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Communication: Dynamic Data System Modeling—Revisited

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Orthodox Box-Jenkins Versus "Dynamic Data System Modeling" Secession— A Rejoinder

Professor Steudel and Wu's reply [8] to my comments [5] is, on the whole, fairly reassuring. I am happy to note that their method, which compulsorily marches in model parameters two by two, is supported by the possibility of a recant, when an overfit occurs. Then, assuming that there does exist a good ARMA (p, q) representation for the data to hand, no troublesome "cancelling" parameter redundancy causing estimation difficulties will arise, when moving from an underfit ARMA $(n, n-1)$ to an overfit ARMA $(n+1, n)$, in the cases where $p = q$ or $q + 2$—compare Anderson [2, pp. 56–57]. Of course, if $p = q + 1$, Steudel and Wu's Dynamic Data System (DDS) Approach should be an ideal procedure. However, what happens for other choices of p and q?

That the authors do not really believe their statement "... any uniformly sampled wide sense stationary stochastic process can be *adequately* (my italics) described by a discrete Autoregressive Moving Average (ARMA) model of order n and $n-1$," given in [7, p. 748] and reiterated in [8], is clear from their discussion of allowing degeneracy to lower order models, when overfits are observed. One might as well say that all such processes follow an ARMA(∞, ∞) model, which is fine formally but, for practical purposes, barren. So what results when $p - q$ is other than 0, 1 or 2?

For instance, consider the example Steudel and Wu give in their defence [8]. The data are apparently clearly overfitted by the ARMA (2, 1) in the "cancelling" redundancy way; for, not only are $\hat{\phi}_2$ and $\hat{\theta}_1$ without significance, but $\hat{\phi}_1$ clearly has a very large standard error. This would seem to strongly support the AR(1) model, originally suggested in [7], until Steudel and Wu, perhaps prompted by my remarks [5], extend the DDS search and allow their n to increase sufficiently so that a "seasonal" lag 3 enters into the calculation. Then all the parameter confidence bounds tighten up and a substantial improvement on the AR(1) fit results. The implication is that one cannot allow the DDS procedure to terminate at low n, even if an "apparently" sound model has been obtained. So how does one know when to shut down?

In the sort of situation Steudel and Wu are investigating, an obvious simple first guess (before making any observations) of what model might be appropriate is

$$(1 - \phi B)(1 - \Phi B^3)n_t = a_t \tag{1}$$

with $0 < \Phi < \phi < 1$. For one may reasonably expect each value to be positively correlated with the previous one, from the preceding shift, and also (but less strongly) with the current shift's previous value. See Anderson [3, p. 132] for a rationale for the "multiplicative" combination of the two effects.

Now (1) can be rewritten as

$$(1 - \phi B - \Phi B^3 + \phi\Phi B^4)n_t = a_t$$

so that, for a Φ only just significant and ϕ not near unity, we have a model not very different from

$$(1 - \phi_1 B - \phi_3 B^3)n_t = a_t \tag{2}$$

with ϕ_1 and ϕ_3 close to ϕ and Φ, respectively. This clearly agrees with Steudel and Wu's extended analysis in [8], where they get

$$(1 - 0.713B - 0.079B^2 - 0.200B^3)n_t = a_t \tag{3}$$

with 0.079 omitted, as it is not significant. But, of course, we should test our simple theory with the observed facts. After all, (2) was based on the simplest of relationships between three *different* shifts, and other more complicated models could equally well have been postulated, in our state of ignorance.

Steudel and Wu [8] obtained (3) by "factoring out" the moving average part from their ARMA (3, 2) fit

$$(1 + 0.107B + 0.128B^2 - 0.831B^3)n_t = (1 + 0.820B + 0.792B^2)a_t. \tag{4}$$

However, whereas (3) clearly has a zero for its autoregressive operator close to unity, since $1 - 0.713 - 0.079 - 0.200 \approx 0$, (4) does not. So (3) must be substantially in error; and, in fact, fully factoring out the MA part from (4) yields

$$(1 - 0.713B + 0.079B^2 - 0.200B^3 + 0.229B^4 + \dots)n_t = a_t$$

where the terms after B^3 are evidently not negligible. We do not pursue this matter of modelling the whole series, except to express the belief that a first fit to (1), provided it was not refuted by the serial correlation and partial correlation plots, would appear to have been a better start than a sequence of ARMA$(n, n-1)$ fits.

However, accepting (4) as a fair representation for the whole series, even though it is probably rather prodigal with parameters, we see from Anderson [1, p. 156] that the three individual shift series should also follow ARMA(3, 2) processes. Without carrying through a full analysis, it is interesting to note that this observation does not necessarily contradict Steudel and Wu's assertion, in [8], that these "skipped" series were correctly modelled by the AR(1) fits of [7]. To see this, rewrite the left of (4) as

$$(1 - 0.862B)(1 + 0.969B + 0.963B^2) \tag{5}$$

which shows that the skipped model will have an autoregressive factor of $(1 - 0.862^3B)$; and so, if the rest of the autoregressive operator approximately cancelled with the moving average one, we would be left with something close to

$$(1 - 0.641B)n_t = a_t. \tag{6}$$

It has to be cancellation though, and not degeneracy, since the two imaginary zeros of (5) are near the unit circle, which means that the skipped model will have its other two autoregressive zeros not far from there. We see that (6) is in remarkable agreement with Steudel and Wu's table 1 [7], which gives an average AR(1) parameter value of 0.648.

My main objection to DDS disappears for non-seasonal series. For then, with moderately lengthed runs of stable data, satisfactory parsimonious ARMA(p, q) fits, with $p + q$ small, should be possible. Box and Jenkins [6] suggest $p + q \leqslant 2$ will often be adequate; and, in such cases, DDS with recant facility will get there and fast.

Finally, a small quibble: contrary to the good book [6], the "Portmanteau" chi-square diagnostic check statistic, suggested by Steudel and Wu [8], is virtually useless, unless it is at least modified to

$$Q^* = N(N+2)\sum_{k=1}^{K}(N-k)^{-1}r_k^2(\hat{a}_t).$$

For instance, see Anderson [4, p. 299].

The main aim of this contribution has been to stimulate an awareness of some of the connections between models for related time series, which are necessarily implied by the very relationships. However, a more important facet is to ensure that the models obtained make sense in their actual context; and I am pleased to note that Steudel and Wu [8] appear to have this as a major objective of their DDS approach.

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