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Communication: Dynamic Data System Modeling—Revisited

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Dynamic Data System Modeling–Revisited

In his comments on our paper, "A Time Series Approach to Queueing Systems with Applications for Modeling Job-Shop In-process Inventories," *Management Sci.*, March 1977, Oliver D. Anderson points out an inadequacy in the modeling results we presented on pp. 748–749. As a defender of the faith, he suggests that the culprit is our modeling strategy, a procedure he views as a Black-Box heresy. Since our modeling strategy is quite new and relatively unknown, especially compared to the Box-Jenkins (1) approach, Anderson's concluding remarks are not surprising.

The modeling procedure we employed is called the Dynamic Data System (DDS) approach (8). Dynamic data, in the form of a time series of observations, are used to develop physically meaningful stochastic difference/differential equations to characterize the underlying mechanism of the system. This approach employs the fact that any uniformly sampled system ordinarily characterized by a linear differential equation can be described adequately by a discrete Autoregressive Moving Average (ARMA) model of order n and $n-1$. The model strategy thus consists of fitting models of the form ARMA $(n, n-1)$ starting with $n = 1$ and incrementing the order of n by one until an adequate model is determined by applying checks of adequacy.

The basic concepts embodied in the DDS approach are not new or unverified. Engineers have used linear differential equations to describe the behavior of physical systems for tens of years. In 1946, Bartlett (3) first pointed out relationships between the continuous differential and the discrete ARMA forms of the equations. Subsequent work has been done by Chu (4), Phadke (6) and Pandit (5).

Utilizing these concepts via the DDS modeling strategy offers distinct advantages. Since the model fitted to the data can be directly related in form to an nth order stochastic differential equation, the behavior of the system can be interpreted in a physical sense by decomposing the fitted model into first and second order differential equations. These simple equation forms have parameters with well known physical meaning such as the time constant, or damping ratio and natural frequency. This approach is particularly advantageous for modeling systems which exhibit high order behavior. (i.e., 8th, 10th order systems) Futhermore, in those cases where the ARMA $(n, n-1)$ would result in an overfit condition, the modeling procedure allows for degenerating the model form into a pure autoregressive, pure moving average, or a lower order mixed model. In fact, the DDS approach is not Black-Box heresy, but rather a modeling strategy for physical systems which employs known relationships to obtain a model which allows for interpreting the behavior of the system in a physically meaningful manner.

To both illustrate the DDS approach and rectify the model inadequacy problem, the details of modeling the series obtained from sampling the inventory process at the end of each shift ($\Delta = 8$ hrs.) are discussed. Fitting an ARMA (1, 0), or simply an AR(1) model, to the series yields the following parameter estimate and 95% confidence interval:

$$\hat{\phi}_1 = 0.759 \qquad (0.677, 0.841).$$

The residual sum of squares (SS) value is,

$$SS = 8190.$$

Incrementing n by 1 and fitting the data to an ARMA (2, 1) model gives the following results:

$$\phi_1 = 0.899 \qquad (0.391, 1.407),$$

$$\phi_2 = -0.009 \qquad (-0.417, 0.399),$$

$$\theta_1 = 0.344 \qquad (-0.157, 0.825),$$

$$\text{SS} = 7707.$$

The ARMA (2, 1) model, however, has poor estimates in that the 95% confidence intervals for ϕ_2 and θ_1 include zero.

Fitting the data to an ARMA (3, 2) model provides:

$$\phi_1 = -0.107 \qquad (-0.195, -0.019),$$

$$\phi_2 = -0.128 \qquad (-0.213, -0.043),$$

$$\phi_3 = 0.831 \qquad (0.755, 0.907),$$

$$\theta_1 = -0.820 \qquad (-0.932, -0.708),$$

$$\theta_2 = -0.792 \qquad (-0.906, -0.678),$$

$$\text{SS} = 5761.$$

In this case, the 95% confidence intervals do not include zero. The adequacy of the model can also be checked by a Chi-square test of the sample autocorrelations of the residuals. To check the adequacy of the model, it is known that if the fitted model is appropriate,

$$Q = N \sum_{k=1}^{K} r_k^2(\hat{a}_t)$$

is approximately distributed as $\chi^2(k - p - q)$ where $r_k(a_t)$ is the kth lag of the sample autocorrelation function for the residuals, and N is the number of series values (2). For the ARMA (3, 2) model with $K = 30$,

$$Q = 18.97 \quad \text{and} \quad \chi^2_{25}(0.05) = 37.7$$

indicating that there is no basis here for questioning the adequacy of the model.

The statistical significance of the reduction in the sum of squares after increasing the order of the model can be checked by an F-criterion (5, 7),

$$F = \frac{A_1 - A_0}{A_0} \cdot \frac{N - r}{S} \sim F(S, N - r)$$

where

A_0 is the smaller sum of squares of the ARMA model,

A_1 is the larger sum of squares of the ARMA model.

$F(S, N - r)$ denotes the F distribution with S and $N - r$ degrees of freedom. (S = the number of additional parameters in the higher order model, r = total number of parameters in the higher order model). Checking the sum of squares, reduction from the AR(1) model to the ARMA (3, 2) model yields

$$F = \frac{8190 - 5761}{5761} * \frac{240 - 5}{4} = 24.77$$

and

$$\underset{4,235}{F(0.05)} = 2.37$$

showing a significant reduction in the sum of squares.

The best model is therefore,

$$(1 + 0.107B + 0.128B^2 - 0.831B^3)\, n_t = (1 + 0.820B + 0.792B^2)\, a_t$$

which appears quite different from the AR(1) model originally reported. However, the model can be simplified in this case by factoring out the quantity,

$$(1 + 0.820B + 0.792B^2)$$

that is,

$$(1 + 0.820B + 0.792B^2)(1 - 0.713B - 0.079B^2 - 0.200B^3)n_t$$
$$\cong (1 + 0.820B + 0.792B^2)a_t$$

or simply

$$(1 - 0.713B - 0.07\overset{0}{\nearrow}9B^2 - 0.200B^3)\, n_t = a_t$$

This model shows that the in-process queue behavior is mainly first order plus a significant factor at lag three indicating the degree to which each of the three shifts are inter-correlated. The absence of this lag factor accounts for the discrepancy in the model parameter estimates. The modeling results given for the three series sampled at the end of first, second, and third shift, however, are correct as reported.

We thank Oliver D. Anderson for his interesting comments and contribution. We would also like to acknowledge a communication from Dr. David W. Scott in which he pointed out that the time series value n_t should be defined more exactly as the observed deviation from the mean value at time t.

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