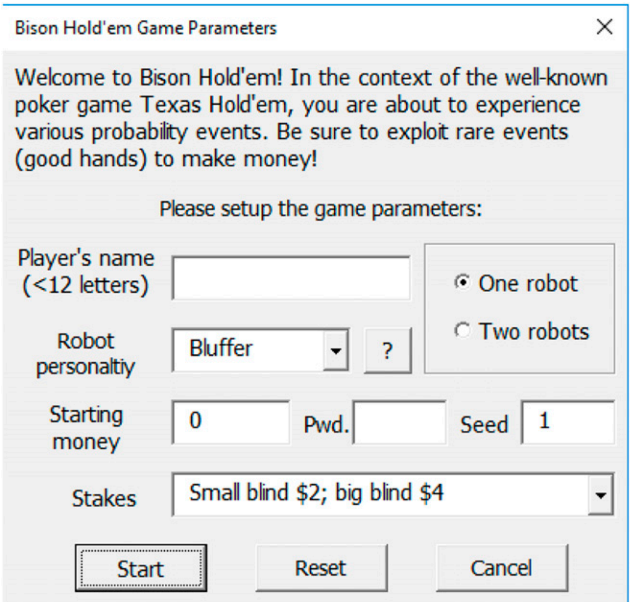
Quiz questions are the heart of this pedagogical game and embody the concepts of probability into relatable questions. Nonetheless, betting mode can be entertaining for it allows the students to use the reward points earned in quiz mode to compete against the robots. When entering betting mode, every student should reset the game to identical parameter settings and use the same seed value if their performances will be compared or evaluated (the seed number controls the random number generator in Excel). The seed value can be entered in the form as shown in Figure 4 that appears right after clicking the reset button.

The game can have one or two robots. Intuitively, the more robots in the game, the harder it is for the students to win. Students choose "bluffers," "rockers," "trickers," "optimists," or "mavericks" as the robots' personality. Figure 5 shows the overview of the personalities (available by clicking on the question mark button in Figure 4). Basically, the personalities determine how the robots adjust their maximum bets. For example, bluffers tend to fearlessly make big bets even if their hands are not that good, whereas the robots assuming the other personalities tend to bet according to what they actually have on hand. Please see Appendix A for a brief overview of how the personalities are coded behind the scene.

Nine hundred dollars is the default reward points assigned to the robots in the beginning, so the students may want to answer enough quiz questions to earn $900. (If needed, the instructor can enter "bayes" in the pwd. field in Figure 4 to set any starting money for all players.) The drop-down list "stakes" (in the bottom row in Figure 4) refers to how much blind bet to set for the game. Players taking turns make the required blind bets so that there are invested interests worth playing for in each game; otherwise, players can keep folding until desired pocket cards appear without facing any consequences, making the game boring. Figure 6 displays the two main message windows for the students to take actions in betting mode. The VBA code is written such that the students, whether they are familiar with the game rules or not, can adhere to the rules throughout the play.

The students can switch back and forth between quiz and betting modes in case they want to earn more reward points, achieving the goal of the game: motivating the learning of probability. Despite a warning message, the game will not stop even if the students run out of the reward points, so the students can play as long as they like. For a better learning experience,

Figure 4. Screenshot of the Input Form for Parameter Entry



we recommend the instructor put students into pairs so they may discuss the game rules, game-play strategy, and problem solving. We estimate this session may take at least 15 minutes. At the end of the class time, the instructor may ask the students to share their final reward points with the class to choose the winners for the prizes.

4.5. Limitations and Potentials

Optimal game-play strategy of Texas hold'em poker is beyond the scope of this paper. There are many websites, studies, and books about the game-play strategy. Chen and Ankenman (2006) scientifically analyzed approaches, such as game theory applicable to the winning strategies for Texas hold'em poker. Ma (2016) detailed optimal game-play strategy in a semester-long course and used some online poker platform to do a live demonstration. For interested instructors and students, we provide a list of websites in Appendix B offering empirical or theoretical winning strategies. Although we do not delve into betting and game-play strategies, our game could be a good tool for practice. To the best of our knowledge, our game is the first of its kind that provides a commercial-free environment for teaching and learning probability in Texas hold'em poker.

Another limitation of the game is the intelligence of the rule-based robots. Although the robots are capable of reacting to student players' actions, the robots do not learn from the student players' behavior or from the robots' own mistakes over time. The robots only play a fixed strategy formulated in the hidden worksheets or VBA codes. The robots demonstrate a human's bounded rationality because they make decisions based on the discrete choice methods with simulation (Train 2009). In the game, the robots' personalities are essentially differentiated by how they perceive the same conditional probability distributions with respect to their own utility functions. Brown and Sandholm (2019) have great information for those who would like to know more about the complexity of designing artificial intelligent robots for poker games.

Figure 5. Screenshot of the Available Robot Personalities

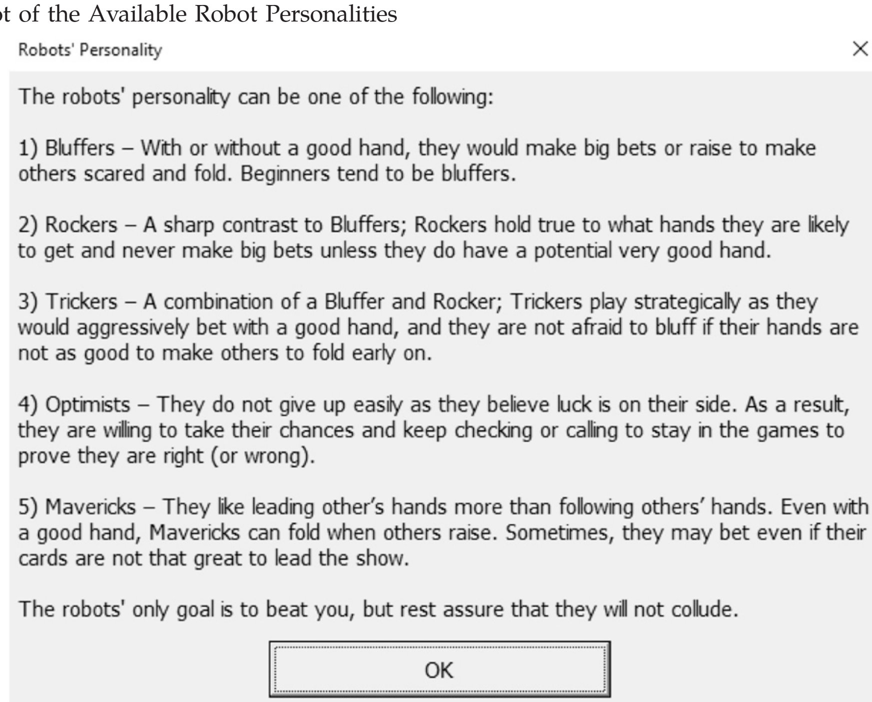
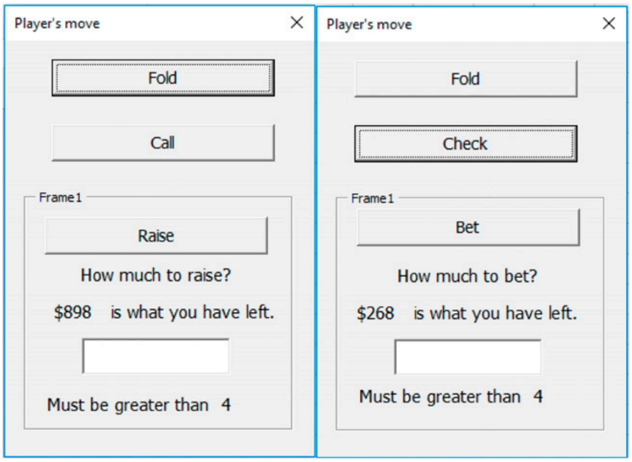


Figure 6. Screenshots of Two Main Message Boxes for the Students to Enter Betting Decisions



As for its potentials, the game can be played after instruction on conditional probability and Bayes' rule as an assessment tool. More broadly, the game can be used as an example for teaching spreadsheet modeling or VBA programming. Although interactive and seemingly complex, the deployed VBA codes are fairly basic. Other business analytics topics, such as utility theory and the flaw of averages could also be potential topics. Beyond teaching, the game could be helpful in research in various disciplines. For example, one can analyze individuals' risk attitudes by investigating betting behaviors in games. Using a simplified poker game and human subjects, Seale and Phelan (2010) found that players did not adopt strategies predicted by the equilibrium solution; instead, the players made considerable "errors": betting when they should have checked, checking when they should have betted, or calling when they should have folded. Research of this sort can take advantage of our game and simplify the research process.

5. Effectiveness Evaluation

To assess the bison hold'em poker game's effectiveness in increasing students' interest in learning probability, we surveyed the undergraduate students in a required first-year introduction to business analytics course. The students were informed that their participation is fully voluntary and no grades would be associated with the anonymous survey results. Figure 7 shows the histograms of the responses to the following survey questions.

1. *I could describe the basics of probability.*
2. *I could describe the basics of conditional probability.*
3. *The activity has helped me relate probability to real-life examples.*
4. *The activity has illustrated conditional probability in interesting ways.*
5. *The activity has increased my interest in knowing more about Bayes' rule.*
6. *I would recommend the activity to others for experientially learning probability.*

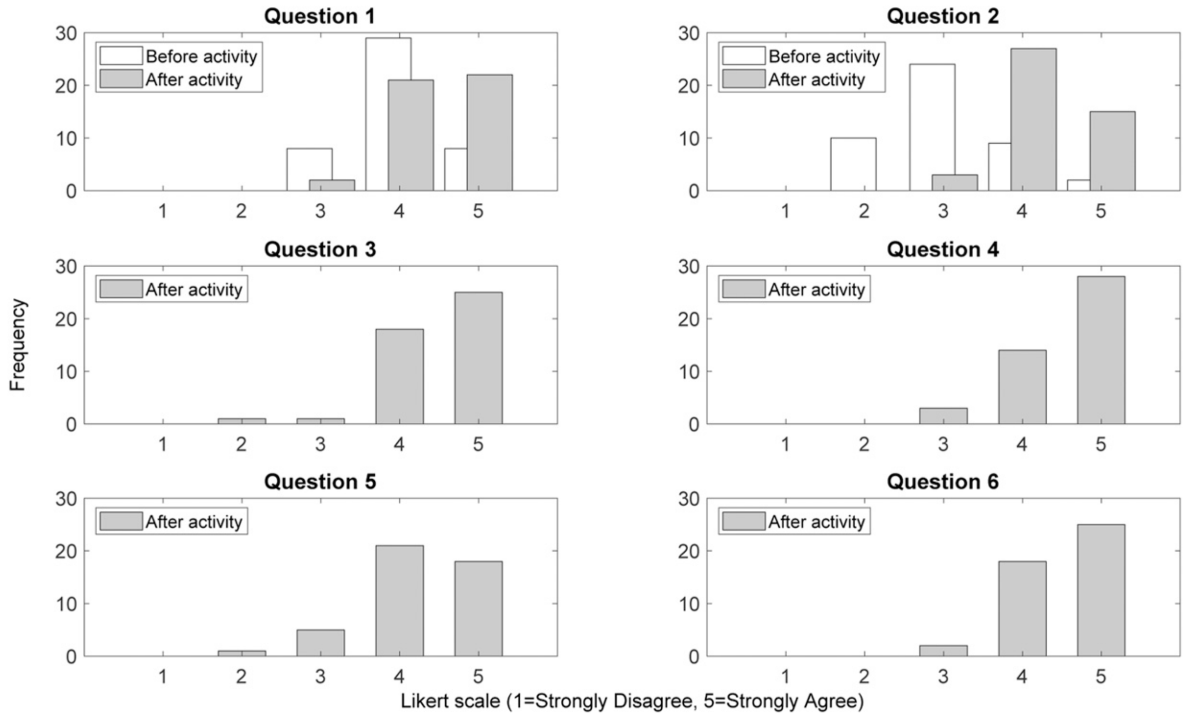
We performed a paired-sample t-test on the responses to the first two survey questions presented to the student both before and after the activity. Questions 3 through 6 were presented to the students only after the activity; to those responses, we performed a t-test for a hypothesized mean three ("undecided"). Please see Appendix C for details of the statistical tests we ran on the responses. Basically, every test shows significant evidence supporting that our game can effectively improve students' learning of probability.

We were concerned that the students' prior experience with Texas hold'em poker might affect the survey responses. Thus, we showed one opening question in the before-activity survey: "*I know how to play the poker game Texas hold'em.*" Grouping the survey responses by "agree" (N = 18) or "disagree" (N = 24), we found no change to the previously reported statistical significance. Therefore, the effectiveness of our game is independent of the students' prior knowledge of Texas hold'em poker. Our game is suitable for students with a high or low level of familiarity with the poker game. Nevertheless, we did observe that the students with prior experience with Texas hold'em poker were more quickly engaged as compared with the students who had little or no experience with Texas hold'em.

We would be bluffing if we claimed that playing the game allows the instructor to skip any regular lectures on the related topics, such as conditional probability or Bayes' rule. The activity is designed to complement rather than replace regular lectures. In fact, the instructors should expect to spend more time than typical on the same topics. The primary goal of the activity is to increase students' interest in learning probability, not to *teach* students probability per se. Nevertheless, we found that the bison hold'em poker game has strong potential for improving students' understanding of conditional probability and Bayes' rule.

Besides the quantitative feedback, we also solicited qualitative feedback from the students regarding their learning experience. A sample of the comments as shown indicate the game was well-received by the students: "Perfect level for a beginner who is not experienced with poker." "The game helped me understand probability better." "One of the best experiential learning I have had in the management school." "I enjoyed it." "Really cool idea!" "The game does a good job of conveying the materials in an interesting and engaging way." "The game was fun!" "Very good game to learn about probability." "Very fun game. I loved it because I love poker." "Fun way to learn more about probability and the Bayes' rule." "I thought it was a unique and interesting way to

Figure 7. Results of the Before and After Activity Surveys



learn about probability." "I think the game was really good. I learned a lot from the game."

In summary, we believe the bison hold'em game is suitable for students of various levels of understanding about probability: for undergraduate students before probability lectures, the game can be used for introduction and motivation; for graduate students or undergraduate students post probability lectures, the game can be used for assessing, diagnosing, or reinforcement. We believe the game introduced in this paper is a valuable contribution to the teaching of business analytics.

6. Conclusion

Offering textbook exercise questions to students to learn probability concepts can be convenient, but not every undergraduate student has sufficient life or work experience to contextualize the questions and grasp the concepts. In this paper, we present a spreadsheet game that can be used to motivate or reinforce students' understanding of the basic concepts of conditional probability and Bayes' rule. The game is designed for undergraduate students in business analytics or related disciplines. The associated activity is to allow students to gain a relatable context and key insights. Beyond teaching probability, our game has the potential to be useful for teaching or research in various disciplines. We believe this interactive pedagogical game would be a great addition to any of the instructors' own lecture materials and can fit in the class of any length of time.

Acknowledgments

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Appendix A: Robot Personalities

Here, we describe the fixed rules by which the robots abide. Honoring the open-access policy of INFORMS Transactions on Education, we set no password on the VBA codes, so anyone can access and modify them to better fit their needs.

Step 0: Ranking the 10 possible hands from the best to the worst, we assign a 100% value under royal flush ($h = 1$), a 90% value under straight flush ($h = 2$), …, 0% under one pair ($h = 9$), and −10% under high card ($h = 10$), where the hand $h \in [1, 10]$ and the underlying values are used by the robots to decide the maximum bet in terms of the percentage of the money on hand. Note that $h = 10$ or −10% means the robot will "fold," $h = 9$ or 0% means the robot will "check," and $h \leq 8$ or any positive percentages means the robot will "call" or "raise," depending on others' betting amounts.

Step 1: Given whatever cards the robots currently have on hand, we run 50 simulations and identify the best hand in each simulation to form a histogram with respect to the 10 hands. We then vary the robots' personalities by altering the histogram in the following ways. To any hands with zero frequency in the histogram

—"Bluffers" set the frequency of those hands to match the maximum frequency in the histogram, thereby inflating the maximum possible bet percentage.

—"Rockers" do not manipulate the histogram.

—"Trickers" increase the frequency of those hands by one, thereby moderately inflating the maximum possible bet percentage.

—"Optimists" set the frequency of those hands to match the maximum frequency in the histogram. Moreover, they truncate the histogram to exclude $h = 10$.

—"Mavericks" increase the frequency of those hands by 0.2, thereby slightly inflating the maximum possible bet percentage.

Step 2: Based on the (altered) histogram, we calculate the cumulative frequency distribution. Let Y denote the random value (percentage) under h. Using inverse transform sampling, we get y, a random realization of Y, for this current betting round.

Step 3: Generating two random numbers u_1 and u_2, both following $Unif[0,1]$, we then get the first round of maximum bet percentage $x_1 = y \cdot u_1$, the second round of maximum bet percentage $x_2 = x_1 \cdot u_2$, and the last round of maximum bet percentage $x_3 = y - x_1 - x_2$. (Under the rule of typical *limit* hold'em poker, each player is allowed to place up to three more bets, i.e., raise, reraise, and cap, after an initial bet during each betting round.)

Additionally, the robots in each betting round randomize the choice of "fold," "check," or "raise." They even respond to the student players' "raise" by increasing the chance of choosing "fold." Note that, although a larger number of simulations in step 1 results in a more representative histogram, the trade-off is that students could grow impatient while waiting to proceed during the game. Furthermore, the robots relying on limited information are prone to make mistakes as humans do, which arguably makes the game more interesting.

Appendix B: Texas Hold'em Websites

Websites with empirical or theoretical strategies for winning in Texas hold'em poker are as follows:

- http://www.natesholdem.com/flippin-switches.php
- http://www.cardfight.com/
- http://www.natesholdem.com/pre-flop-odds.php
- https://caniwin.com/
- https://www.cs.indiana.edu/~kapadia/nofoldem/index.html
- https://www.cardschat.com/poker-stove.php\#using.

Appendix C: Effectiveness Analyses

For question 1, results show that the students were better able to describe the basics of probability based upon the significant rating increase from before the activity ($M = 4.00$, $SD = 0.60$) to after the activity ($M = 4.44$, $SD = 0.59$), $t(44) = 4.52$, $p < 0.001$. For question 2, results show that the students were better able to describe the basics of conditional probability based upon the significant rating increase from before the activity ($M = 3.07$, $SD = 0.78$) to after the activity ($M = 4.27$, $SD = 0.58$), $t(44) = 10.24$, $p < 0.001$.

For question 3, results show that the activity helped the students relate probability to real-life examples ($M = 4.49$, $SD = 0.66$), $t(44) = 15.10$, $p < 0.001$. For question 4, results show that the students agreed the activity illustrated conditional probability in interesting ways ($M = 4.56$, $SD = 0.62$), $t(44) = 16.73$, $p < 0.001$. For question 5, results show that the activity increased the students' interests in knowing more about Bayes' rule ($M = 4.24$, $SD = 0.74$), $t(44) = 11.23$, $p < 0.001$. For question 6, results show that the students would recommend the activity to others for experientially learning probability ($M = 4.51$, $SD = 0.59$), $t(44) = 17.22$, $p < 0.001$.

Omitted in the main body of the paper for conciseness, the following additional survey questions were also presented to the students both before and after the activity:

7. I could describe the basics of Bayes' rule.
8. Peter has a deck of playing cards; each suit (i.e., spade, heart, diamond, club) has 13 cards: 1 (ace), 2, . . ., 10, 11 (jack), 12 (queen), and 13 (king). Peter draws a card. What is the probability that the card is a king, given that the suit of the card is heart?
9. Peter and Mary meet today to play poker. Data suggest there is in the past an 80% chance a player will lose the game given the player cannot focus. Peter has lost 20% of all the games played with Mary. When playing with Mary, there is a 10% chance Peter cannot focus. For today's game, what is the probability that Peter failed to focus, given he lost the game?

For question 7, we performed a paired-sample t-test on the responses and found that the students gained some confidence in describing the basics of Bayes' rule based upon a significant rating increase from before the activity ($M = 1.73$, $SD = 0.92$) to after the activity ($M = 3.89$, $SD = 0.78$), $t(43) = 13.86$, $p < 0.001$. For the last two questions, not every student answered them correctly. Nonetheless, of the students who answered both questions incorrectly before the activity, nearly 30% corrected their answers after the activity, which can be considered a significant improvement. Therefore, we believe the bison hold'em poker game could improve students' understanding of conditional probability and Bayes' rule.

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