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Reply—On Stationarity and Optimality in Arborescent Production/Inventory Systems

Stephen C. Graves, Leroy B. Schwarz,

single-cycle policy. In this stationary cycling policy production occurs at the warehouse when there is positive inventory there. A simple shifting of production leads to an obviously better nonstationary cycling policy. This shifting of production will work for any general 1 warehouse-m retailer example which leads us to the following conclusion:

Observation. If the overall optimal solution is a stationary cycling policy, then it is a single cycle policy.

Schwarz has previously demonstrated [2] that single-cycle policies are optimal for the single retailer and identical retailers cases. Further research is clearly necessary to demonstrate whether single-cycle policies are of value in a wider range of cases.[1]

[1] This research was supported in part by the Office of Naval Research under contract N00014-75-C-1172, Task NR 024-335.

References

1. Graves, Stephen C. and Schwarz, Leroy B., "Single Cycle Continuous Review Policies for Arborescent Production/Inventory Systems," *Management Sci.*, Vol. 23, No. 5 (January 1977), pp. 529–540.
2. Schwarz, L. B., "A Simple Continuous Review Deterministic One-Warehouse N-Retailer Inventory Problem," *Management Sci.*, Vol. 19, No. 5 (January 1973), pp. 555–566.

REPLY*

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ON STATIONARITY AND OPTIMALITY IN ARBORESCENT PRODUCTION/INVENTORY SYSTEMS†

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(INVENTORY PRODUCTION; INVENTORY/PRODUCTION–DETERMINISTIC MODELS)

We are indebted to Jack Muckstadt and Howard Singer for calling our attention to two errors in [1]. First, the proof of P4 presented assumed that $\inf(I_j(t) \mid t_i > t_k) > 0$, or, in other words, that the greatest lower bound on an infinite set of positive numbers is positive. This incorrect assumption invalidates the proof of P4. Fortunately, it does not invalidate P4 itself. A revised proof is presented below. Second, the lemma for Theorem 1 and Theorem 1 itself implicitly assumed that properties P1–P5 (in particular P1 and P4) of optimal policies in general apply to optimal *stationary* policies as well. Revised statements of Theorem 1 are provided below.

Proof of P4. Consider a policy P' such that retailer j violates P4. Define time zero to be the last simultaneous production point for j. Let $I_j(t)$ be the echelon inventory of j at time t, and $I_j^{\min} = \inf_{i>1}\{I_j(t_i)\}$ for t_i being the ith production point for $p(j)$ and $t_1 = 0$. There are two cases to consider: Case 1 in which $I_j^{\min}\beta 0$; and Case 2 in which $I_j^{\min} = 0$. If $I_j^{\min} > 0$, the original proof applies. If $I_j^{\min} = 0$, define $\bar{Q}_j = \sup_i\{Q_j(\tau_i')\}$ for τ_i' being the ith production point for j and $\tau_1' = 0$. Clearly

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$\overline{Q}_j < \infty$; otherwise the cost of P' would be infinite. Choose $\overline{T} \geqslant \max\{\tau_2'; [h_j/h_{p(j)}]\cdot \overline{Q}_j/D_j\}$. Define $t_k = \min\{t_i \mid t_i \geqslant \overline{T}\}$, and let $\epsilon = \min_{i=2,\ldots,k}\{I_j(t_i)\} > 0$. Define $\tau_l' = \min_i\{\tau_i' \mid \tau_i' \geqslant t_k\}$ and $\tau_m' = \min_i\{\tau_i' \mid Q_j(\tau_i') > \epsilon\}$. Note that $l > m$ by definition.

Consider policy P which is identical to P' at $p(j)$ except at time $O(t_k)$, $p(j)$ produces ϵ less (more) than in P'. At j policy P produces at times τ_i where $\tau_i = \tau_i'$ for $i = 1, 2, \ldots, m$ and for $i > l$, while $\tau_i = \tau_i' - \epsilon/D_j$ for $i = m+1, \ldots, l$; accordingly the production quantities for policy P have $Q_j(\tau_m) = Q_j(\tau_m') - \epsilon$ and $Q_j(\tau_l) = Q_j(\tau_l') + \epsilon$, while $Q_j(\tau_i) = Q_j(\tau_i')$ otherwise. Policy P has a simultaneous production point by construction at one of the times $\tau_{m+1}, \ldots, \tau_{l-1}$. Furthermore, P has costs no larger than P'. To see this note that echelon holding costs are decreased on the interval $(0, t_k)$ by $\epsilon \cdot t_k \cdot h_{p(j)}$ and increased on the interval (t_k, ∞) by $\epsilon \cdot Q_j(\tau_l') \cdot h_j/D_j$. However, by definition $Q_j(\tau_l') \leqslant \overline{Q}_j$ and by the choice of t_k, we have $\epsilon \cdot t_k \cdot h_{p(j)} \geqslant \epsilon \cdot Q_j(\tau_l') \cdot h_j/D_j$. Finally, since P has the same number of order points as P', P is no more costly than P'. By repeated application of this construction plan, we can find a policy with no greater cost than P', for which the time interval between successive simultaneous production points is finite. Q.E.D.

Theorem 1 is correctly stated as follows: *Given zero initial inventories, or equivalent, if there exists an optimal policy which is stationary, that policy is a single cycle policy.* See [1, p. 540] for proof.

Reference

1. Graves, Stephen C. and Schwarz, Leroy B., "Single Cycle Continuous Review Policies for Arborescent Production/Inventory Systems," *Management Sci.*, Vol. 23, No. 5 (January 1977), pp. 529–540.

Notes* VI

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SENSITIVITY TO DISTRIBUTIONS IN INVENTORY SYSTEMS†

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The optimal decisions and costs of several inventory systems with the s, S policy are presented showing how they are affected by different distributions of demand, different shortage costs and different leadtimes.

The numerical results seem to imply that in many cases the optimal decisions depend on the means and standard deviations of demand but not on the specific forms of the distributions.

(INVENTORY/PRODUCTION—PARAMETRIC ANALYSIS)

Undocumented numerical solutions of many inventory systems seem to indicate that the precise form of the distribution of demand in a given system is not essential for the determination of the optimal decisions in the system. In many cases the optimal decisions only depend on the mean demand and its standard deviation.

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