Management Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

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MANAGEMENT SCIENCE
Vol. 24, No. 16, December 1978
Printed in U.S.A.

COMMENTS ON "SINGLE CYCLE CONTINUOUS REVIEW POLICIES FOR ARBORESCENT PRODUCTION/INVENTORY SYSTEMS"†

JOHN A. MUCKSTADT‡ AND HOWARD M. SINGER§

(INVENTORY/PRODUCTION; INVENTORY/PRODUCTION–DETERMINISTIC MODELS)

In the paper by Graves and Schwarz [1], we observed an error in their discussion of optimal policies for the one warehouse-m retailer system. Specifically, Theorem 1, which states that the optimal stationary policy is a single-cycle policy, is incorrect.

The proofs for property P4, the Lemma for Theorem 1, and Theorem 1 are in error. Property P4 states that the time interval between successive simultaneous production points for all stages is finite for an optimal policy. The proof presented implicitly assumes that the infimum of an infinite set of positive echelon inventory levels is a positive quantity. This need not be the case. The Lemma for Theorem 1 states that given zero initial inventories, the optimal stationary policy is a renewal policy. The proofs of the Lemma and Theorem 1 rely on the application of the properties P4 and P1 respectively. Both of these results, however, deal with optimal *stationary* policies and, unfortunately, it is not necessarily the case that the same properties which hold for overall optimal policies are valid for optimal stationary policies as well.

The following example demonstrates that a single-cycle policy need not be the best stationary policy. Consider the following one warehouse-two retailer system with costs

* All Notes are refereed.

† Accepted by Warren H. Hausman; received March 10, 1977. This paper was with the authors 6 months for 2 revisions.

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0025-1909/78/2416/1766$01.25

and demand rates as follows:

$$K_3 = 0.1, \quad K_2 = 99.9, \quad K_1 = 99.9,$$
$$h_3 = 1, \quad h_2 = 199, \quad h_1 = 99,$$
$$D_3 = 2, \quad D_2 = 1, \quad D_1 = 1,$$

where the subscript 3 corresponds to the warehouse and the subscripts 1 and 2 to the retailers. The optimal single-cycle policy is $(n_{31} = 2, n_{32} = 3)$ with $Q_3 \cong 5.824$ and cost $z_c = \$343.13$ per time unit. Let us now examine the non-stationary policy where the warehouse and retailer 1 are treated independently of the warehouse and retailer 2. This "separate retailing policy" [2] yields order quantities $Q_{31} = Q_1 = \sqrt{2}$ and $Q_{32} = Q_2 = 1$ with average cost $Z_R = \$341.42$ per time unit. Consider now the stationary policy based on the separate retailing policy above with $Q_1 = \sqrt{2}$, $Q_2 = 1$ and $Q_3 = 1 + \sqrt{2}$. In this policy, retailer 1 produces every $\sqrt{2}$ time units, retailer 2 produces every time unit and the warehouse produces Q_3 whenever its on-hand inventory is insufficient to meet a retailer's demand requirements. Note although this policy is stationary, it cannot be the overall optimal policy because it violates property P1 which states that the warehouse produces only when its inventory is zero. To evaluate the average cost for this policy, it should be noted that the average inventory at the warehouse is $(1 + \sqrt{2})/2$ and that in the long run, the warehouse sets-up once every $(1 + \sqrt{2})/2$ time units. Thus the average cost per unit time for this policy is

$$Z_s = \underbrace{0.1\left(\frac{2}{1+\sqrt{2}}\right) + 1\left(\frac{1+\sqrt{2}}{2}\right)}_{\text{warehouse costs}} + \underbrace{\frac{99.9}{1} + \frac{99.9}{\sqrt{2}}}_{\text{retailer set-up costs}} + \underbrace{100\left(\frac{\sqrt{2}}{2}\right) + 200\left(\frac{1}{2}\right)}_{\text{retailer holding costs}}$$
$$= \$342.54.$$

Thus the single-cycle policy is not always the optimal stationary policy since in the above example, there exists a stationary policy which is not single-cycle but which has lower cost than the best single-cycle solution.

The parameters for the example obviously are very unrealistic and they were specially contrived to demonstate the nonoptimality of single-cycle policies. In addition, the stationary policy presented above, which is not a single-cycle policy, has no *regeneration points*. At no point in time other than zero do the retailers run out of stock simultaneously since the production quantities are 1 and $\sqrt{2}$ and there is a unit demand rate at each retailer. In any actual application, rational production quantities will be assumed and regeneration points must occur.

One would hope that among the class of stationary cycling policies (i.e. constant lot sizes and regeneration points) that a single-cycle policy would be best. This, however, is also not the case. Consider the following one warehouse-two retailer system with costs and demand rates as follows:

$$K_3 = 1, \quad K_2 = 17, \quad K_1 = 17,$$
$$h_3 = 1, \quad h_2 = 10, \quad h_1 = 22,$$
$$D_3 = 2, \quad D_2 = 1, \quad D_1 = 1,$$

where the subscript 3 corresponds to the warehouse and the subscripts 1 and 2 to the retailers.

The stationary cycling policy in which the warehouse and retailer 2 both produce twice in each cycle and retailer 1 produces three times, has lower cost than the best single-cycle policy. In this stationary cycling policy production occurs at the warehouse when there is positive inventory there. A simple shifting of production leads to an obviously better nonstationary cycling policy. This shifting of production will work for any general 1 warehouse-m retailer example which leads us to the following conclusion:

Observation. If the overall optimal solution is a stationary cycling policy, then it is a single cycle policy.

Schwarz has previously demonstrated [2] that single-cycle policies are optimal for the single retailer and identical retailers cases. Further research is clearly necessary to demonstrate whether single-cycle policies are of value in a wider range of cases.[1]

[1] This research was supported in part by the Office of Naval Research under contract N00014-75-C-1172, Task NR 024-335.

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REPLY*

MANAGEMENT SCIENCE
Vol. 24, No. 16, December 1978
Printed in U.S.A.

ON STATIONARITY AND OPTIMALITY IN ARBORESCENT PRODUCTION/INVENTORY SYSTEMS†

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(INVENTORY PRODUCTION; INVENTORY/PRODUCTION–DETERMINISTIC MODELS)

We are indebted to Jack Muckstadt and Howard Singer for calling our attention to two errors in [1]. First, the proof of P4 presented assumed that $\inf(I_j(t) \mid t_i > t_k) > 0$, or, in other words, that the greatest lower bound on an infinite set of positive numbers is positive. This incorrect assumption invalidates the proof of P4. Fortunately, it does not invalidate P4 itself. A revised proof is presented below. Second, the lemma for Theorem 1 and Theorem 1 itself implicitly assumed that properties P1–P5 (in particular P1 and P4) of optimal policies in general apply to optimal *stationary* policies as well. Revised statements of Theorem 1 are provided below.

PROOF OF P4. Consider a policy P' such that retailer j violates P4. Define time zero to be the last simultaneous production point for j. Let $I_j(t)$ be the echelon inventory of j at time t, and $I_j^{\min} = \inf_{i>1}\{I_j(t_i)\}$ for t_i being the ith production point for $p(j)$ and $t_1 = 0$. There are two cases to consider: Case 1 in which $I_j^{\min} \beta 0$; and Case 2 in which $I_j^{\min} = 0$. If $I_j^{\min} > 0$, the original proof applies. If $I_j^{\min} = 0$, define $\overline{Q}_j = \sup_i\{Q_j(\tau_i')\}$ for τ_i' being the ith production point for j and $\tau_1' = 0$. Clearly

* This Reply has been refereed.

† Accepted by Warren H. Hausman; received July 11, 1978.

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0025-1909/78/2416/1768$01.25