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Note—On "Further Results on Planning Horizons in the Production Smoothing Problem"

Robert E. Johnson, John O. McClain,

energy was spent and the variance did not decrease at all; see Hopmans and Kleijnen (1978).

Readers interested in more details about steady-state simulation, variance reduction, etc., are referred to Kleijnen (1978).

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Notes*
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ON "FURTHER RESULTS ON PLANNING HORIZONS IN THE PRODUCTION SMOOTHING PROBLEM"†

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(INVENTORY/PRODUCTION—PLANNING HORIZONS: INVENTORY/PRODUCTION—PRODUCTION SMOOTHING)

In a recent paper [1], Lee and Orr present some results based on a study of the Modigliani-Hohn (M-H) production/inventory model. There are some ambiguities and errors in the paper which, when corrected, diminish the value of its principal results.

1. In §1 of [1] a planning horizon is defined, but the definition given is that of a *strong* horizon; i.e., one that is unchanging regardless of future conditions. As discussed in Modigliani and Hohn [2, p. 64] and Lieber [3, p. 325] the planning horizon in the M-H model is a *weak* horizon, which is dependent on knowing bounds on the demand beyond the horizon period. The Wagner-Whitin algorithm, on the other hand, provides a strong horizon. The authors fail to draw this distinction in footnote 1 of [1, p. 490], where they associate the M-H and Wagner-Whitin models. §5 of [1, p. 498], also lacks the clear distinction between weak and strong horizons.

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2. There is a wide literature on planning horizons indicating the importance of the nature of the cost functions and the conditions for a horizon. In the case of the M-H model the marginal production cost $F(x)$ is assumed to be nonnegative, monotone increasing and continuous [2, p. 47]. Contrary to footnote 1, this assumption excludes the fixed charge cost component which is basic to the Wagner-Whitin inventory model. Hence the nature of the planning horizons obtained for the two models also differ.

3. In §2 of [1, p. 392], it is stated that "... it is necessary that the desired terminal inventory be zero, or else a horizon is *not* located ..." by the conditions given in Theorem 1, p. 492. However, this is an assertion that the *converse* of Theorem 1 is true, which is incorrect. As noted by Modigliani and Hohn [2], if ending inventory has any *fixed* value Z, the problem has the same structure as one with zero ending inventory, and may be transformed by treating Z as a demand in period T, rather than as inventory. That is, in the notation of [1], let

a) $S_T' = S_T + Z$,
b) $Z' = 0$,
c) total cost $C' = C + \alpha \cdot Z$.

Then the new problem has zero ending inventory, and differs from the Modigliani-Hohn structure only by the added constant $\alpha \cdot Z$ in the objective function, representing the fixed cost of carrying the terminal inventory one period. Consequently, the Modigliani-Hohn algorithm may be applied to search for a planning horizon in the transformed problem. Therefore, contrary to the assertion of the paper, $Z = 0$ is *not* required to find a M-H horizon. (Two simple counter examples are given below, which reinforce our assertion.)

4. Theorem 2 [1, p. 494] is incorrect, and the statement "The proof follows from the definition of a horizon ..." is correspondingly incorrect. Two counter examples of Theorem 2 will show this.

a) If inventory cost is infinite, no inventory will be accumulated, regardless of demand, except as required for terminal inventory. Thus there is a horizon at the first period "nonnegativity bottleneck" for any value of Z. This contradicts the "$Z = 0$" and "$Z = B$" parts of Theorem 2.

b) If inventory cost is zero, and $Z = 0$, $h_0 = 0$, $\lambda = 1$, and $S_t = 0$ except for $S_T > BT/(T-1)$, then the optimal solution is $x_t = B/(T-1)$ for $t \leqslant T-1$ and $x_T = S_T - B$. A weak horizon then occurs at the "warehouse bottleneck" at $T-1$. This contradicts the remainder of the theorem, since terminal inventory is between zero and B.

To the extent that Theorems 3 and 4 depend on Theorem 2, they are also incorrect.

5. §2 reads in part "... a value can be assigned to the terminal state h_T, which can lead to the discovery of a horizon at period N, $N < T$; *and the horizon renders the terminal state h_T, and everything that lies beyond it in time, inconsequential to the current decision!*" (italics theirs). The meaning and value of this statement are not clear. As noted above, $h_T = Z = 0$ is not a necessary condition for the existence of a horizon. Also, the discovery of a M-H *weak* horizon is always conditional on bounds on future demand levels, so demand past period T is never inconsequential to the current decision.

6. In §5 of [1], Lee and Orr state that the T period problem must be solved in order to find a planning horizon. This, they say, renders the results useless. This is not the case. The M-H approach requires data beyond the sought-for planning horizon, but only requires a bounding condition on an extrapolation of the candidate solution, not the solution of the T-period optimization problem. This approach reduces computation and the required accuracy of distant forecasts.

7. Several typographical errors also appear in this paper:

1) Equation (7), p. 491 should read:

$$\partial \mathfrak{L} / \partial x_j = \lambda^j f(x_j) + \alpha \sum_{t=j}^{T} \lambda^t - \sum_{t=j}^{T} \theta_t > 0.$$

2) Equation (8″), p. 491 should read:

$$\partial \mathfrak{L} / \partial \theta_T = Z - h_0 - X_T + S_T = 0, \qquad \theta_T \geqslant 0.$$

3) Equation (9), p. 492:

$$f(x_j) + \alpha - \theta_j / \lambda^j = \lambda f(x_{j+1}).$$

4) Equation (11), p. 494:

$$\partial \mathfrak{L} / \partial x_j = \lambda^j f(x_j) + \alpha \sum_{t=j}^{T} \lambda^t - \sum_{t=j}^{T} (\theta_t + \phi_t) > 0.$$

5) Line 7, p. 494:

$$f(x_j) + \alpha - (\theta_j + \phi_j)/\lambda^j = \lambda f(X_{j+1}).$$

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