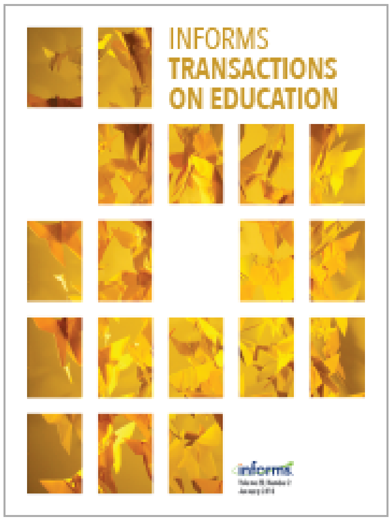

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Teaching Data Envelopment Analysis using Applichem—New Perspective on a Popular Operations Case

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Abstract

This paper reports an innovative use of a popular operations management case to illustrate the usefulness and application of a powerful analysis technique called Data Envelopment Analysis (DEA). DEA is often the only available tool that enables managers to evaluate the relative efficiencies of various business units in the presence of multiple performance measures. Applichem is a case study about a multinational chemical company, with six manufacturing plants located all over the world. The manufacturing plants' efficiencies are highly varied and in the presence of excessive capacity, management is having a difficult time determining which plants must be shut down. This case is often studied from an optimization perspective, with the objective of matching customer demands with plant capacities at the lowest possible cost. The case involves multiple measures of performance (e.g. labor cost, material cost, etc.), which makes it ideal for introducing and demonstrating DEA from a practical perspective. This paper details how such an analysis can be presented in a business classroom.

1. Introduction

Case studies are an integral part of the modern business curriculum and the fields of operations management and decision sciences are no exception. Cases help students learn to appreciate how they can take a situation from the business world, identify the inherent issues, formulate the appropriate decision problem, gather the relevant data, and obtain a solution. A typical case study also enables a discussion on how a solution from a decision model can be and should be adapted to suit the business situation.

Flaherty (1986) is one such case that was written and distributed by the Harvard Business School. This case details the situation in a multinational chemical company that manufactures and sells a product that is protected by patent and copyright restrictions. While it faces competition from a few companies, it has significant market share in the industry, and more importantly, it has a stable customer base. The case is centered on managing the company's network of manufacturing locations and the customer requirements in the presence of labor, material, and utility costs. These manufacturing plants, due to their location, age and working conditions (education, wages, etc.), have different efficiencies and the students are expected to determine the lowest cost strategy for matching customer demands and the manufacturing capacities. Inherent in this computation is deciding which manufacturing plants should be shut down. Exchange rate fluctuations, governmental regulations on manufacturing and transportation, the work environment (e.g., unionized or not, incentives, etc.), and cultural factors further complicate the issue and should be included in the discussion.

It is not surprising that, due to its emphasis on international issues and the opportunity it presents to highlight operations as a top-management issue, this case has been used extensively in graduate and undergraduate business classrooms. We use it in our operations elective focused on Supply Chain Management, Materials Planning, and Logistics. Students in this undergraduate course are mostly seniors and a few juniors and all of them possess excellent quantitative skills and are in interested in operations management careers. They have experience with optimization and simulation and associated spreadsheet-based tools for using them effectively. As such they are keenly interested in identifying tools that will help them make decisions in the business world which they will soon join. I know of at least twenty other top ranked business schools that use the Applichem case in their curriculum. Initially, I used this case in the manner outlined in the teaching note (Flaherty, 1992). Later, I discovered that this case is ideal for introducing and demonstrating Data Envelopment Analysis (DEA) to business students. DEA (see Charnes et al., 1994) is a powerful, linear programming based technique that can be used to compare numerous Decision Making Units (DMUs) and evaluate their relative efficiencies with respect to multiple measures of performance. I noticed that the varied performance measures (raw material cost, labor cost, energy cost, etc.) make it difficult to compare the six manufacturing plants in the Applichem case study head-to-head. DEA provided an effective tool for such a comparison and we have applied that technique to the case data. This paper details how the application, results, and managerial insights gained from DEA are used as a basis for classroom discussion.

The rest of this paper is organized as follows. Section 2 provides an expanded summary of the Applichem case along with some details on the suggestions provided in the teaching note. Sections 3 and 4 describe DEA via an example from the banking industry. Section 5 describes how DEA can be used to analyze the Applichem situation and provides detailed analyses, results, and managerial insights. Section 6 briefly discusses incorporating these results into the class room and also contains some concluding remarks.

2. Applichem Case

Applichem (Flaherty, 1986) is a multinational company in the specialty chemicals business. One of its principal products is called "Release-ease" and is used for cleaning stainless steel molds. This product was developed in response to demand from the plastic molding industry and has enjoyed tremendous success since its introduction in 1952. By 1982, the year this case is set in, the sales and profit growth had leveled off and there was no longer an active corporate program for improving the product or the manufacturing process. Applichem is currently manufacturing Release-ease at six locations: Gary (Indiana, USA), Frankfurt (Germany), Sunchem (Japan), Canada, Mexico, and Venezuela. The plants' names are the ones used in the original case. The product is demanded and sold all over the world and is shipped (the case details the current allocation/transportation strategy) from these manufacturing locations to all the customer regions. The manufacturing equipment at these plants is not of the same age and has not been maintained at a uniform standard. As a result, these plants demonstrate varying levels of efficiency in their production of Release-ease.

In 1982, it was apparent that the sales of Release-ease had leveled off and there was little chance of another surge in demand. In addition, there was excess manufacturing capacity as only 79.9 out of 100.7 million pounds of capacity was being utilized. The current allocation strategy appeared to be inefficient (e.g. the North America region was exporting 14.2 million pounds and importing 12.4 million pounds per year). It was necessary to analyze the system costs and determine an effective strategy for managing this network of manufacturing locations. Thus, a cross-plant productivity study was commenced and the data necessary for performing that analysis were compiled. Exhibits 2 to 6 in the case show production, labor, utility, and transportation costs. From these data, it becomes apparent that there is no single measure that comprehensively captures the efficiency of a plant. Some possible measures of performance are: (1) Cost per pound of Release-ease; (2) Labor cost per pound of Release-ease; and (3) Utility cost per pound of Release-ease.

The teaching note (Flaherty, 1992) (available to the faculty) associated with the case encourages the students and the instructor to perform a number of tasks in the process of analyzing the situation at Applichem. They include analyses that are as simple as comparison of ratios and ones that are complicated such as formulating and solving the cost minimization problem associated with production and distribution of "Release-Ease". In addition, it recommends that the students try to grasp the qualitative impact of various business conditions on the decisions that need to be made.

While these recommended analyses provide significant teaching material from the case and highlight the complexities involved in managing an internationally distributed operations environment, they fail to motivate the student to generate a relative efficiency rating of the plants. Such a rating is immensely useful to identify the most efficient plant, and also to determine how far away the inefficient ones are. For example, we will show that while Gary and Venezuela are almost identical in terms of cost per pound, Gary is much more efficient than Venezuela. In addition, if an inefficient plant is to be kept open and its efficiency is to be improved, it is not easy to identify the plant that it must try to emulate. We will show that Gary must emulate Frankfurt in curtailing the material costs, and in order to reduce the labor costs, it could look to the Mexico plant. Such identification is crucial for communicating best practices and technology transfer. In order to make these decisions about possible plant shutdowns and technology transfers, it is useful to employ Data Envelopment Analysis (DEA). The next section provides a brief introduction of DEA and the following section illustrates DEA with a banking example.

3. Data Envelopment Analysis - A Brief Introduction

Originally introduced by Farrell (1957) and enhanced by Charnes et al. (1978), (see Forsund and Sarafoglu (2000) for a history of its origins) Data Envelopment Analysis, sometimes called frontier analysis, is a linear programming based technique for evaluating relative efficiencies of DMUs. An organization is typically made of many smaller teams, groups, or units who often perform similar tasks with varying degrees of success. As an example, a bank typically has many branches of different sizes. It is often necessary to compare the performances of these branches, for example to. (i) evaluate management performance, (ii) determine whether some branches should be shut down, or (iii) identify best practices,. Because branches have different levels of inputs (number of employees, costs, etc.) and outputs (business transactions, personal transactions, loan activities, etc.), it is difficult to compare them. DEA was developed to overcome these difficulties. The technique has achieved tremendous success and has been widely applied in banking, health care, education, the military, and other fields (see Charnes et al., 1994). Its application in manufacturing and logistics has been limited and this endeavor is in part an attempt to change that. A first step in that process is to introduce DEA in Operations Management curricula. I searched for case studies suitable for that purpose but found only one: Modell's DEA (van Ryzin and Mahajan, 1998) which focuses on the performance of retail stores. I found no cases that applied DEA in a manufacturing/logistics environment. Adapting the Applichem case, which is already popular, was an easy way to correct that.

Data Envelopment Analysis works as follows. Consider *K* DMUs, each having *M* inputs and *N* outputs. Let X_i^k be the level of input *i* and Y_j^k the level of output *j* at DMU *k*. Without loss of generality, we assume that lower inputs and higher outputs are better. The relative efficiency of DMU *k*, denoted by w_k, can be computed by solving the following linear program.

$$\text{Maximize} \quad w_k = \sum_{j=1}^{N} \beta_j Y_j^k$$

Subject To:

$$\sum_{i=1}^{M} \alpha_i X_i^k = 1$$

$$\sum_{j=1}^{N} \beta_j Y_j^t - \sum_{i=1}^{M} \alpha_i X_i^t \le 0 \quad \forall t = 1,2,\ldots,K$$

$$\alpha_i, \beta_j \ge 0$$

The basic idea is to use the weights α and β to convert the multiple inputs into a single "virtual input" and the multiple outputs into a single "virtual output". The ratio of the virtual output to the virtual input determines the efficiency associated with the DMU. In addition, when the efficiency of a DMU is being computed, the weights are determined in such a way that its virtual input is set equal to 1. The resulting virtual output for that DMU determines its relative efficiency. It is worth noting that this efficiency rating is only relative and that a rating of 1 for a DMU does not indicate that there is no room for improvement. The second constraint ensures that no DMU has efficiency greater than 1 by restricting the virtual output to be no larger than the virtual input. Due to the presence of multiple measures of performance, each DMU would presumably like to choose the weights that put it in the best light and this linear programming formulation does just that. That is, when solving for DMU *k*, the weights chosen are the ones that result in that DMU getting the highest efficiency possible. If a DMU, even with such a self-centered selection of weights does not score a rating of 1, then it must be inefficient. To complete the analysis, *k* linear programs (one for each DMU) need to be solved and the relative efficiencies of the DMUs can be tabulated.

4. A Banking Example

The following example is adapted from Beasley (2000). We wish to analyze the performance of four bank branches. Each of which has two inputs (number of staff and operating cost) and two outputs (number of personal transactions and number of commercial transactions). Table 1 contains the data on these performance measures.

Table 1: The data for the banking example used to illustrate DEA.

Branch Name	Staff Size	Annual Cost (Millions of US $)	Personal Transactions (Thousands)	Business Transactions (Thousands)
Croydon	18	1.1	125	50
Dorking	16	0.8	44	20
Redhill	17	1.1	80	55
Reigate	11	0.4	23	12

It is easily noted that Croydon and Redhill have higher staff counts and costs while processing relatively larger number of personal and business transactions. On the other hand, Reigate has fewer employees, significantly lower annual cost, and processes many fewer transactions. The question that needs to be addressed is whether the lower number of transactions coming out of Reigate are justified based on their lower headcount and operating costs. The answer to that question is not immediately obvious and necessitates the use of a sophisticated technique such as DEA. The linear programming formulation to compute the relative efficiency of Reigate is:

$$\text{Maximize } 23\beta_1 + 12\beta_2$$

Subject To:

$$11\alpha_1 + 0.4\alpha_2 = 1$$

$$125\beta_1 + 50\beta_2 - 18\alpha_1 - 1.1\alpha_2 \le 0$$

$$44\beta_1 + 20\beta_2 - 16\alpha_1 - 0.8\alpha_2 \le 0$$

$$80\beta_1 + 55\beta_2 - 17\alpha_1 - 1.1\alpha_2 \le 0$$

$$23\beta_1 + 12\beta_2 - 11\alpha_1 - 0.4\alpha_2 \le 0$$

$$\alpha_1, \alpha_2, \beta_1, \beta_2 \ge 0$$

Solving this linear program reveals that Reigate has an efficiency rating of 62.65% and that Croydon and Redhill, the closest ones with relative efficiency ratings of 100% are its peer references. That is, in order to improve its efficiency, it must study the techniques and strategies being used at Croydon and Redhill, and try to implement them. If Reigate wishes to be rated at 100% efficiency, the DEA procedure can identify the closest point on the efficient frontier that it should move towards. DEA identifies the closest point on the efficient frontier. To reach that point, Reigate must reduce its staff count to 4 and annual cost to 0.25 millions. Any other point on the efficient frontier will also result in Reigate being labeled as efficient. Similar analysis for the other three locations reveals that Croydon and Redhill are rated at 100% efficiency and Dorking is rated at 53.57%. If Dorking wants to achieve a 100% rating, it must reduce its input levels to 7 and 0.43 respectively.

5. Using DEA at Applichem

As detailed in the previous section, DEA is a powerful analytical technique that can be used to compare the relative efficiencies of many decision making units in the presence of multiple measures of performance. The setting at Applichem fits very well into that category and it makes perfect sense to analyze the situation using DEA. The six factories, namely Gary, Frankfurt, Mexico, Venezuela, Canada, and Sunchem, are the decision making units. There are many ways (in terms of the combination of the inputs and outputs) in which DEA could be applied to the Applichem situation. The specific combination of inputs and outputs to use should be driven by the corporate strategy which will drive the operational strategy, as such guidance from and the support of the top management is key to the success of any DEA application. For illustrative purposes, we will pick one possible combination and discuss the results. The inputs we consider are annual material costs, annual labor costs, and the annual energy costs. The output is millions of pounds of releaseease produced. Using the data provided in exhibits 2, 3, 4, and 6 of the case, all the relevant annual data regarding these inputs and the output can be compiled. Table 2 presents these data in a summary fashion.

Table 2: The Applichem data that was used in the DEA.

	Inputs			Output
Plant	Labor Cost (Million US $)	Material Cost (Million US $)	Utility Cost (Million US $)	Production (Million Pounds)
Gary	4.34	16.30	0.21	14.0
Frankfurt	4.64	2.53	0.04	38.0
Mexico	0.40	4.77	0.11	17.2
Venezuela	0.70	29.14	0.42	4.1
Canada	2.02	14.41	0.27	2.6
Sunchem	1.65	6.15	0.15	4.0

Before illustrating how DEA can be used to analyze this data, it is worth understanding how one, who does not know DEA, may approach it. It would be tempting to compute a per-pound cost in the areas of labor, materials, and utility. The resulting ratios are given in the table 3.

Table 3: The Applichem data in terms of ratios.

Plant	Labor Cost ($ per pound)	Material Cost ($ per pound)	Utility Cost ($ per pound)	Total Cost ($ per pound)
Gary	0.31	1.03	0.019	1.359
Frankfurt	0.12	0.77	0.011	0.901
Mexico	0.02	0.95	0.012	0.982
Venezuela	0.17	1.16	0.027	1.357
Canada	0.78	0.97	0.015	1.765
Sunchem	0.41	1.54	0.038	1.988

From these ratios, it is easy to see that the Mexico plant dominates all others in the area of labor costs, while the Frankfurt dominates all others on the material cost and the utility cost fronts. For the other plants, there is no area in which they dominate all others. Thus there is no clear indication of superiority between the Mexico and Frankfurt plants and they are both efficient in their own way. The other four plants are easily dominated by the Mexico plant, the Frankfurt plant, or a combination of the two. Thus they are all considered to be inefficient. How inefficient are they? How would one rank them in terms of their inefficiency? These are questions that cannot be easily answered with some simple calculations and it is necessary to use a technique such as DEA. The following picture illustrates these per-pound costs in a graph. For ease of presentation, we represent only the labor and material costs.

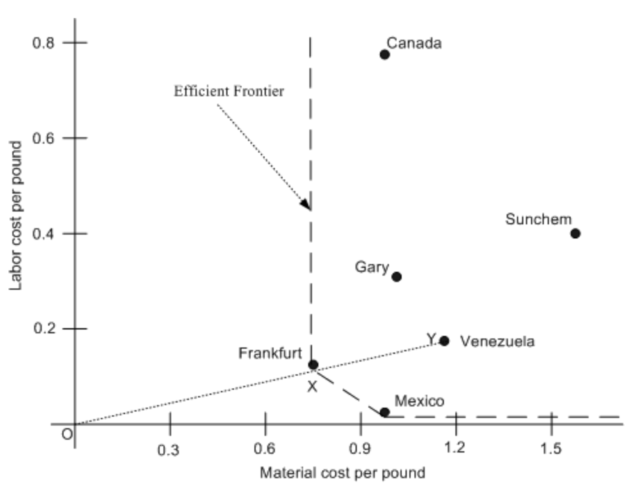

Figure 1: Plot of labor cost per pound versus material cost per pound.

Notice from Figure 1 that the Mexico plant, Frankfurt plant, or combinations of the two dominate all the other plants. Thus an efficient frontier can be drawn as shown in the picture. All the plants on the efficient frontier, in this case two, will be rated at 100% efficiency while the others will be given a lower relative efficiency rating. That does not mean that there is no room for improvement at the Mexico and Frankfurt plants. It only means that they are efficient relative to the other plants. The relative efficiency of a plant is determined by the distance from the efficient frontier. As an example, the relative efficiency of the Venezuela plant will be computed as the ratio of OX to OY where O is the origin of the graph; X is the point on the efficient frontier that is inline with the Venezuela plant which is represented by Y. It turns out that the relative efficiency of Venezuela plant is 67.1% while the Canada, Gary, and Sunchem plants are rated at 78.8%, 74.5%, and 49.9% respectively. To understand how DEA works, it is necessary to take a closer look at the resulting efficiency ratings and determine how they may have come about. The best candidates for such a study are the Gary and Venezuela plants. Although the Gary plan has a higher total cost per pound, its efficiency rating is higher than that of Venezuela. To understand this, one needs to consider the competitive advantages that these factories are striving for. It appears that the Venezuela plant competes on the labor cost while the Gary plant competes on material cost. However Venezuela labor costs are much higher than the Mexico (the leader on that dimension) labor costs where as the difference in the material costs between Frankfurt (the leader in that dimension) is relatively lower. As a result it will take a smaller change for the Gary plant to reach the efficient frontier when compared to Venezuela. Thus the higher efficiency rating for the Gary plant.

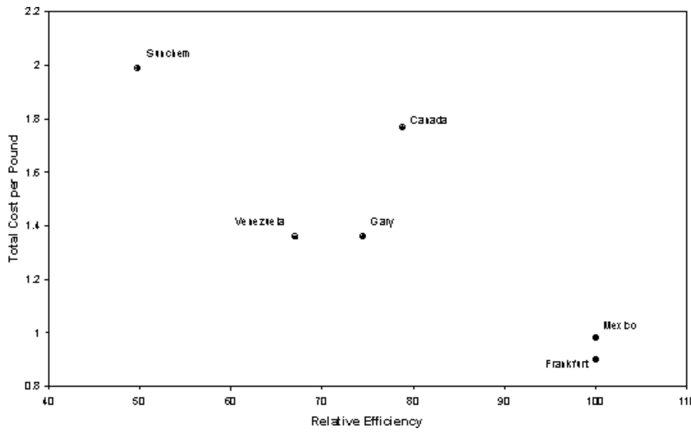

Figure 2: Plot of the total production cost per pound versus relative efficiency at the various plants.

It is interesting to compare the results from the traditional (total cost per pound) analysis done above and the efficiency ratings computed by DEA, as illustrated in Figure 2. Notice that while the total cost per pound was almost indistinguishable (1.359 versus 1.357) between Gary and Venezuela, DEA indicated that Gary was more efficient than Venezuela (74.5% versus 67.1%). Similarly, on the cost per pound basis Canada appears to place itself in 5 th place whereas based on the relative efficiencies, it is third on the list. Sunchem is placed at the bottom of the list in both the analyses.

When there are only two inputs and a single output, as was the case above, one can use ratios and a graph to manually identify the efficient frontier and compute the relative efficiencies of plant that are away from it. In the presence of several inputs and many outputs, such a manual computation is not possible. We need to use a systematic, efficient mechanism such as Data Envelopment Analysis. Consider the situation when we try to incorporate Utility costs and the yields (ratio of actual pounds of active ingredient to theoretical pounds of active ingredient) into the analysis. While the raw material costs included costs associated with four ingredients, one of those four ingredients was considered the most important and was labeled the active ingredient. It is conceivable that the yield associated with the active ingredient could be a key indicator of performance. So we decided to include it in the analysis. With these many inputs and outputs, it is not possible to draw a simple graph such as the one above. In this case, we can use commercially available software (see Barr (2004) for a detailed analysis of DEA tools, their capabilities, and costs) to compute the relative efficiencies. It turns out that Canada, Venezuela, Mexico, and Frankfurt are now considered to be efficient. Gary and Sunchem are rated at the efficiencies of 76.9% and 72.6% respectively.

In the Applichem case, one of the main complaints from the Gary plant management was that this cross-plant comparative study was being performed at a time when the exchange rates are not in its favor. In order to evaluate the validity of this concern, we performed an analysis in which the labor costs were computed at the exchange rates in 1977. Since the Peso had experienced significant devaluation in the period 1977-82, the labor costs, in this analysis, for the Mexico plant were significantly (more than 4 times) higher. The labor costs at Canada and Frankfurt plants went up a little bit, while the others remained almost unchanged. The resulting efficiencies for the Gary and Sunchem plants were 76.9% and 73.7% respectively, while the other four maintained their 100% efficiency. Thus it appears that exchange rates by themselves do not explain the inefficiencies at the Gary plant.

Based on this analysis, it appears that the Sunchem plant is the most inefficient plant and could be an ideal candidate for closing down. All the other plants, while being inefficient, are not so far from the efficient frontier. They can be improved and made to move toward the efficient frontier by evaluating the practices at the efficient plants and possibly implementing them in-house. The DEA analysis gives the target input levels (assuming that the outputs remain same) that must be achieved in order for the Gary plant (or any other inefficient plant) wants to achieve 100% efficiency. The labor, material, and utility costs must be brought down to 1.71, 10.74, and 0.16 respectively. The levels to which these measures must be reduced in order to be labeled efficient is one of the most useful outputs of a DEA analysis. In order to reduce the material costs, the management at Gary may wish to study the procedures implemented at Frankfurt. Similarly, in order to reduce the labor costs, they could look at the processes at the Mexico plant. While, it is possible that the lower Mexican wages may not be implementable at the Gary plant, there might be some other techniques that could help them on this issue.

6. Use in the Classroom

In this section, we identify three possible classroom settings (graduate or senior level business course, graduate or senior level Operations Research or Industrial Engineering course, Doctoral level seminar course) and give suggestions on its use. We have had much success in presenting this analysis in a senior level undergraduate business course. While we have not directly used the approaches described for the other settings, discussions with colleagues lead us to believe the proposed strategies will be effective.

6.1. Business Course

In a typical business classroom in which we have the most experience, the students are more interested in general managerial or strategic insights and less focused on technical detail. Keeping that in mind, we recommend that the discussion should be less mathematical. We usually present this case and analysis in a 75 minute session. After a 15-20 minute discussion of the case environment (business, current strategy, competitors, etc.), the students could be led to understand the necessity for evaluating the efficiencies of the various plants. When such a comparison is attempted, quickly students realize that simple techniques such as ratios and two-dimensional plots are inadequate or misleading. They will be eager to look for a more sophisticated technique that can address the complexities associated with multiple inputs and multiple outputs. That would be a good time to introduce DEA, probably with an example, and demonstrate a commercially available tool (we use DEA Frontier (Zhu, 2002)). Then the Applichem data could be input into the software and the results presented to the class. This could be followed by a discussion of other analyses that could be done, the reliability of the efficiency ratings generated, and the plants that are prime candidates for closing down. Such a presentation in the class room is aimed towards informing the students of the availability of DEA. It certainly will not result in the students becoming immediate users of it, but will enable them to consider using it when the right situation arises.

6.2. Operations Research or Industrial Engineering

Graduate and senior level students in Operations Research or Industrial Engineering or more technically oriented curricula should be in a position to formulate and solve linear programs. For such students, it is possible to provide more details on DEA procedures and how they can be implemented. These students and their courses are more geared towards the use and application of these techniques and there is less interest in the business aspects of the case. Thus, after the students have had a chance to read the case and understand DEA, they could be assigned to apply DEA to the Applichem data without formally introducing them to commercial DEA software. Thus, they would be responsible for formulating the necessary linear programs and solving them using generic mathematical program solvers such as CPLEX, GAMS, XA, etc. They could also be urged to tabulate the sensitivity results from the linear programs and study how they could be used to identify target levels, the efficient frontier, and other DEA-specific information.

6.3. Doctoral Seminar

The doctoral seminar students will not only be interested in the state of the art in DEA, but will be eager to extend it. Thus the use of this case in this setting should be different. In addition to applying DEA to the specific Applichem data, they could be asked to extend this analysis in different directions. They could be directed to define different measures of relative efficiency and interpret the usefulness (or lack there of) of the results. They could also look at the volatility of the efficiency ratings to changing data, or possible errors in the data, and identify a strategy for determining statistically valid ranges for these efficiencies or computing ratings that are more "robust". Since the use of DEA necessitates solving a sequence of linear programs, the computational requirements could be quite significant. However, since these linear programs are all somewhat related, it is possible to perform this task more efficiently by identifying mechanisms (e.g., row and column manipulations in the simplex tableau) by which the solution for a DMU can be used to quickly solve the linear program associated with another DMU. The doctoral students could be urged to identify such "speed-up" techniques, implement them, and tabulate their effectiveness.

6.4. Conclusions

In this paper, we have demonstrated how an instructor can introduce DEA into the classroom by using it to analyze data from Applichem, a popular case in operations management. Traditional analysis of this case has focused on formulating an appropriate mathematical program for determining the optimal strategy for matching capacities and demands. We have realized that the resulting solutions are rather non-intuitive and provide no means of guidance if the poorly performing plants were to focus on improvement. Data Envelopment Analysis, with its ability to provide relative efficiency ratings, peer reference units, and target input/output levels provides the students with a much better picture of the relative performance of the various units. The students are then able to make better informed choices about plant shutdowns, implementing best practices, and technology transfer. We have also detailed how the case discussion can be adapted to objectives of the course, the technical abilities of the students, and the availability of resources.

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