Management Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Notes—An Index of Health: An Application in Accidents

John Morton Krall,

MANAGEMENT SCIENCE
Vol. 18, No. 12, August, 1972
Printed in U.S.A.

Notes*

AN INDEX OF HEALTH: AN APPLICATION IN ACCIDENTS†‡

JOHN MORTON KRALL§

West Virginia University

This paper presents an index of health which is based on a multiple decrement model. The index is defined in terms of costs and benefits which are weighted sums of losses and gains experienced in a population due to poor or good health. The index of health is then applied to a farm accident problem. The purpose of the index is to serve as an aid to administrators in making decisions concerning which types of accidents should receive larger resource allocations in the effort toward elimination.

Introduction

Many different indices are currently being used to call attention to the need for research, treatment, or control of a particular disease or group of diseases.

This paper proposes, in summary form, a health index designed as management tool for policy-makers in their task of making health allocation decisions. The index, which is defined in tems of *costs* of illness relative to *benefits* derived from absence of illness, includes the indices of Rice [8] and Chiang [1] as special cases [5, pp. 17–20].

An illustrative application, that of mortality from accidents on farms, is presented in order to highlight use of the index. This will serve to point toward the need for research in the development of algorithms for selecting optimal allocations to produce a maximum index of health for a given budget of health resources.

Formulas for Costs of Illness, C, and Benefits, B

Let j denote a population group, such as an age group, and let P_j denote the size of the population j. Further, let $v = 1/(1 + r)$, where r is the annual interest rate, and consider K mutually exclusive decrements (either morbidity or mortality). We make the following definition for the cost of illness to population j:

$$(1)\quad C_j = P_j \sum_{i=0}^{w} v^{i+n} \left(\sum_{k=1}^{K} {}_iW_{jk}\ {}_{i|}{}_nq_j^k\right), \qquad i = 0, n, 2n, \cdots, w, \quad k = 1, 2, \cdots, K,$$

where n is a nonnegative number, ${}_iW_{jk}$ is a weight (loss) associated with decrement k and population j, after survival of i years ($i = 0$ is understood if i is omitted), and ${}_nq_j^k$ is the probability that a life in population j will move to decrement k within n years. The symbol w denotes the maximum future years for which losses will be included in C_j, and ${}_{i|}{}_nq_j^k = {}_ip_j \cdot {}_nq_{j+i}^k$ where ${}_ip_j$ is the probability that a life in population j will survive i years ($i = 0, n, 2n, \cdots, w$). [4]

C_j may be characterized as the present expected value of future losses due to morbidity and mortality which will be suffered by population j. Here the future has been subdivided into intervals of time each n years long, and it is assumed that losses occur

* Notes are refereed.

† Received August 1971; revised March 1972.

‡ This research was supported in part by PHS Fellowship 5-F01-GM32657.

§ The author gratefully acknowledges the assistance of Dr. Paul E. Leaverton and Dr. James C. Hickman of The University of Iowa for their contributions and suggestions as thesis advisors.

at the end of an interval. The probabilities of decrement, ${}_nq_j{}^k$, and P_j, the size of the population, must be found in existing tables or developed from known data. However, the interest rate, r, and the weights W_{jk} are the choice of the index user.

The cost is based on the K decrements and the relative importance (losses W_{jk}) which the user attaches to those decrements.

We now make the following definition for benefits derived from population j:

$$B_j = P_j \sum_{i=0}^{w} {}_iE_j\, v^{i+n}\, {}_{i+n}p_j\,, \qquad i = 0, n, 2n, \cdots, w, \tag{2}$$

where ${}_iE_j$ is a gain associated with active members of population j for period of time i. This is specified by the user of the index, and should be expressed in the same units as the losses, W, in the cost formula. B_j may be characterized as the total *present expected value* of the contributions of population j. It is the sum of future potential contributions discounted with interest and survival, that is, the *benefit* of the contributions (gains) of the population j which has P_j current members. Note that if ${}_iE_j = 1$ and $v = 1$, B_j becomes an estimate of the expected number of healthy life years to be lived by population j.

The index of health for population j is then defined:

$$I_j = 1 - C_j/B_j\,.$$

If many population groups are combined in one population, a total index I is defined:

$$I = 1 - \sum_{\text{all } j} C_j / \sum_{\text{all } j} B_j\,. \tag{3}$$

We observe that if cost increases (a greater proportion of P_j experiences sickness or death), then benefits will decrease (a smaller proportion of P_j contributes benefits), and the index, I, will decrease.

Thus the cost of illness is inversely related to the index. Since the total cost of illness is a meaningful figure, we may wish to observe the *changes* in the cost of illness as well as the index itself when making an allocation decision.

An Illustrative Example

1. *Farm Accident Data—Construction of the Health Index*

In order to illustrate the use of the index defined previously, an example using real data was constructed. The source data are for "Fatal Accidents on Farms by Major Causes or Agencies of Death, Males, United States, 1965" contained in [9]. It will be necessary to calculate the index under our multiple decrement (farm accidents) model, to observe the changes in the index corresponding to different resource allocations applied to decrease specific accident rates, and to choose an optimal allocation. This example appears in detail in "An Index of Health and the Allocation of Health Resources." [5, pp. 81–113] The major results are given here.

Cost and benefits were considered for one year only $\binom{n=1}{w=0}$ so that the health index is:

$$I = 1 - \sum_j C_j / \sum_j B_j\,,$$

where $C_j = P_j\, v \sum_{k=1}^{K} W_{jk}\, q_j{}^k$, $B_j = P_j\, v\, E_j P_j$ in this example.

2. *Choice of Input Parameters*

There were eleven causes of death which were included: machinery, drownings, firearms, falls, blows, animals, burns, electric current, poisonings, lightning and other. The international cause-of-death codes are given in Table 1. The source data were combined with appropriate population data to form a Multiple Decrement Table,

q_j^k, (Table 1), with seventeen age groups and twelve decrements ($K = 12$). Note: the twelfth decrement is for all deaths which were not accidental deaths on farms.

Interpolated population figures, P_j, which were used for the seventeen age groups are given in Table 2.

The choice of income weights, E_j, was derived from data labelled "Income for Farmers and Farm Managers, 1959" found in a U. S. Commerce Department publication [7]. The actual choice of interpolated and assigned weights is given in Table 2.

TABLE 1

*Multiple Decrement Table**
Fatal Accidents on Farms, Males, 1965
(rates per 100,000)

Age Class	**912	929	919	900–4	910
	Machinery	Drownings	Firearms	Falls	Blows
0–4	5.761	7.017	0.129	0.249	0.767
5–9	5.192	6.878	1.041	0.633	0.648
10–14	6.316	8.335	4.500	0.274	0.280
15–19	10.824	11.070	7.123	0.946	1.290
20–24	13.951	5.202	7.705	1.389	1.137
25–29	9.443	4.980	2.550	2.128	1.814
30–34	15.374	3.679	3.301	1.427	1.828
35–39	14.601	1.987	3.425	1.211	3.719
40–44	14.567	2.956	3.243	0.790	3.769
45–49	14.962	1.519	3.124	1.419	2.488
50–54	19.941	1.228	3.818	2.187	2.682
55–59	23.893	1.899	3.382	3.040	2.851
60–64	25.327	1.975	3.714	5.724	4.007
65–69	35.732	2.175	3.184	6.585	4.359
70–74	26.749	4.525	2.980	6.765	2.474
75–79	23.662	2.621	1.435	12.594	1.428
80+	14.302	4.089	0.895	12.211	5.352

Age class	Decrement 927–8	916–8	914	870–95
	Animals	Burns	Electricity	Poisonings
0–4	0.579	0.589	0	0.715
5–9	0.469	1.194	0	0.120
10–14	0.127	0.775	1.169	0.262
15–19	0.875	0.298	2.186	0.603
20–24	0.258	0.786	3.559	1.591
25–29	0.984	0.669	2.649	1.014
30–34	0	0.336	3.428	0.682
35–39	0.560	1.431	2.911	1.159
40–44	0.975	1.737	1.967	0.503
45–49	1.126	1.528	1.298	0.775
50–54	0.404	0.619	0.699	1.251
55–59	1.877	2.151	2.433	0.241
60–64	2.789	1.136	0.322	0
65–69	3.226	4.017	0.414	0.368
70–74	2.682	4.103	0.516	0.459
75–79	3.883	5.938	0	0
80+	3.232	6.576	0	0

* Source Data [9]: The source data were combined with appropriate population to form the table above. [5, pp. 128–132]

** Numbers after causes of death are category numbers of the Seventh Revision of the International Lists, 1951 (Series E)

TABLE 1—*Continued*

Age class	Decrement 935	Other Farm	All Other
	Lightning		
0–4	0	0.935	619.248
5–9	0.118	1.658	32.101
10–14	0.639	3.073	26.305
15–19	1.176	1.326	98.654
20–24	0.777	1.818	148.522
25–29	1.325	2.320	154.803
30–34	0.667	1.335	180.845
35–39	0.566	1.702	268.538
40–44	0.984	1.478	431.302
45–49	0.567	1.517	712.552
50–54	1.020	1.636	1175.14
55–59	0.474	2.843	1830.75
60–64	0.280	0.846	2727.67
65–69	1.084	2.175	4143.10
70–74	0	4.066	5785.65
75–79	0.650	3.271	8046.22
80+	0	4.899	11512.7

TABLE 2

Age	Farm Population Males 1965* (thousands)	Farmers and Farm Managers Average Annual Income Males 1959** Amount	Present Value of Future Income Farmers and Farm Managers Males 1965*** Amount
0–4	739	$0	$37065.553
5–9	731	0	45267.945
10–14	676	140	55178.629
15–19	587	1185.2	64080.875
20–24	333	2365.6	69057.601
25–29	261	3311.2	68867.346
30–34	259	3805.8	64546.035
35–39	305	3869.2	57953.176
40–44	351	3768.8	50384.159
45–49	455	3542.2	42442.996
50–54	421	3300.2	34636.288
55–59	362	3055.2	27009.919
60–64	303	2794.2	19483.107
65–69	234	2300.0	14111.649
70–74	186	1800.0	8900.017
75–79	127	1300.0	4756.980
80+	100	$800.0	$1567.916
Total	6430		

* Source Data [10]: Figures in the table above were derived from the source data with interpolation methods. [5, pp. 134–135]

** Source Data [7]: Figures in the table above were derived from the source data. [5, p. 137]

*** Source Data [7]: Figures in the table above were derived from the source data. [5, p. 139]

It was decided to use the present value of a man's future earnings for W_{jk}, $k = 1, 2, \cdots, 11$, since the decrements are mortality decrements. For $k = 12$, let $W_{j,12} = 0$, since the twelfth decrement is not a type of accident on farms. The weights, W_{jk}, were derived for each age group, j, from income and survival data and are given in Table 2.

A choice of $r = 0.04$ was made for the interest rate. It should be noted that various choices should be examined to see what effect they would have on the index and costs therein.

3. *Index of Health, I*

The data described above and exhibited in Tables 1–2 provide the input to the Index of Health, I. The result was as follows where $I' = (I–0.9)10$:

Total Cost (Millions)	*Benefit (Millions)*	*Index*	I'
\$83.77	\$11,393	0.992648	0.92648

This index, calculated with original source data, will serve as a basis for comparison to other indices calculated under slightly altered conditions (see next section). The relatively high value of the index may, in part, be attributed to the fact that farm accidents as a whole make up a rather small proportion of the total deaths in the farm population. The value of the total cost does itself provide an estimate of the impact of these decrements (farm accidents) upon the (farm) population.

4. *Index of Health after Changes in Some of the Decrements*

We now illustrate the changes which occur in the index if some or all of the decrement rates are changed as a result of resource allocations against specific decrements. If (q_j^k) denotes the original multiple decrement table and changes are imposed on some or all of these rates, then an entirely new multiple decrement table, say $(q_j^k)^*$, must be constructed. Greville [3] and Chiang [2] discuss methods for doing this; the method used in this paper is an elaboration on earlier methods and described in Krall and Hickman [6]. The new table, $(q_j^k)^*$, is used to calculate a new index, I^*, from formulas (1), (2), (3), where constants P_j, W_{jk}, v, and E_j remain the same. The result of four examples is given below:

1. Suppose machinery deaths are eliminated: $q_j^1 = 0$, all j.

Cost	*Benefit*	*Index*	*Adj. Index* I'
\$51.31	\$11,395	0.995498	0.95498

2. Suppose drowning deaths are eliminated: $q_j^2 = 0$, all j.

Cost	*Benefit*	*Index*	*Adj. Index,* I'
\$68.39	\$11,395	0.993997	0.93997

3. Suppose machinery and drowning deaths are eliminated: $q_j^1 = q_j^2 = 0$, all j.

Cost	*Benefit*	*Index*	*Adj. Index,* I'
\$35.93	\$11,396	0.996847	0.96847

4. Suppose all except "other" deaths are eliminated: $q_j^k = 0$, all j, $k = 1, 2, \cdots, 10$.

Cost	*Benefit*	*Index*	*Adj. Index,* I'
\$4.90	\$11,397	0.999570	0.99570

(Cost and Benefit in millions of dollars.)

The results indicate that if machinery were eliminated, an increase in the index (and a corresponding decrease in the cost) would occur which is superior to that for elimination of drownings. Elimination of both would be even better, and of course, elimination of all accidents except "other" would be the best of these four choices. If there were only four allocations available which would produce the four results above, then the use of the criterion which was previously defined, namely maximum improvement (change) in the health index, results in an order of preference for 4, 3, 1, and 2.

As a practical matter, complete elimination of specific causes cannot be obtained, and the above four choices are not the only possible resource allocations and associated changes in decrements. The final solution to the farm accident example must include completion of (1), (2), and (3) below. Further research, including more on this example [5], is underway to facilitate these objectives:

(1) finding ways to associate or measure the amount of effort used to reduce losses in each decrement group,

(2) developing methods for predicting the *partial* reduction in decrement rates for given effort allocations, and

(3) developing an algorithm for selecting the optimal set of allocations which will produce a maximum index of health for a given budget of health resources.

References

1. Chiang, C. L., "An Index of Health, Mathematical Models," *Vital and Health Statistics*, National Center for Health Statistics, Series 2, No. 5 (May 1965).
2. ——, "On the Probability of Death from Specific Causes in the Presence of Competing Risks," *Fourth Berkeley Symposium on Mathematical Statistics and Probability*, Vol. 4 (1961), p. 169.
3. Greville, T., "Mortality Tables Analyzed by Cause of Death," *The Record*, American Institute of Actuaries, Vol. 37 (1948), p. 283.
4. Jordan, C. W., Jr., *Society of Actuaries' Textbook on Life Contingencies*, Edition 1, Society of Actuaries, Chicago, 1952.
5. Krall, J. M., "An Index of Health and the Allocation of Health Resources," Doctoral dissertation, University of Iowa, Iowa City, February 1969.
6. —— and Hickman, J. C., "Adjusting Multiple Decrement Tables," *Transactions, Society of Actuaries*, Vol. 22 (1970), p. 163.
7. Miller, H. P. and Hornseth, R. A., "Present Value of Estimated Lifetime Earnings," Technical Paper 16, Bureau of the Census, U. S. Department of Commerce, U. S. Government Printing Office, Washington, D. C., 1967.
8. Rice, D., "Estimating the Cost of Illness," *Health Economic Series*, No. 6, U. S. Department of Health, Education and Welfare, Public Health Service, May 1966.
9. Tabulations entitled "Deaths from Nontransport Accidents on Farms, by Cause of Death, Age, Color, and Sex, U. S. and Each State, 1965"—furnished January 27, 1967, by the National Vital Statistics Division, USDHEW.
10. U. S. Government Printing Office, *Farm Population of the United States, 1965*, Series Census, ERS p. 27, No. 36 (April 1966), Washington, D. C.