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Comments on "Dynamic Version of the Economic Lot Size Model"

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What Happened

It is an unexpected pleasure to find that our dynamic lot size article is selected as one of the 10 most influential publications in the first half century of *Management Science*, especially since the selection process reflects the assessment of current INFORMS members. I am deeply appreciative of the honor.

It is an article that was written in the good old days. It was published in the same year that it was submitted.

Here is the background of what happened. In 1956 Thomson M. Whitin was a faculty member at MIT's Sloan School. I was a doctoral candidate in MIT's economics department, and concomitantly Tom's research assistant. Tom was preparing the second edition of his acclaimed book *The Theory of Inventory Management* (Princeton Press). In the new edition, Tom included recent research that he had published and he proposed our working together on an appendix that would contain unpublished research. He explained to me the pivotal importance of the classic economic lot size model and kindly suggested the opportunity to make a contribution by generalizing the EOQ model to represent an environment with time variation in the parameter values.

Tom and I shared a professional interest in microeconomic theory, and for the new appendix, we posited a model that included the impact of pricing and variable manufacturing (or purchasing) cost. The dynamic lot size model was one of several cases that we discussed in the new appendix.

Shortly thereafter, we extended our collaborative research. We dropped the pricing and production cost aspects so that only demand, setup cost, and unit holding cost varied. We derived key results about planning horizons, an idea that already had received some attention by scholars at the Carnegie Institute of Technology.

Key words: inventory replenishment; EOQ; dynamic programming; planning horizon

Our Mind-Set Then

By 1956, many extremely important contributions to stochastic inventory models had been published—several of the authors were later honored with Nobel prizes. The form of an *optimal* policy when stocking costs included setup (fixed) ordering costs had not yet been established.

The standard continuous time EOQ model, with its stationary assumptions, permitted framing the model as an unbounded horizon optimization using the criterion cost per unit time. We had to consider whether to assume an optimization for a finite number of periods or an unbounded horizon. We accepted the typical formulation of the EOQ model in which all demand is met on time.

We knew that the model could be formulated as a so-called fixed charge linear program. But that was of minor importance in 1956 when mixed integer programs lacked a promising algorithmic approach.

We also understood that the model could be formulated as a classic dynamic programming optimization, with the state variable being entering inventory. That was a satisfactory starting point but, in 1956, it was not particularly attractive computationally except as a last resort. Whereas the EOQ model was a practical option for real applications, dynamic programming, by and large, was not.

We started down the analytic path of searching for a way to determine the periods in which an order is placed, that is, a setup cost is incurred. The idea that succeeded is establishing that there always exists an optimal solution in which no inventory is carried into a period when a setup cost is incurred. Adapting a dynamic programming formulation to consider only such policies provides the solution to our model. In that context, the state space reduces to the dichotomy of entering inventory being either 0 or positive, and the decision space being restricted to the set of $1, 2, \ldots,$ periods of accumulated future demand.

Typically dynamic programming recursions are initiated by solving the optimization in the final period;

that is, by optimizing a one-period model at the end of the planning horizon. Then a backward recursion is pursued until a solution to the first period order quantity is obtained. A backward recursion is an especially felicitous approach when demands are stochastic.

In the dynamic economic lot size model, with deterministic demands, the optimization is initiated with the first period. Our paper provides a planning horizon result that, in its simplest case, enables the forward optimization process to be terminated whenever it is optimal to order in the latest period considered so far. For example, consider a 50-period model. Suppose that when periods $1, 2, \ldots, 5$ only are considered, it is optimal to order in period 5. Then the optimal values for periods $1, 2, 3,$ and 4 do not depend on demands in periods $6, 7, \ldots, 50$. The optimization for a four-period planning horizon is appropriate. Depending on the model's parameter values, the length of the first planning horizon may be relatively short.

A Modeling Aficionado's Delight

The *Management Science* process for nominating this paper indicated that the paper received 592 article citations. This level of attention may be related to the model's being at the crossroads of a multitude of analytic formulations.

The core constraints of the cost minimization model are that cumulative quantities ordered from periods 1 to k, for each k, must be at least as large as cumulative demand over the same time span.

Here are several formulations, each of which provides a particular focus. I am indebted to my friend Arthur F. Veinott Jr. for suggesting most of these ideas.

As was mentioned above, one model formulation is a standard mixed integer linear programming model. The LP variables are the amounts ordered and the binary variables ensure that a setup cost is incurred whenever an order is placed. Using an MIP model, all decisions are considered simultaneously, and the computational difficulty is finding optimal values for the binary variables.

Also mentioned above is a standard dynamic programming model. Like the MIP model, the form of an optimal policy is not exploited in the standard formulation. Computationally this formulation involves finding an optimal order quantity *strategy* that is conditional on each possible amount of entering inventory at each period.

If the model is formulated as minimizing a concave function over a closed convex set, the property that there is an optimal solution at an extreme point reveals the *form* of an optimal policy (as described earlier).

The dynamic programming formulation that exploits the form of an optimal policy can be equivalently stated as finding a minimum cost path from node 1 to node N (end of the horizon) in a classic acyclic network described as containing all arcs from node i to node j, for each i and all j larger than i. An arc cost is comprised of the setup cost at i and the implied carrying costs for placing an order to satisfy demand from period i until the next reorder in period j. This model can be solved, of course, by using standard LP, network optimization, or a one-pass network recursion (for the dual variables).

The above network formation is tantamount to describing a standard renewal model, such as is applicable for equipment replacement. Hence the only difference between the dynamic economic lot size model and the standard equipment replacement model is the calculation of the arc costs.

Today, every one of the above formulations can serve as a final-examination exercise in a modeling course that employs Excel's Solver tool (actually, the recursive formulations require only familiar Excel formulas and there is no need to invoke Solver).

Comparable to other core models in operations research, what seem to be minor changes in assumptions can make the model properties and solution process much more complex. An example is adding each period upper limits on production capacity or total inventory at the end of a period.

Not-Exactly-Optimal Formulations

Many authors have extended the model in a variety of different directions. Here are two paths that merit special mention.

In one, the dynamic properties of the decisions are considered by embedding the model in a situation with a rolling horizon. For example, the model is solved for N periods, the first period's production level is implemented, and then the model is solved again, one period later, using a rolling horizon. The motivation is that the demand values actually may be forecasts and subject to revision. The importance of this research is that it demonstrates that a policy that is only approximately optimal for a finite horizon may outperform the dynamic lot size model when the two are compared in a situation with a rolling horizon. This research explored hierarchies of optimization and added valuable sophistication to the understanding of dynamic operations models.

The other path was suggested by practitioners. These authors concluded that calculating an optimal (s, S) policy in a dynamic stochastic-demand environment (for example, a situation that has shifting mean demands) was too difficult to implement. Instead, they used point *demand forecasts* as if they were known values and applied the dynamic economic lot size model. From the calculated results, they determined how large the first order should be, based on when the likely subsequent order would be placed. This initial value from the model is augmented by an uncertainty hedge (safety stock) to avoid stock out due to unanticipated stochastic demand. In essence, this practitioners' approach was tantamount to a simple approximation for an optimal (s, S) policy.

The Next Half-Century

Of course the nomination process for the next 50th anniversary of *Management Science* is far off. But a candidate may be a paper that provides an effective solution to a dynamic economic lot size model that encompasses a multiechelon environment.