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LETTERS TO THE EDITOR

Letters should be addressed to the Editor, Graduate School of Business, 401 Uris Hall, Columbia University, New York, New York 10027.

Concerning Simulation and Sixty Years of Queuing Theory

Professor Bhat, in his excellent paper, "Sixty Years of Queuing Theory," (*Management Science*, February 1969) stresses the difficulties encountered by "operational people" in applying results of behavioral queuing studies done by "theoreticians." Everyone having worked in this field who did not have an extensive course on complex variables in school will remember the gruesome feeling when at the end of his derivation, he is faced with the inversion of an untabulated La Place transform. At this stage most operational people will give up and simulate the queue in order to get an answer, particularly in multi-server problems.

My observation is that many, at least industrial, queuing systems have a state space that is completely defined by the number of customers in the system. We know that balance equations can be set up for the steady state description of these systems by using a node-arc representation of the events in the state space.

For a variety of cases, the equations can be written out and the ensuing system of linear equations easily solved analytically. However, often the behavior of the queue is subject to complex rules, and the analytical formulation of the balance equations becomes not only tedious and awkward, but nearly impossible. Therefore, in such cases, it seems preferable to generate and solve the balance equations simultaneously. Then one needs only to specify the policy governing the system.

Given that the system is in state m, consider that it may be changed in two ways:

(a) Service to a customer is terminated and he departs; the number of customers in the system decreases by one.

(b) Service to a customer is still going on while a new customer arrives. The number of customers in the system increases by one.

To use the technique, one writes a program that generates each state one by one. Every time a state is generated with its associated probability of occurrence, one computes the probabilities to enter the other states. Adding together all probabilities to enter a state yields the probability of the state's occurrence. Thereby, each new value of a state probability is not computed at once, but gradually put together as the computer deals with each state and the possible transitions from this state. Iterating this scheme the same as one does the analytically formulated equations, yields the desired solution.

It has also to be pointed out that for complex systems the calculation of the various characteristics of the queue (or queues) is not straightforward; e.g., computing the expected waiting time of a customer or the probability that he has to wait for service after his arrival.

If one specifies that no arrivals occur and counts the time required to serve all but the last customer in the system at a particular state and doing this for all states, yields the expected waiting time for FIFO queues after weighing each time with the appropriate state probability. The probability that one has to wait for service is obtained by simply adding all probabilities associated with states in which customers wait for service.

Properties of LIFO queues require more computation time due to the many possibilities and this approach may become economically inferior to the simulation of the system.

The scheme requires double precision if many states are involved, and in systems where an infinite length of the queue is possible, some thought also has to be given to the finite number of states which one has to compute in order to get an acceptably low level of bias. But overall, this technique appears to be superior to simulation for queues with a state space defined by the number of customers in the system.

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