Management Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Quantum Economic Advantage

Francesco Bova, Avi Goldfarb, Roger G. Melko

informs
https://pubsonline.informs.org/journal/mnsc

MANAGEMENT SCIENCE
Vol. 69, No. 2, February 2023, pp. 1116–1126
ISSN 0025-1909 (print), ISSN 1526-5501 (online)

Quantum Economic Advantage

Francesco Bova,[a,b] Avi Goldfarb,[a,b,*] Roger G. Melko[b,c]

[a] Creative Destruction Laboratory, Rotman School of Management, University of Toronto, Toronto, Ontario M5S 3E6, Canada; [b] Creative Destruction Lab, Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada; [c] Department of Physics & Astronomy, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

*Corresponding author

Contact: francesco.bova@rotman.utoronto.ca, https://orcid.org/0000-0003-3365-2294 (FB); agoldfarb@rotman.utoronto.ca, https://orcid.org/0000-0002-8919-5814 (AG); rmelko@perimeterinstitute.ca, https://orcid.org/0000-0002-5505-8176 (RGM)

Received: January 28, 2022
Revised: August 12, 2022
Accepted: September 15, 2022
Published Online in Articles in Advance: December 2, 2022

https://doi.org/10.1287/mnsc.2022.4578

Abstract. A quantum computer exhibits quantum advantage when it can perform a calculation that a classical computer is unable to complete. It follows that a company with a quantum computer would be a monopolist in the market for such a calculation if its only competitor was a company with a classical computer. Conversely, economic outcomes are unclear if quantum computers do not exhibit a quantum advantage, but classical and quantum computers have different cost structures. We model a Cournot duopoly where a quantum computing company competes against a classical computing company. The model features an asymmetric variable cost structure between the two companies and the potential for an asymmetric fixed cost structure, where each firm can invest in scaling its hardware to expand its respective market. We find that even if (1) the companies can complete identical calculations, and thus there is no quantum advantage, and (2) it is more expensive to scale the quantum computer, the quantum computing company may be more profitable and also invest more in market creation due to efficiency gains from using quantum algorithms. Finally, we provide examples of settings where the classical computer can also perform a calculation, but not in a cost-effective enough manner to be commercially viable. In such a setting, the quantum computing company becomes a monopolist despite exhibiting no quantum advantage. Taken together, quantum computers may not need to display a quantum advantage to be able to generate a quantum economic advantage for the companies that deploy them.

History: Accepted by D. J. Wu, information systems.

Funding: R. G. Melko is supported by the Natural Sciences and Engineering Research Council of Canada, Canada Research Chair program, and the Perimeter Institute for Theoretical Physics. Research at the Perimeter Institute is supported in part by the Government of Canada through the Department of Innovation, Science and Economic Development Canada and by the Province of Ontario through the Ministry of Colleges and Universities. A. Goldfarb is supported by the Sloan Foundation and the Social Sciences and Humanities Council of Canada.

Supplemental Material: The online appendix is available at https://doi.org/10.1287/mnsc.2022.4578.

Keywords: quantum computing • quantum advantage • information technology

1. Introduction

We live in an exciting era of optimism in the field of quantum computing where the first claims of quantum *supremacy* or *advantage*[1] are being made on a number of innovative quantum devices. We say that a quantum computer has a *quantum advantage* if it can perform some calculation, no matter how arbitrary, that a conventional (or "classical") computer cannot complete. Demonstrating quantum advantage has been a major pursuit of a large community of scientists and engineers working on different types of quantum computers in academia and industry and will be considered a watershed moment in the history of science when and if it is achieved (National Academics of Sciences, Engineering, and Medicine 2019). This pursuit has also led to a growing discussion of the business opportunities that may arise from quantum computers (Bova et al. 2021, Ruane et al. 2022) and quantum's potential to disrupt or replace classical computing (Cusumano 2018, Yang et al. 2022). It has also coincided with billions of dollars of public and private funding in the quantum computing industry.[2]

In 2019, Martinis' landmark experiment at Google was the first to make a strong claim for quantum advantage (Arute et al. 2019). Using a programmable 53-qubit superconducting qubit device called Sycamore, the experiment produced a probability distribution by sampling a random quantum circuit that was thought to be impossible to simulate classically at the time. Google's claim motivated several groups to repeat the equivalent classical calculation using new algorithms and powerful hardware, thereby moving the bar which defined quantum advantage for that particular calculation (Huang et al. 2021, Pan and Zhang 2021). Subsequently, another collaboration using a 56-qubit superconducting device called Zuchongzhi seemingly put the calculation out of reach for classical computers once again (Wu et al. 2021).

The previous example illustrates the challenges in illustrating a quantum advantage. These challenges arise in part because the benchmark for quantum advantage keeps moving. It is also important to note that calculations like those done by Sycamore and Zuchongzhi, as well as others like the 2022 Xanadu demonstration (Madsen et al. 2022), although certainly groundbreaking achievements, involve esoteric mathematical calculations that currently have no practical applications (Madsen et al. (2022) demonstrated quantum advantage in Gaussian boson sampling). It stands to reason that finding a business application that can display quantum advantage may be even more challenging to do. With uncertainty about the ability of quantum computers to generate a quantum advantage over classical approaches on problems of practical interest, natural questions arise as to whether a quantum computer can still generate economic value without being able to generate a quantum advantage.

In this paper, we argue that quantum computers can still generate economic value even when they do not provide a quantum advantage over classical computers. This outcome arises because there are asymmetries in the cost structure between the classical and quantum computer. To illustrate the intuition, we consider a strategic game, using a duopoly model where a quantum computing company and classical computing company compete. In this way, our paper builds on well-established literature in information systems that uses game theoretic models to understand how technological change might impact new and existing industries (e.g., Bakos and Nault (1997); Zhu and Iansiti (2012) and Adner et al. (2020) on platforms; Niculescu and Wu (2014) on zero marginal cost software; and Dellarocas et al. (2013) on hyperlinks).

In the model, each firm's cost structure is influenced by two different factors, which are driven by differences between quantum and classical computers. The first factor is the ability of quantum algorithms to speed up certain processes relative to algorithms that are run on classical hardware, where the expected efficiency brought on by quantum algorithms leads to a variable cost advantage for quantum computers. In some cases, this efficiency advantage has been formally proven in the quantum computing literature. For example, Grover (1996) proved that quantum computers can achieve a quadratic speedup for a wide range of unstructured search problems. These unstructured search problems apply to a variety of commercial applications including portfolio optimization, foreign exchange arbitrage, and credit scoring in finance (Orús et al. 2019), identifying the sources of failure in advanced manufacturing (Bova et al. 2021), and chemical engineering (Cao et al. 2019). Although we also illustrate the robustness of our approach for very general models of efficiency advantage, we use the functional form provided by Grover's algorithm in our main analysis, specifically a quadratic speedup (Grover 1996).

The second factor is the cost to scale each hardware, where the more qubits (bits) a quantum (classical) computer has, the larger the problems it can solve. By solving larger problems, the market for potential applications grows. For classical computers, it is well documented that the improved performance of computer hardware directly impacted the scope of applications and demand for computing (Ceruzzi 2003, Campbell-Kelly 2004). Campbell-Kelly (2004, chapter 4), for example, explicitly identifies the faster processing speed of IBM's 1401 and 360/30 (relative to the 650) as a driver of the rise of the software industry. We expect the same for quantum computers. For example, it has already been argued that increasing the number of qubits in a quantum annealer enables a wider variety of applications in material discovery (Genin et al. 2019). More generally, as one chief executive officer (CEO) put it, "It's natural that the larger the number of qubits, the larger problems we can solve."[3]

Notably, it is currently less expensive to scale a classical computer than it is to scale a quantum computer (Ball 2021).[4] It follows that the ability for classical computers to scale more cheaply may lead to a cost advantage for classical computers. We assume that scaling costs in each case are strictly convex. To implement a tractable functional form in our model, we further assume for both quantum and classical computers that costs to scaling are quadratic. This is consistent with standard assumptions on the costs and returns to research and development (Reinganum 1989, Sutton 1998). We then model the difference in scaling costs as a difference in the coefficient on the quadratic term. This allows us to capture the idea that adding qubits to a quantum computer may be proportionately more costly than adding bits to a classical computer, without assuming that this difference grows as computers gets bigger.

In our model, customers purchase a solution to a computational problem. The offerings provided by the quantum computing company and classical computing company can be differentiated or homogeneous. When the offerings are perfectly differentiated, one computer can solve a problem that the other computer cannot solve and vice versa. For example, a sufficiently coherent quantum computer running a quantum algorithm could solve certain problems that classical computers cannot solve in several human lifetimes (e.g., the factoring of very large numbers into their respective primes; Shor 1999). In this setting, the quantum computing company is the monopolist in the market to solve the problem that generated the quantum advantage.

When the firms' offerings become more homogeneous, both types of computers can output the same solution to solve the same problem. This represents the case in which the quantum computer does not exhibit a quantum advantage over its classical competitor, in the sense that it is feasible to use either computer to solve the problem. An example of such a problem is given by Genin et al. (2019), who demonstrate that small molecule quantum chemistry calculations can be solved relatively quickly on a quantum machine although the calculations are also feasible on a classical machine.

Regardless of the level of differentiation between the firms' offerings, there may still be asymmetric costs to generating a solution across the two firms. As we noted previously, the principal factor that impacts the variable cost structure of each company's offering is the speed in which the solution is generated. In our simplified model, we model variable costs as the amount of time it takes to output a solution.

Additionally, there is a fixed cost component to each firm's profit function. Each firm can choose to invest a fixed amount in market creating investments. These market creating investments lead to improvements in each firm's underlying hardware which allow their respective computers to solve larger, more intractable problems. For the quantum computing company, these investments may represent the development of higher quality qubits, improvements in entanglement, and enhancements in error correction. Investments in these areas should allow the quantum computer to solve increasingly larger and more intractable problems. Similarly, the classical computer company might make investments to build compute capabilities that will allow the classical firm to solve increasingly larger problems.[5] Taken together, as investments in each company's hardware increase, the size of the problems each company can assess become bigger, and in turn, the size of each firm's addressable market also become bigger.

We model competition as a Cournot duopoly, where each firm sets quantity simultaneously. These quantities, in part, determine the price of each firm's offering. In our setting, quantities can be thought of as a commitment to the amount of computing time each company makes available to consumers to solve challenging problems. This assumption implies that each firm makes a finite amount of computing time available to prospective consumers. This implication is consistent with observations from the current computing landscape, as we currently do not have ubiquitous accessibility to either quantum computers or classical supercomputers to solve these computationally challenging problems.[6]

As the cost to solving challenging problems is decreasing in both the speedup associated with certain algorithms (which may favor quantum computers) and the size of and ability to scale the hardware (which may currently favor classical computers), it is not clear which of the two architectures will ultimately have a lower cost structure in aggregate. In this way, we focus on two forces separate from quantum advantage as typically defined. First, consistent with our model, we assume that, for a given scale, quantum computers perform some calculations faster than classical computers, although such calculations are feasible on both types of computers. Second, our model allows for the assumption that quantum computers are currently more expensive to scale than classical computers. As noted previously, these two assumptions are based on formally proven efficiencies of quantum computing (Grover 1996) and evidence on the costs of adding bits and qubits to computers thus far (Ball 2021). These two forces determine the market opportunity for quantum and classical computing.

Our analysis illustrates that even when there is no quantum advantage and it is cheaper to scale the classical computer, the quantum computing company can still have a greater incentive to invest in building its market and can still be more profitable than the classical computing company. These outcomes arise due to the efficiency of the quantum computing company's variable cost structure. We also illustrate that the quantum computing company can still be a monopolist even if it does not exhibit a quantum advantage in settings where a classical computing company can complete a task, but not in a manner that is cost effective enough to be commercially viable. In aggregate, our results suggest that a quantum computing company can generate a quantum economic advantage without exhibiting a quantum advantage.

Finally, we also explore how advances in areas like error correction (which should make quantum computers easier to scale in the future) and the creation of quantum-inspired algorithms (which should mitigate the speed up advantage of quantum computers over classical computers for certain problems) affect our insights.

The paper proceeds as follows. In the second section we employ a model of differentiated Cournot competition and assess optimal investment and profitability outcomes. In the third section, we provide additional analysis and context for the main results and extend our model to incorporate a more general variable cost structure. In the final section, we conclude.

2. Cournot Duopoly Model

We model competition between a quantum computing company and a classical computing company. The competition occurs in two stages. In the first stage, each firm invests in the scale of the computer they build. In the second stage, the companies compete in quantity of computations.

As we previously noted, we assume that the duopoly is comprised of a company that has created a quantum computer and a company that has created a classical computer. The quantum computing company is labeled firm 1 and the classical computing company is labeled firm 2.

We begin by reprising the commonly cited inverse demand functions for a Cournot duopoly that are generated in Singh and Vives (1984, p. 547) when a representative consumer maximizes its quadratic utility function.

$$p_1 = \alpha_1 - \beta_1 q_1 - \gamma q_2$$
$$p_2 = \alpha_2 - \beta_2 q_2 - \gamma q_1$$

The variables p_1 and p_2 are the prices that each firm can charge for their respective offerings, and α_1 and α_2 represent the intercept for each firm's respective demand. We set $\alpha_1 = a + x_1$ and $\alpha_2 = a + x_2$, where a is an exogenous demand intercept that is common to both firms, and x_1 and x_2 are endogenous demand parameters that are specific to each firm, respectively. We discuss these endogenous parameters more later. We assume that $\beta_1 = \beta_2 = 1$, and thus the firms have a common slope for demand for each of their respective offerings. As in Singh and Vives (1984) γ is the measure of product differentiation and we assume $\gamma \in [0,1]$.[7]

As γ gets closer to zero, the product offerings become perfectly differentiated. A value of $\gamma = 0$ implies that the quantum computer has a quantum advantage; in other words, it can complete a calculation that is effectively impossible for the classical computer. As a result, the two firms operate in separate markets, with the quantum firm facing no competition from the classical firm in its market. For example, the quantum computer might be applying Shor's algorithm to factor numbers into primes in ways that are impossible on a classical computer. A value of $\gamma = 0$ could also occur for a classical computer that is calculating any number of applications that are challenging for a quantum computer to do effectively.[8]

As γ gets closer to one, the product offerings become more homogeneous. A value of $\gamma = 1$ implies that the quantum computer and classical computer are supplying identical offerings, and thus, the quantum computer does not offer a quantum advantage over its classical counterpart.[9]

In the Cournot setup, each firm sets its firm-specific quantity, q_i, to maximize profits (Cournot and Fisher 1929). As we previously noted, quantities can be thought of as the amount of computing time each company makes available to consumers to solve challenging problems. The price for each firm's offerings arise as a function of the chosen quantities. Following our previous assumptions, the final inverse demand function for the quantum computing firm is $p_1 = a + x_1 - q_1 - \gamma q_2$, and the inverse demand function for the classical computing firm is $p_2 = a + x_2 - q_2 - \gamma q_1$. In other words, following standard economics, each firm's price is decreasing in the quantity it produces and the quantity produced by its competitor provided the offerings are not perfectly differentiated. The revenue generated by the quantum computing firm and the classical computing firm are $p_1 q_1$ and $p_2 q_2$, respectively.

Singh and Vives (1984) assume a constant variable cost structure for each firm. The firms in our model also have a constant variable cost structure, but the variable cost structure is asymmetric across the two firms. Each firm's variable cost structure assumptions are informed by the speed at which each firm's computer can solve a problem. As discussed previously, we assume that the quantum computer can complete a process with less resources (e.g., in a timelier manner) than a classical computer by running a quantum algorithm. For the purpose of our exercise, we focus on one particular quantum algorithm, Grover's algorithm, which allows for a quadratic speed up over classical algorithms for several types of problems related to unstructured searches (Grover 1996).

A quadratic speed up implies that the quantum computer can complete specific processes in square-root the number of steps compared with the equivalent classical algorithm. For the purpose of model tractability, we assume that the time it takes to complete a calculation is the sole driver of each firm's variable cost structure. To incorporate the impact of a quadratic speedup, we first set the classical computer's variable cost to c^2. The quantum computer, using Grover's algorithm, can complete a process in square root the number of steps as the classical computer, which leads to the variable cost function $\sqrt{c^2} = c$. We assume that $a > c^2 > c > 1$ to ensure the quantum firm has a natural cost advantage over the classical computer due to the quadratic speedup brought on by the use of Grover's algorithm and that the common demand intercept is larger than either firm's variable cost. We also assume that each firm's demand intercept is sufficiently large relative to its variable cost to ensure that the resulting optimal quantities are strictly positive. Taken together, the variable cost base for the quantum company is calculated as cq_1 and for the classical company is $c^2 q_2$.

Next, we model each company's ability to scale its respective computer to solve more complex problems. First, we include an endogenous, convex, fixed cost investment that allows each firm to increase the size of the market its hardware has access to. Each firm's respective investment positively impacts their respective demand intercepts. The fixed cost investment is $B_1 x_1^2/2$ and $B_2 x_2^2/2$ for the quantum and classical firm, respectively, where x_1 and x_2 are investment choice variables, and B_1 and B_2 are positive coefficients that can take different values. B_1 and B_2 taking different values reflect that it may be more costly to scale one type of computer than the other. As we previously noted, these investments generate an increase in the size of the market (e.g., the intercept for demand) by x_1 and x_2 for the quantum and classical computing company, respectively. An increase in market size is driven by an increase in the size of the problem that each firm's hardware can assess.

If, for example, $B_1 > B_2$, then it is more costly to scale the hardware (and in turn, expand the market) for a quantum computer than it is for a classical computer. This outcome would, in part, offset the natural variable cost advantage that a quantum computer gleans via the quadratic speed up brought on by Grover's algorithm. Taken together, although a quantum computer may be able to provide a result in a more timely manner, it may be more costly for the firm to build a computer to achieve this more timely result. This may impede its ability to create or expand the market for its service. Thus, the net benefits to quantum computing from a cost perspective are unclear. The full profit function for each company is represented as

$$\pi_1 = q_1(a + x_1 - q_1 - \gamma q_2) - cq_1 - B_1\frac{x_1^2}{2}, \tag{1}$$

$$\pi_2 = q_2(a + x_2 - q_2 - \gamma q_1) - c^2 q_2 - B_2\frac{x_2^2}{2}. \tag{2}$$

In each profit function, the first term captures the firm's revenue, the second term captures the firm's variable cost base, and the third term captures the firm's fixed cost base. The sequence for firm decisions proceeds in two stages. In the first stage, Firm 1 sets investment x_1 to maximize its profits, and Firm 2 sets x_2 to maximize its profit simultaneously. Both firms then observe each other's choice of x. In the second stage, Firm 1 sets q_1 to maximize its profits, and Firm 2 sets q_2 to maximize its profits. We use backward induction to solve the program. Taking the first-order condition for Firm 1 (Firm 2) with respect to q_1 (q_2) and then solving simultaneously yields the following optimal quantities:

$$q_1^* = \frac{2(a - c + x_1) - (a - c^2 + x_2)\gamma}{4 - \gamma^2}, \tag{3}$$

$$q_2^* = \frac{2(a - c^2 + x_2) - (a - c + x_1)\gamma}{4 - \gamma^2}. \tag{4}$$

As expected, each firm's optimal quantities are increasing in the common demand intercept and their own investments.

Next, we plug the optimal quantities from (3) and (4) into the profit functions in Equations (1) and (2) to generate $\pi_1(q_1^*, q_2^*)$ and $\pi_2(q_1^*, q_2^*)$. We simultaneously set x_1 to maximize $\pi_1(q_1^*, q_2^*)$ and x_2 to maximize $\pi_2(q_1^*, q_2^*)$. In Lemma 1, we define the conditions that ensure that both $\pi_1(q_1^*, q_2^*)$ and $\pi_2(q_1^*, q_2^*)$ are concave in x_1 x_2, respectively, and that the resulting optimal investment levels x_1^* and x_2^* are both greater than zero.

Lemma 1. *For $\pi_1(q_1^*, q_2^*)$ and $\pi_2(q_1^*, q_2^*)$ to be concave in x_1 and x_2, respectively, and for x_1^* and $x_2^* > 0$, B_1, B_2, and the intercept (a) need to be sufficiently high.*

Taking the first-order condition for $\pi_1(q_1^*, q_2^*)$ ($\pi_2(q_1^*, q_2^*)$) with respect to x_1 (x_2) and then solving simultaneously yields the optimal investments x_1^* and x_2^*:

$$x_1^* = \frac{4[a(4 - B_2(\gamma-2)^2(2+\gamma)) + c(B_2(c\gamma - 2)(\gamma^2 - 4) - 4)]}{B_1(\gamma^2-4)(B_2(\gamma^2-4)^2 - 8) - 8(2 + B_2(\gamma^2-4))}, \tag{5}$$

$$x_2^* = \frac{4[-4c^2 - B_1 c(2c - \gamma)(\gamma^2 - 4) + a(4 - B_1(\gamma-2)^2(2+\gamma))]}{B_1(\gamma^2-4)(B_2(\gamma^2-4)^2 - 8) - 8(2 + B_2(\gamma^2-4))}. \tag{6}$$

Plugging optimal quantities from Equations (3) and (4) and optimal investments from Equations (5) and (6) into the profit functions in (1) and (2) yields the following optimal profit functions:

$$\pi_1^* = \frac{B_1(B_1(\gamma^2-4)^2 - 8)[a(-4 + B_2(\gamma-2)^2(2+\gamma)) + c(4 - B_2(c\gamma-2)(\gamma^2-4))]^2}{[B_1(\gamma^2-4)(B_2(\gamma^2-4)^2-8) - 8(2 + B_2(\gamma^2-4))]^2}$$

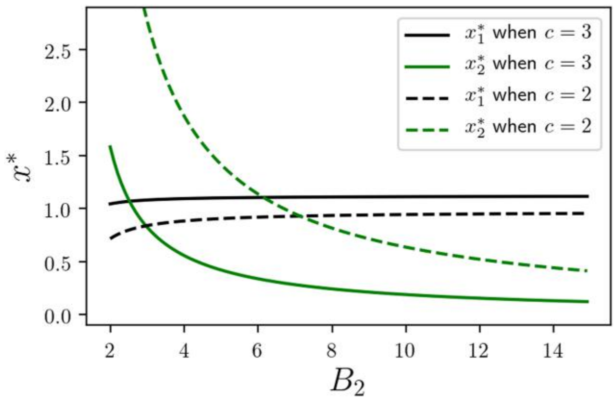
$$\pi_2^* = \frac{B_2(B_2(\gamma^2-4)^2 - 8)[4c^2 + B_1 c(2c-\gamma)(\gamma^2-4) + a(-4 + B_1(\gamma-2)^2(2+\gamma))]^2}{[B_1(\gamma^2-4)(B_2(\gamma^2-4)^2-8) - 8(2 + B_2(\gamma^2-4))]^2}.$$

Proposition 1. *Given the assumptions in Lemma 1, the quantum computing company is profitable for all γ.*

Each firm's profits are strictly positive provided each firm's concavity conditions (defined in Lemma 1) are met. Each firm's profits are a function of a (size of the common market intercept), c (cost to run a program on a quantum computer), B_1 (investment efficiency of a quantum computer), B_2 (investment efficiency of a classical computer), and γ (differentiation of product offerings).

When the quantum company has a quantum advantage over the classical firm, $\gamma = 0$, and the quantum company extracts monopoly rents in its market as the classical computer cannot compete with the quantum computer. Perhaps more interestingly, even in cases where there is no quantum advantage (a setting where the offerings are homogeneous; i.e., $\gamma = 1$), the quantum company is still profitable even though it is not a monopolist.[10]

Figure 1. Optimal Investments



To provide more insight, we assess optimal investments and the resulting optimal profits numerically in Figures 1 and 2, respectively. For both Figures 1 and 2, we set $a = 20$, $B_1 = 10$, and $\gamma = 1$. Notably, in a setting where $\gamma = 1$, the offerings are sufficiently homogeneous so that the quantum computer does not observe a quantum advantage. For both figures, we vary B_2 along the horizontal axis. Finally, we graph outcomes for both firms by varying the variable cost, c, to be either two or three.

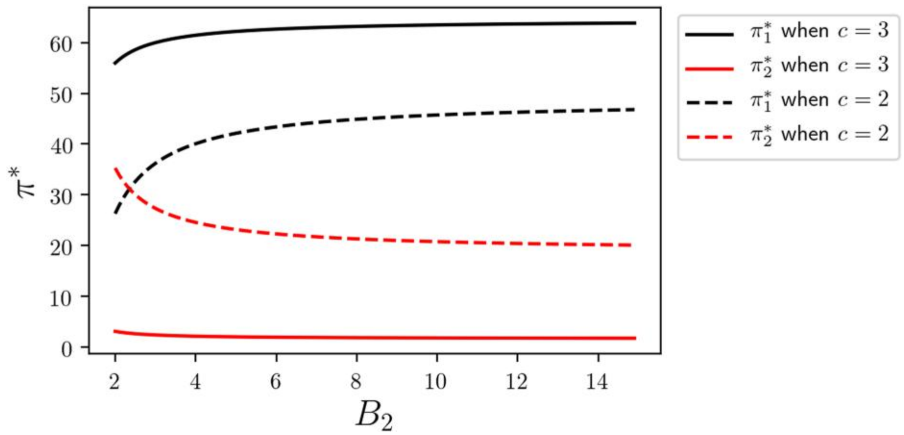
The figures provide some interesting insights. First, there are settings where the classical computing company is more profitable than the quantum computing company (e.g., in the parameter space where the red dashed line approaches the y axis in Figure 2). In general, the classical firm is more profitable than the quantum firm when (1) variable costs are comparatively low (in some cases when $c = 2$, but never when $c = 3$), and thus the benefit of the quantum quadratic speed up is diminished (i.e., these may not be tasks that an analyst needs a quantum computer to solve); and (2) the cost to scale is much lower for the classical firm than the quantum firm (i.e., B_2 much lower than B_1). These outcomes map well into the current computing ecosystem and provide predictions for the future. For example, the cost to scale a classic computer is currently much lower than the cost to scale a quantum computer and thus B_2 is currently much lower than B_1. Separately, classical computers have no problems handling smaller, more tractable problems (i.e., problems where c^2 is still comparatively small). In such a setting, we would expect classical computers to be more profitable than quantum computers and this is in fact what we observe today. At some point in the future, however, if the difference between B_2 and B_1 gets smaller and the computing ecosystem attempts to tackle larger, more intractable problems (i.e., c increases), then the model predicts that the quantum computing company would become the more profitable of the two companies even in settings where it does not display a quantum advantage.

Additionally, in Figure 1, x_2 is not always greater than x_1 when $B_2 < B_1$. In these settings, despite it being cheaper to scale the classical computer ex ante, there are still circumstances where we observe greater investments in market creation by the quantum computing company (i.e., $x_2 < x_1$ when $B_2 < B_1$), ex post.

Proposition 2. *There are settings where the quantum computer may invest more in market creation even in circumstances where it is less costly to scale a classical computer than a quantum computer.*

This outcome arises because of the asymmetries in variable cost structures across the two companies. To illustrate the intuition behind this observation, we use a simplified model to isolate the benefit to increasing the intercept of market demand on each firm's profits, absent the impact on fixed costs. We do this by first removing market-creating investments from the optimization programs in (1) and (2) by setting $x_1 = x_2 = 0$.

Figure 2. Optimal Profits



This yields Profit Functions (7) and (8):

$$\pi_1 = q_1(a - q_1 - \gamma q_2) - cq_1, \tag{7}$$

$$\pi_2 = q_2(a - q_2 - \gamma q_1) - c^2 q_2. \tag{8}$$

Next, we optimize (7) and (8) with respect to q_1 and q_2, respectively, and solve simultaneously. Optimal quantities are as follows:

$$q_1^* = \frac{a(2-\gamma) - c(2 - c\gamma)}{(4-\gamma^2)}, \tag{9}$$

$$q_2^* = \frac{a(2-\gamma) - c(2c - \gamma)}{(4-\gamma^2)}. \tag{10}$$

Note that $q_2^* > 0$ if $a > c(2c-\gamma)/(2-\gamma)$. Thus, this is a necessary condition to ensure that $q_1^*, q_2^* > 0$ for this model set up. Plugging the optimal quantities from (9) and (10) into (7) and (8), we get the optimal profits:

$$\pi_1^* = \frac{(a(2-\gamma) - c(2 - c\gamma))^2}{(4-\gamma^2)^2}, \tag{11}$$

$$\pi_2^* = \frac{(a(2-\gamma) - c(2c - \gamma))^2}{(4-\gamma^2)^2}. \tag{12}$$

If we differentiate π_1^* and π_2^* in Equations (11) and (12) with respect to a, we get

$$\frac{\partial \pi_1^*}{\partial a} = \frac{2(2-\gamma)(a(2-\gamma) - c(2 - c\gamma))}{(4-\gamma^2)^2}$$

and

$$\frac{\partial \pi_2^*}{\partial a} = \frac{2(2-\gamma)(a(2-\gamma) - c(2c - \gamma))}{(4-\gamma^2)^2},$$

respectively. Additionally, $\partial\pi_1^*/\partial a > \partial\pi_2^*/\partial a > 0$ provided $q_1^*, q_2^* > 0$. Thus, (1) the quantum computing company's profits in (11) and classical computing company's profits in (12) increase as the intercept a increases (as expected), and (2) all else equal, an increase in a has a bigger impact on improving profits for the quantum computing company than on improving profits for the classical company. Said another way, all else equal, an increase in the intercept will have a smaller impact on improving the profitability for the firm with the higher variable cost structure (in this case, the classical computing firm).

Taken together, even in some settings where the cost of market creation is lower for the classical company (i.e., $B_2 < B_1$), the benefits to market creation (i.e., increase in the demand intercept) is also lower because of the classical computer's higher variable cost structure ($c^2 > c$). These competing tensions lead to instances where $x_2 < x_1$ even in cases where $B_2 < B_1$. We can conclude that having a lower cost to scaling may not necessarily lead to greater market creating investments because the classical firm is still a comparatively high variable cost producer.

Finally, as Figure 2 shows, there are many circumstances where the quantum computing company is more profitable than the classical computing company even when $\gamma = 1$ (i.e., where competition is at its most intense) and $B_2 < B_1$ (i.e., it is cheaper to scale the classical computer). This outcome arises because the quantum computing company's lower variable cost structure has both a direct effect on improving profitability by reducing the firm's cost base, and an indirect effect on improving profitability by generating greater benefits to investing in market creation.

3. Discussion

3.1. Quantum Speedups and Quantum vs. Economic Advantage

In the prior section, the quantum computing company is a monopolist when $\gamma = 0$, that is, when it has a quantum advantage over the classical computing company. Next, we illustrate a situation where the quantum computing company can still be a monopolist even when $\gamma = 1$, that is, when the classical computing company can also complete a task that the quantum computing company can complete.

To illustrate the point, we use a simplified example. Starting with Equations (1) and (2), we first assume that each firm in the market makes no market-creating investments or $x_1 = x_2 = 0$. As a result, both companies face a common demand intercept of a. Next, we assume that $\gamma = 1$ and, as such, both firms provide identical offerings.

With these adjusted assumptions, we provide a numerical example to illustrate how a quantum computing company can still end up as a monopolist even if it does not display a quantum advantage. First, we assume the intercept for demand is $a = 20$. Next, we note that when $c = 2$, the variable cost base for the quantum firm is $c = 2$ and for the classical firm is $c^2 = 4$. When $c = 3$, the variable cost base for the quantum firm is $c = 3$ and for the classical firm is $c^2 = 9$. Finally, if we increase the value for c to $c = 5$, the variable cost base for the quantum firm is $c = 5$ and for the classical firm is $c^2 = 25$. In this last case, the variable cost for the classical firm is greater than the common intercept of the demand curve (i.e., $a = 20$). When the classical computing firm's variable cost is 25, there is no retail price that could be set for the classical computing company's service that would allow the firm to be profitable (i.e., any price less than 25 would lead to a loss and any price greater than 20 would result in no consumer demand). Therefore, we have a situation where the classical computer can complete a task in a somewhat timely manner compared with the quantum computer (i.e., 25 steps versus 5 steps), and hence, there is no quantum advantage, but where the classical computer company's offering may not be commercially viable because its variable cost base is greater than the demand intercept. This is another example of economic advantage without quantum advantage. In this example, the asymmetric cost structure would result in the quantum computing company becoming a monopolist in the market despite providing the same solution as its classical competitor.

Corollary 1. *When the classical computer can perform a calculation, but not in a timely enough manner to be commercially viable, the quantum computing company becomes a monopolist despite not exhibiting a quantum advantage.*

Examples of this corollary might be most apparent in a setting where one requires computational results relatively quickly. For example, real-time transactions in financial markets or complex queries to large cloud databases may require response times in seconds as opposed to minutes (or even hours). Quadratic speedups such as Grover's algorithm may be sufficient to deliver such speedups, as could other algorithms discussed later. Thus, in cases like this, although there may be no strict quantum advantage (in the sense that the equivalent calculation is still possible on classical hardware), quantum economic advantage is nonetheless achieved.

3.2. Other Quantum Algorithms and Quantum-Inspired Algorithms

We next explore how technological advances will affect the relative variable costs c and c^2. In the main model, we use Grover's algorithm to illustrate the impact of a quantum speedup on the quantum computing company's variable cost structure. With respect to Grover's algorithm, there are two important points to discuss. First, Grover's algorithm provides a *provable* improvement for a class of problems related to unstructured search when it is run on a coherent quantum computer. Second, Grover's algorithm scales polynomially with the number of qubits (i.e., $\sqrt{N}$ time for quantum versus N for classical). Despite the fact that a quadratic speedup could be argued to be only a modest improvement, we illustrated earlier that it is sufficient to promote a robust quantum economic advantage in certain cases. Nevertheless, our application of Grover's algorithm in the model should bias the analysis away from finding a quantum economic advantage, as there are a large variety of quantum algorithms that offer a *much larger* speed up over their classical counterparts. The most well known of these algorithms is Shor's algorithm for factoring, which reduces the computational cost from scaling exponentially in N to scaling polynomially in N (Shor 1999). Shor's algorithm run on a coherent quantum computer would be so effective that it puts our standard Rivest–Shamir–Adleman (RSA) public key encryption protocols at risk. For example, factoring a 2,048-bit RSA key (the size recommended by National Institute of Standards and Technology) is estimated to take billions of years or more using conventional computers, whereas a fully fault-tolerant quantum computer with a sufficient number of logical qubits could complete the task in seconds to days (Van Meter and Horsman 2013).

In contrast to Grover's and Shor's algorithms, many quantum speedups have not been mathematically proven. These other quantum algorithms remain open to competition from "quantum-inspired" approaches, which are algorithmic improvements inspired by the study of quantum algorithms that can be applied to classical computers. As these new quantum-inspired algorithms continue to be developed, a quantum computer's variable cost advantage related to the timeliness of its speed up will presumably be mitigated for certain applications. For example, it was believed for some time that a particular quantum algorithm would give an exponential speedup for a certain type of machine learning problem relevant for recommendation systems (Kerenidis and Prakash 2017), like those used by Amazon and Netflix. However, Ewin Tang developed a classical algorithm that, inspired by a deep understanding of the quantum speedup, was proven to be capable of performing the same calculation on a normal computer without the need for quantum hardware (Tang 2019).

Although quantum-inspired algorithms might mitigate the speed up advantage of quantum computers relative to classical computers for some problems, there are classes of problems (like those addressable by Shor's algorithm) that should only be solvable by quantum computers in a timely manner. Therefore, our model uses the mathematically proven quadratic speedup from Grover's algorithm as the basis for the difference in variable costs and our results apply even if quantum-inspired algorithms mitigate the usefulness of other quantum algorithms.

Nevertheless, we illustrate that, provided the quantum computer company has any variable cost advantage over the classical computer company, the general tenor of our results continue to hold.

Proposition 3. *If we replace the variable cost of the quantum computer company, c, with c_1, and replace the variable cost of the classical computer company, c^2, with c_2, and set $c_2 > c_1$, the general tenor of the results in Lemma 1, Proposition 1, and Proposition 2 continue to hold (proof in the online appendix).*

3.3. Advances in Quantum Computing

It is important to note that, although classical computers are currently easier to scale than their quantum counterparts, there are reasons to expect that the cost to scale each architecture will change over time. It is possible that if these costs change over time, so too will the gap between B_1 and B_2.

Historically, classical computers have scaled more efficiently over time following a pattern predicted by Moore's law (Moore 2006). Moore's law is the observation that the number of bits n_i in a classical computer (transistors) doubles every two years:

$$n_i = n_0 2^{(y_i - y_0)/T_2}, \tag{13}$$

where n_0 is the number of transistors in some reference year y_0, and $T_2 = 2$ is the doubling time. This can be inverted to say how the cost per transistor decreases as a function of year. The quantum version of Moore's law is sometimes called Rose's law, after D-Wave's founder Geordie Rose in 2002. It can be presumed to have the same form as Equation (13), with different values of n_0 and T_2. Further simplifications in the comparison could be to assume that $T_2 = 2$ for quantum or to say that T_2 for classical is slowing down as we approach the "end" of Moore's law for CMOS architectures (Track et al. 2017).

Notably, our model incorporates the cost to scale each architecture at a specific point in time. As Moore's law implies that the increase in the ability to scale classical computers will eventually slow down, it may also be reasonable to predict that the differences in scaling efficiency between a classical and quantum computer will continue to decrease over time and that the gap between B_1 and B_2 might get smaller over time. If this future is realized, it may have a material impact on the profitability of quantum computers relative to classical computers. When the difference between B_1 and B_2 gets smaller, the difference in profitability between the quantum and classical computer becomes larger, even in settings where no quantum advantage is observed.

4. Conclusion

Our model emphasized the marginal advantages of quantum computing over classical computing. Using a simplified example, the results show that it is possible for quantum computers to be worth deploying even if quantum advantage is never achieved. The relative usefulness of quantum computers depends on how the benefit of faster calculation compares to the higher cost of scaling.

This outcome does not imply that quantum computers will be immediately useful for a wide range of applications. Instead, the results suggest that quantum advantage is not the appropriate benchmark for a commercially viable quantum computer. Economics emphasizes decisions at the margins. Hence, fast calculations that are nevertheless feasible on classical computers may be the key to unlocking the potential of the quantum computing industry.

Acknowledgments

We thank seminar participants at the Massachusetts Institute of Technology Digital Initiative for helpful comments and the participants in the Creative Destruction Lab quantum stream who helped us understand the business aspects of quantum computing.

Appendix A. Proof of Lemma 1

For $\pi_1(q_1^*, q_2^*)$ and $\pi_2(q_1^*, q_2^*)$ to be concave in x_1 and x_2, respectively, and for x_1^* and $x_2^* > 0$, B_1, B_2, and a need to be sufficiently high. We plug the optimal quantity from each company from (3) and (4) into the profit functions in Equations (1) and (2) and take the second-order derivatives with respect to x_1 and x_2, respectively. This yields the following outcomes:

$$\frac{\partial^2 \pi_1(q_1^*, q_2^*)}{\partial x_1^2} = \frac{8 - B_1(-4+\gamma^2)^2}{(-4+\gamma^2)^2}, \tag{A.1}$$

$$\frac{\partial^2 \pi_2(q_1^*, q_2^*)}{\partial x_2^2} = \frac{8 - B_2(-4+\gamma^2)^2}{(-4+\gamma^2)^2}. \tag{A.2}$$

For each respective second order derivative to be negative, B_1 and B_2 need to be sufficiently high, respectively. Specifically,

$$B_1 > \hat{\hat{B}}_1 = \frac{8}{(-4+\gamma^2)^2}, \tag{A.3}$$

$$B_2 > \hat{\hat{B}}_2 = \frac{8}{(-4+\gamma^2)^2}. \tag{A.4}$$

Thus, we assume (A.3) and (A.4) as they are necessary conditions for $\pi_1(q_1^*, q_2^*)$ ($\pi_2(q_1^*, q_2^*)$) to be concave in x_1 (x_2).

Next, we assess the numerator for x_1^* from (5). We define the numerator for x_1^* as

$$D = 4[a(4 - B_2(\gamma-2)^2(2+\gamma)) + c(B_2(c\gamma-2)(\gamma^2-4)-4)].$$

Taking the first derivative of D with respect to B_2, we get

$$\frac{\partial D}{\partial B_2} = -4(-4+\gamma^2)(a(-2+\gamma) + c(2-c\gamma)).$$

Note that $\partial D/\partial B_2 < 0$ for all γ and for $a > c^2 > c > 1$. Solving for the value of B_2 that leads the numerator to equal zero will give us the value for B_2, over which the numerator is negative. For the numerator, that value is

$$\hat{B}_2 = \frac{4(a-c)}{(-4+\gamma^2)(a(-2+\gamma) + c(2-c\gamma))}.$$

Thus, the numerator for x_1^* will be negative if $B_2 > \hat{B}_2$.

Next, we assess the numerator for x_2^* from (6). We define the numerator for x_2^* as

$$E = 4[-4c^2 - B_1 c(2c-\gamma)(\gamma^2-4) + a(4 - B_1(\gamma-2)^2(2+\gamma))].$$

Taking the first derivative of E with respect to B_1, we get

$$\frac{\partial E}{\partial B_1} = -4(c(2c-\gamma) + a(-2+\gamma))(-4+\gamma^2).$$

Note that $\partial E/\partial B_1 < 0$ for all γ provided that $a > a^* = c(-2c+\gamma)/(-2+\gamma)$. This constraint approaches $a^* = c^2$ as $\gamma \to 0$. Thus, conditional on $a > a^*$, the numerator is decreasing in B_1. Solving for the value of B_1 that leads the numerator to equal zero will give us the value for B_1 over which the numerator is negative. For the numerator, that value is

$$\hat{B}_1 = \frac{4(a-c^2)}{(c(2c-\gamma) + a(-2+\gamma))(-4+\gamma^2)}.$$

Thus, the numerator for x_2^* will be negative if $B_1 > \hat{B}_1$ and $a > a^*$. Next, x_1^* and x_2^* both have the same denominator.

We define this common denominator as

$$F = B_1(\gamma^2-4)(B_2(\gamma^2-4)^2-8)-8(2+B_2(\gamma^2-4)).$$

Taking the first derivative of F with respect to B_2, we get

$$\frac{\partial F}{\partial B_2} = -8(-4+\gamma^2)+B_1(-4+\gamma^2)^3.$$

Note that $\partial F/\partial B_2 < 0$ provided $B_1 > \hat{\hat{B}}_1$. As we assume $B_1 > \hat{\hat{B}}_1$ in (A.3), F is decreasing in B_2. Also

$$\frac{\partial F}{\partial B_1} = (-4+\gamma^2)(-8+B_2(-4+\gamma^2)^2).$$

Note that $\partial F/\partial B_1 < 0$ provided $B_2 > \hat{\hat{B}}_2$. As we assume $B_2 > \hat{\hat{B}}_2$ in (A.4), F is also decreasing in B_1.

Next,

$$\hat{B}_2-\hat{\hat{B}}_2 = \frac{4(c(2c-\gamma)+a(-2+\gamma))\gamma}{(-4+\gamma^2)^2(a(-2+\gamma)+c(2-c\gamma))} > 0$$

if $a > a^*$, as previously assumed. Thus, $\partial F/\partial B_1 < 0$ when $B_2 \geq \hat{B}_2$ provided $a > a^*$. Similarly,

$$\hat{B}_1-\hat{\hat{B}}_1 = \frac{4\gamma(a(-2+\gamma)+c(2-c\gamma))}{(c(2c-\gamma)+a(-2+\gamma))(-4+\gamma^2)^2} > 0$$

if $a > a^*$. Thus, $\partial F/\partial B_2 < 0$ when $B_1 \geq \hat{B}_1$ provided $a > a^*$. Finally, $F=0$ when we set $B_1 = \hat{B}_1$ and $B_2 = \hat{B}_2$. As we assume that $a > a^*$, we know that F is decreasing in B_1 when $B_1 \geq \hat{B}_1$ and B_2 when $B_2 \geq \hat{B}_2$. Thus, $F < 0$ if $B_1 > \hat{B}_1$, $B_2 > \hat{B}_2$, and $a > a^*$.

Taken together, when $B_1 > \hat{B}_1, B_2 > \hat{B}_2$, and $a > a^* = c(-2c+\gamma)/(-2+\gamma)$, the numerators and denominators of both x_1^* and x_2^* are negative, and hence x_1^* and x_2^* are strictly positive.

In turn, $x_1^*, x_2^* > 0$ and $\pi_1(q_1^*, q_2^*)$ and $\pi_2(q_1^*, q_2^*)$ will be concave in x_1 and x_2, respectively, provided $B_1 > B_1^* = Max[\hat{B}_1, \hat{\hat{B}}_1]$, $B_2 > B_2^* = Max[\hat{B}_2, \hat{\hat{B}}_2]$, and $a > a^*$.

Appendix B. Proof of Proposition 1

Given the assumptions in Lemma 1, the quantum computing company is profitable for all γ. Optimal profits in (11) are

$$\pi_1^* = \frac{B_1(B_1(\gamma^2-4)^2-8)[a(-4+B_2(\gamma-2)^2(2+\gamma)) + c(4-B_2(c\gamma-2)(\gamma^2-4))]^2}{[B_1(\gamma^2-4)(B_2(\gamma^2-4)^2-8)-8(2+B_2(\gamma^2-4))]^2}.$$

Rearranging the terms in the previous equation yields

$$\pi_1^* = \frac{B_1(-8+B_1(-4+\gamma^2)^2)x_1^{*2}}{16}.$$

Given the assumptions in Lemma 1, $x_1^{*2} > 0$. Additionally, as we assume in Equation (A.3) in Lemma 1, we know that $B_1 > \hat{\hat{B}}_1 = 8/(-4+\gamma^2)^2 > 0$, and thus the first two terms in the numerator are also positive. Thus, optimal profits are positive for all γ given the assumptions made in Lemma 1.

Appendix C. Proof of Proposition 2

We generate the conditions where $x_1^* > x_2^*$. First, we calculate $x_1^*-x_2^*$. Doing so yields the following expression:

$$\frac{4(4(-1+c)c-B_2(-4+\gamma^2)(a(-2+\gamma)+c(2-c\gamma)) + B_1(c(2c-\gamma)+a(-2+\gamma))(-4+\gamma^2))}{B_1(\gamma^2-4)(B_2(\gamma^2-4)^2-8)-8(2+B_2(\gamma^2-4))}.$$

Given the assumptions in Lemma 1, the denominator of the previous expression is negative. Thus, for $x_1^* > x_2^*$, the numerator also needs to be negative. Notably,

- $4(-1+c)c > 0$ as we assume $c > 1$
- $-B_2(-4+\gamma^2)(a(-2+\gamma)+c(2-c\gamma)) < 0$ as we assume $a > c^2 > c > 1$
- $+B_1(c(2c-\gamma)+a(-2+\gamma))(-4+\gamma^2)) > 0$ as we assume $a > a^* = c(-2c+\gamma)/(-2+\gamma)$ is a necessary condition to get x_1^*, $x_2^* > 0$.

Thus, $x_1^* > x_2^*$ if

$$4(-1+c)c+B_1(c(2c-\gamma)+a(-2+\gamma))(-4+\gamma^2) < B_2(-4+\gamma^2)(a(-2+\gamma)+c(2-c\gamma)),$$

or

$$\frac{4(-1+c)c+B_1(c(2c-\gamma)+a(-2+\gamma))(-4+\gamma^2)}{(-4+\gamma^2)(a(-2+\gamma)+c(2-c\gamma))} < B_2.$$

Finally, we illustrate via a numerical example that there are instances where $x_1^* > x_2^*$ even when $B_1 > B_2$ that do not violate any of the assumptions in the paper including those in Lemma 1: We set $B_1 = 10$, $B_2 = 8$, $c=2$, $\gamma=1$, and $a=20$ (similar to the parameters in our figures). With these inputs,

$$B_1 = 10 > B_2 = 8,$$
$$\frac{4(-1+c)c+B_1(c(2c-\gamma)+a(-2+\gamma))(-4+\gamma^2))}{(-4+\gamma^2)(a(-2+\gamma)+c(2-c\gamma))} = \frac{107}{15} < B_2 = 8,$$
$$x_1^* = \frac{102}{109} > x_2^* = \frac{89}{109}.$$

Endnotes

[1] We adopt the latter terminology in this article (Mueck et al. 2020).

[2] For example, a 2022 McKinsey report emphasizes tens of billions in public funds for quantum research and commercialization, along with rapidly growing private investment in quantum startups (https://www.mckinsey.com/business-functions/mckinsey-digital/our-insights/quantum-computing-funding-remains-strong-but-talent-gap-raises-concern).

[3] Interview on August 5, 2022, with Michael Helander, CEO of OTI Lumionics, which uses quantum technology for materials discovery.

[4] Helander also contrasted recent progress in the scale of classical and quantum computers. "GPUs and tensor processing units have given 10× to 100× improvements in the last 5 years," whereas" real progress in quantum computers have been slow" (interview August 5, 2022).

[5] For example, the increases in the amount of compute used in AI training (https://openai.com/blog/ai-and-compute/) and NVIDIA's investments in GPU scaling (https://developer.nvidia.com/blog/scaling-out-the-deep-learning-cloud-efficiently/).

[6] Our modeling assumptions on fixed capacity, duopoly competition, and differences in variable costs reflect similar assumptions in other information systems contexts, such as Choudhary and Vithayathil (2013) and Fazli et al. (2018) on cloud computing and Abhishek et al. (2016) and Zhang (2009) on online retail.

[7] This demand structure is commonly used in the management and economics literature (Anand and Goyal 2009, Abhishek et al. 2016, Bustamante and Frésard 2021).

[8] In an interview (on July 18, 2022), University of British Columbia Professor and quantum computer scientist Olivia Di Matteo said there are certain problems, such as multiplication, for which there are known classical algorithms that scale better than the best-known quantum algorithms. In addition, she noted theoretical proofs that "There are also some examples of problems that are known not to scale better" on a quantum computer, such as sorting.

[9] We model this as a duopoly, but our emphasis is on the strategic implications for the quantum computing company. We do not emphasize the scenario in which the classical computer can solve problems that the quantum computer cannot. We also abstract away from firms that build both quantum and classical computers. Although these are potentially interesting extensions, they take the focus away from the impact of competition from classical computers on the usefulness of quantum computing.

[10] Similar outcomes may also arise in Bertrand competition settings where each firm sets its retail price directly. For example, Tirole (1988, p. 210–211) notes that in certain Bertrand settings where there are unit cost asymmetries between competitors, the low unit cost competitor can take the whole market by charging epsilon lower than the high unit cost competitor's unit cost.

References

Abhishek V, Jerath K, Zhang JZ (2016) Agency selling or reselling? Channel structures in electronic retailing. *Management Sci.* 62(8):2259–2280.

Adner R, Chen J, Zhu F (2020) Frenemies in platform markets: Heterogeneous profit foci as drivers of compatibility decisions. *Management Sci.* 66(6):2432–2451.

Anand KS, Goyal M (2009) Strategic information management under leakage in a supply chain. *Management Sci.* 55(3):438–452.

Arute F, Arya K, Babbush R, Bacon D, Bardin JC, Barends R, Biswas R, et al. (2019) Quantum supremacy using a programmable superconducting processor. *Nature* 7779:505–510.

Bakos Y, Nault B (1997) Ownership and investment in electronic networks. *Inform. Systems Res.* 8(4):321–341.

Ball P (2021) First quantum computer to pack 100 qubits enters crowded race. *Nature* 599:542.

Bova F, Goldfarb A, Melko R (2021) Commercial applications of quantum computing. *EPJ Quantum Tech.* 8:1.

Bustamante MC, Frésard L (2021) Does firm investment respond to peers' investment? *Management Sci.* 67(8):4703–4724.

Campbell-Kelly M (2004) *From Airline Reservations to Sonic the Hedgehog: A History of the Software Industry* (MIT Press, Cambridge, MA).

Cao Y, Romero J, Olson JP, Degroote M, Johnson PD, Kieferová M, Kivlichan ID, et al. (2019) Quantum chemistry in the age of quantum computing. *Chemical Rev.* 119(19):10856–10915.

Ceruzzi P (2003) *A History of Modern Computing* (MIT Press, Cambridge, MA).

Choudhary V, Vithayathil J (2013) The impact of cloud computing: Should the IT department be organized as a cost center or a profit center? *J. Management Inform. Systems* 30(2):67–100.

Cournot A, Fisher I (1929) *Researches into the Mathematical Principles of the Theory of Wealth* (Macmillan Company, New York).

Cusumano M (2018) Technology strategy and management: The business of quantum computing. *Comm. ACM* 61(10):20–22.

Dellarocas C, Katona Z, Rand W (2013) Media, aggregators, and the link economy: Strategic hyperlink formation in content networks. *Management Sci.* 59(10):2360–2379.

Fazli A, Sayedi A, Shulman JD (2018) The effects of autoscaling in cloud computing. *Management Sci.* 64(11):5149–5163.

Genin SN, Ryabinkin IG, Izmaylov AF (2019) Quantum chemistry on quantum annealers. Preprint, submitted January 15, https://arxiv.org/abs/1901.04715.

Grover LK (1996) A fast quantum mechanical algorithm for database search. *Proc. 28th Annual ACM Sympos. Theory Comput.* (ACM), 212–219.

Huang C, Zhang F, Newman M, Ni X, Ding D, Cai J, Gao X, et al. (2021) Efficient parallelization of tensor network contraction for simulating quantum computation. *Nature Comput. Sci.* 1(9):578–587.

Kerenidis I, Prakash A (2017) Quantum recommendation systems. Papadimitriou CH, ed. *Proc. 8th Innovations in Theoretical Comput. Sci. Conf.*, *Leibniz Internat. Proc. Informatics* (Schloss Dagstuhl, Wadern, Germany), 49:1–21.

Madsen LS, Laudenbach F, Askarani MF, Rortais F, Vincent T, Bulmer JFF, Miatto FM, et al. (2022) Quantum computational advantage with a programmable photonic processor. *Nature* 606(7912):75–81.

Moore GE (2006) Cramming more components onto integrated circuits. *IEEE Solid-State Circuits Soc. Newsletter* 11.3:33–35.

Mueck L, Palacios-Berraquero C, Persaud DM (2020) Towards a quantum advantage. *Phys. World* 33(2):17–17.

National Academics of Sciences, Engineering, and Medicine (2019) *Quantum Computing: Progress and Prospects* (The National Academies Press, Washington, DC).

Niculescu MF, Wu DJ (2014) Economics of free under perpetual licensing: Implications for the software industry. *Inform. Systems Res.* 25(1):173–199.

Orús R, Mugel S, Lizaso E (2019) Quantum computing for finance: Overview and prospects. *Rev. Physics.* 4:100028.

Pan F, Zhang P (2021) Simulating the Sycamore quantum supremacy circuits. Preprint, submitted March 4, https://arxiv.org/abs/2013.03074.

Reinganum J (1989) The timing of innovation: Research, development, and diffusion. Schmalensee R, Willig R, eds. *Handbook of Industrial Organization*, vol. 1, 1st ed. (Elsevier, New York), 849–908.

Ruane J, McAfee A, Oliver WD (2022) Quantum computing for business leaders. *Harvard Bus. Rev.* https://hbr.org/2022/01/quantum-computing-for-business-leaders.

Shor PW (1999) Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. *SIAM Rev.* 41(2):303–332.

Singh N, Vives X (1984) Price and quantity competition in a differentiated duopoly. *RAND J. Econom.* 15(4):546–554.

Sutton J (1998) *Technology and Market Structure* (MIT Press, Cambridge, MA).

Tang E (2019) A quantum-inspired classical algorithm for recommendation systems. *Proc. 51st Annual ACM SIGACT Sympos. Theory Comput.* (ACM), 217–228.

Tirole J (1988) *The Theory of Industrial Organization*, vol. 1, 1st ed. (MIT Press, Cambridge, MA).

Track E, Forbes N, Strawn G (2017) The end of Moore's law. *Comput. Sci. Engrg.* 19(2):4–6.

Van Meter R, Horsman C (2013) A blueprint for building a quantum computer. *Commun.* ACM 56(10):84–93.

Wu Y, Bao W-S, Cao S, Chen F, Chen M-C, Chen X, Chung T-H, et al. (2021) Strong quantum computational advantage using a superconducting quantum processor. *Phys. Rev. Lett.* 127: 180501.

Yang J, Chesbrough H, Hurmelinna-Laukkanen P (2022) How to appropriate value from general-purpose technology by applying open innovation. *California Management Rev*. 64(3): 24–48.

Zhang X (2009) Retailers' multichannel and price advertising strategies. *Marketing Sci.* 28(46):1080–1094.

Zhu F, Iansiti M (2012) Entry into platform-based markets. *Strategic Management J.* 33(1):88–106.