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EFFECTS OF CENTRALIZATION ON EXPECTED COSTS IN A MULTI-LOCATION NEWSBOY PROBLEM†

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This paper concerns a multilocation newsboy problem with normal demand at each location and identical linear holding and penalty cost functions at each location. Consolidation of demand from several facilities is considered, and an expression is derived for the resulting expected holding and penalty costs as a function of the demand parameters for each location (means, variances, and correlation coefficients). The expression is used to demonstrate that (i) the expected holding and penalty costs in a decentralized system exceed those in a centralized system; (ii) the magnitude of the saving depends on the correlation of demands; and (iii) if demands are identical and uncorrelated, the costs increase as the square root of the number of consolidated demands.
(FACILITIES/EQUIPMENT PLANNING; INVENTORY/PRODUCTION–OPERATING CHARACTERISTICS; INVENTORY/PRODUCTION–STOCHASTIC MODELS)

A major wholesaler of steel products has 23 distribution centers scattered about the country. The firm is considering increasing the level of centralization in its inventory system. The process of centralization obviously influences a number of cost factors. This paper considers what might be called statistical economies of scale. It shows how under quite reasonable assumptions one can incorporate the usual holding and penalty costs into a model involving centralization and then examines the effects of the centralization process.

The basic model considers a single-period single product model with N individual sources of demand, e.g., the distribution centers for the steel wholesaler. The problem is a multi-location newsboy problem with an opportunity for centralization. Let ξ_i be the demand at location i and assume that ξ_i has a normal distribution with $E(\xi_i) = \mu_i$ and the variance of $\xi_i = \sigma_i^2$. Let σ_{ij} be the covariance of ξ_i and ξ_j and ρ_{ij} be the correlation coefficient of ξ_i and ξ_j. Normality is assumed because of its frequent applicability in applied settings and because in this case it yields particularly simple analytic results. We also assume that the coefficient of variation of demand, σ_i/μ_i, at each location is sufficiently small so that the probability of negative demand is negligible.

Assume that a separate inventory is held for each source of demand (at each distribution center) and that holding and penalty costs are constant over all sources of demand. In particular assume that linear holding and penalty costs are assessed for inventory on hand at the end of the period. If $H_i(y)$ is the expected holding and penalty cost at source i when y items are on hand at the beginning of the period then

$$H_i(y) = \int_{-\infty}^{y} h\cdot(y-\xi)\phi_i(\xi)\,d\xi + \int_{y}^{\infty} p\cdot(\xi-y)\phi_i(\xi)\,d\xi \qquad (1)$$

where $\phi_i(\cdot)$ is the density function of ξ_i. [The symbol $\Phi_i(\cdot)$ denotes the cumulative

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distribution function of ξ_i.] With rather standard steps it follows that

$$H_i(y) = hy - h\mu_i + (h+p)\sigma_i R\left(\frac{y-\mu_i}{\sigma_i}\right), \tag{2}$$

where $R(u)$ is the right-hand unit normal linear-loss integral that is tabulated and widely used in the inventory literature [1], i.e.,

$$R(u) = \int_u^\infty (w-u)\frac{1}{\sqrt{2\pi}}e^{-w^2/2}\,dw. \tag{3}$$

It is well known that if one wishes to minimize the expected holding and penalty cost at source i then one should start the period with $\bar{y}_i$ items on hand where $\bar{y}_i$ is the smallest y such that

$$\Phi_i(y) \geq \frac{p}{h+p}. \tag{4}$$

That is, the optimal initial inventory at source i is defined by a certain fractile point of the demand distribution at demand source i. Since p and h are constant across all locations, the same fractile point is optimal at each source of demand. Let $\bar{z}$ be the $(p/p+h)$th fractile of the standard normal distribution. Then it follows that

$$\bar{y}_i = \mu_i + \bar{z}\sigma_i. \tag{5}$$

and that

$$H_i(\bar{y}_i) = \left[h\bar{z} + (h+p)R(\bar{z})\right]\sigma_i. \tag{6}$$

For notational convenience, let

$$K = \left[h\bar{z} + (h+p)R(\bar{z})\right]. \tag{7}$$

This result provides a mechanism for analyzing the question of inventory centralization.

To provide a particular focus for an analysis of the effects of centralization, consider the option of satisfying the demand for all N sources of demand from one central warehouse. Let ξ_T be the demand at this warehouse where

$$\xi_T = \sum_{i=1}^{N} \xi_i. \tag{8}$$

Clearly then ξ_T is $N(\mu_T, \sigma_T)$ where

$$\mu_T = \sum_{i=1}^{N} \mu_i,$$

$$\sigma_T = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_{ij}}. \tag{9}$$

$H_C(\bar{y}_T)$ is the optimal value of the expected holding and penalty cost for the central location then

$$H_C(\bar{y}_T) = K\sigma_T. \tag{10}$$

Now consider two extreme configurations:

(i) a completely decentralized system, i.e., a system in which a separate inventory is kept to satisfy the demand at each source of demand;

(ii) a completely centralized system, i.e., a system in which all demands are satisfied from one central warehouse.

Then in the decentralized system, the cost for each source is given by

$$H_i(\bar{y}_i) = K\sigma_i, \tag{11}$$

and the total cost, TC_D is given by

$$TC_D = K\sum_{i=1}^{N}\sigma_i. \tag{12}$$

For the centralized system the total cost, TC_C, is given by

$$TC_C = K\sigma_T = K\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_{ij}} \ . \tag{13}$$

For comparison purposes it is convenient to rewrite TC_C in the form

$$TC_C = K\sqrt{\sum_{i=1}^{N}\sigma_i^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\sigma_i\sigma_j\rho_{ij}} \ . \tag{14}$$

Hence, the magnitude of the effect of centralization on the expected holding and penalty cost depends on the correlation between the demands that are centralized. From the two total cost expressions above, we can quickly establish the following results:

(i) $TC_D \geq TC_C$.

(ii) If $\rho_{ij} = 1$ for all i and j then $TC_D = TC_C$.

To more clearly understand the role of correlation in determining the effect of centralization it is useful to define a quantity, v, as follows

$$v = 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\sigma_i\sigma_j\rho_{ij}. \tag{15}$$

Then by definition

$$\sigma_T^2 = \sum_{i=1}^{N}\sigma_i^2 + v, \tag{16}$$

and v can be thought of as the contribution of centralization to the variance of the resulting demand. From the facts that $\sigma_T^2 \geq 0$ and $\rho_{ij} \leq 1$ it follows that

$$-\sum_{i=1}^{N}\sigma_i^2 \leq v \leq 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\sigma_i\sigma_j. \tag{17}$$

From (14) and (15) we know that

$$TC_C = K\sqrt{\sum_{i=1}^{N}\sigma_i^2 + v} \ . \tag{18}$$

Thus, TC_C is a concave function of v, and furthermore, the function assumes the value zero when

$$v = -\sum_{i=1}^{N}\sigma_i^2 \tag{19}$$

and TC_D when

$$v = 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\sigma_i\sigma_j. \qquad (20)$$

Note that if $v = 0$, i.e., if the demands are uncorrelated, then TC_C/TC_D is the square root of the ratio of the sum of the squares of the standard deviations to the square of the sum of the standard deviations.

To the extent that warehouse size is a result of centralization, the expression for TC_C when demands are uncorrelated ($\sigma_{ij} = 0$ for $i \neq j$) provides a convenient vehicle for investigating how costs increase as a function of warehouse size. In this case

$$TC_C = K\sqrt{\sum_{i=1}^{N}\sigma_i^2}\ . \qquad (21)$$

For simplicity, assume that σ_i^2 is constant over all locations, i.e., $\sigma_i^2 = \sigma^2$. Then

$$TC_C = K\sigma\sqrt{N}\ . \qquad (22)$$

Hence, the expected holding and penalty cost increases as the square root of warehouse size.

This paper has demonstrated that under quite reasonable assumptions the consolidation of demand can reduce the total expected holding and penalty cost in an inventory system. More importantly, the general approach and expression (6) are useful in investigating various questions concerning the design and operation of inventory systems.[1]

[1] The author is indebted to David Baier of Joseph T. Ryerson & Sons, Inc., for introducing the author to this problem and to Linus Schrage for his helpful comments on an earlier draft of this paper.

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