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Note*

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ON "A PARSIMONIOUS DESCRIPTION OF THE HENDRY SYSTEM"†

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The Hendry market structure and partitioning theory and methodology are explained and contrasted with a strictly empirical approach in this note.
(MARKETING-COMPETITION; PROBABILITY-ENTROPY)

1. Introduction

The Hendry Corporation has developed an innovative system for analyzing consumer behavior in order to suggest marketing strategies which will be appropriate for various circumstances. The Hendry system—HendroDynamics—is being used by a number of large, well-known companies which sell frequently purchased consumer products. Kalwani and Morrison [8] have provided a discussion of two related aspects of the Hendry system—brand switching and market partitioning. They call attention to the fact that the brand switching aspect of the Hendry system stems from an assumption that there is a heterogeneous population of consumers choosing among brands on the basis of the multinomial probability model—the zero-order process—and provide a useful discussion of the algebra of brand switching. While the Kalwani and Morrison discussion of the brand switching component of the Hendry system is illuminating, their description of the market partitioning methodology in the Hendry system is incomplete. Kalwani and Morrison attempted to produce a strictly empirical approach which would achieve the same results as the Hendry partitioning methodology [3], [5] without resorting to an "entropy"-based derivation. Their reasons seemed to be: (1) to show that controversy over the entropy mathematics in HendroDynamics is not a solid basis for criticizing the Hendry models, since entropy was not needed to develop their partitioning methodology, and (2) to conclude that the Hendry partitioning model amounts to a *description* of direct competing sets, not a *theory* of consumer choice. Kalwani and Morrison discussed two commonly found partitioning structures but did not discuss a third important structure.[1] Since different market structures can have diametrically opposed marketing implications it is important to understand how theory, on the one hand, and an empirical algorithm on the other hand, work in

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[1] Since only limited aspects of the Hendry system have been (privately) published, it is not surprising that various misimpressions exist about it. The Kalwani and Morrison discussion provides a valuable opportunity to clarify the important concept of market partitioning.

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discriminating between competing hypotheses about structure. Our purpose here is to: (1) provide a description of the Hendry market partitioning rationale and methodology, and (2) show the contrast between the theoretically based Hendry approach and the empirical approach of Kalwani and Morrison.

2. The Switching Constant

If a randomly chosen consumer chooses among g alternatives in a set with the probability vector $[\tilde{\theta}_1, \tilde{\theta}_2, \ldots, \tilde{\theta}_g]$ and the expectation over the population of the random variable $\tilde{\theta}_i$ is the market share of the alternative i, $E(\tilde{\theta}_i) = \theta_i$, then the stationarity assumption implies that in equilibrium the fraction of the population switching from i to j and conversely will be $E(\tilde{\theta}_i\tilde{\theta}_j) = E(\tilde{\theta}_j\tilde{\theta}_i)$. An essential property of the Hendry scheme is that switching will be proportional to shares:

$$E(\tilde{\theta}_i\tilde{\theta}_j) = K_w\theta_i\theta_j, \qquad i \neq j, \tag{1}$$

where K_w is independent of i and j. Bass, Jeuland, and Wright [2] have shown that if the multivariate distribution of $[\tilde{\theta}_1, \tilde{\theta}_2, \ldots, \tilde{\theta}_g]$ is Dirichlet, then (1) will hold.[2] The switching constant, K_w, will lie between zero and one and $\mathrm{Var}(\tilde{\theta}_i) = (1 - K_w)\theta_i(1 - \theta_i)$, $i = 1, 2, \ldots, g$. Bass [1] has shown that $(1 - K_w)$ is the correlation of successive purchases of a brand and, because it is independent of brand, has termed it as the "product class brand loyalty factor."

If (1) holds, the fraction of the population switching from i will be $K_w\theta_i\sum_{j\neq i}\theta_j = K_w\theta_i(1 - \theta_i)$ and total switching in the population will be $K_w\sum_{i=1}^{g}\theta_i(1 - \theta_i)$. Therefore

$$K_w = \text{Total Switching} \Big/ \sum_{i=1}^{g} \theta_i(1 - \theta_i). \tag{2}$$

Hence, if empirical observations exist for the fraction of the market which switches and for market shares, an empirical basis for estimating the switching constant is given by (2). Kalwani and Morrison suggest an algorithm in which K_w's are estimated empirically for different partitions in a given hypothesized partitioning structure. If the empirical switching constants for all pairs of brands in the hypothesized structure are well approximated by the K_w from (2), they indicate that the correct partitioning structure has been identified. If, however, the switching patterns are found to be inconsistent, it is suggested that a new hypothesized partitioning structure be set up and examined. This algorithm will not necessarily lead to the same conclusion about the partitioning structure of a market as the Hendry theory would suggest.

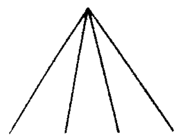
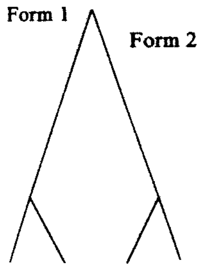
3. Theoretical Structures

Kalwani and Morrison have pointed out that from a managerial viewpoint the hierarchy of market structure relationships—partitioning—is one of the most important aspects of the Hendry approach. Different partitioning structures will suggest different marketing strategies. The purpose of studying market structure is to develop marketing strategies which will be appropriate for different types of market conditions. Shown in Table 1 are strategy implications which Hendry suggests are appropri-

[2] The Dirichlet distribution discussed in most textbooks is actually a special case of a more general distribution. In the general case (1) will not hold. Thus in general the stationarity assumption implies only the symmetry $E(\tilde{\theta}_i\tilde{\theta}_j) = E(\tilde{\theta}_j\tilde{\theta}_i)$.

TABLE 1

Three Partitioning Structures and Their Marketing Strategy Implications

Partitioning Structure		Switching Characteristics	Hendry Selected Marketing Implications
1. Brand-Primary  A B C D	On each trial consumers choose between brands Brands	Switching is proportional to share among all brands in the market.	A brand's share, loyalty, and marketing support profiles should be evaluated within the entire market rather than within brands of the same form.
2. Form-Primary Form 1 Form 2  A B C D	On each trial consumers first choose a form and then select a brand within the chosen form. Brands	Switching is much higher among alternatives within the same partition, relative to share.	A brand's share, loyalty, and marketing support profiles should be evaluated within its partition, rather than relative to all brands. Each brand's advertising elasticity is greater for moderate spending changes, with saturation points reached more quickly (vs. a brand-primary structure) because there are fewer competing brands. Spending increases by a brand in a form also increase primary demand for the form. A new form of an existing brand will get its "fair" share of that form's partition and should be marketed as a separate brand rather than a line extension.
3. Mixed		Switching among alternatives in the same form set is in the same proportion to share as if a form partition existed; however, switching across form sets is lower compared to brand primary switching patterns.	Spending increases by a brand in a form set do not increase primary demand for that set. A brand's total franchise share will be higher if all forms are introduced simultaneously, rather than sequentially. In a sequential introduction, the second form of a brand introduced will almost completely cannibalize the existing brand/form.

ate for different structures. Some of the implications have intuitive appeal, but others appear to require additional support. They apparently are based in part upon theory and in part upon generalization from experience.

The first step in the study of market partitions is the identification of hypothesized alternative structures. Attributes of the brands are used to limit the partitioning alternatives considered. Three commonly found structures are: a brand-primary market, a form-primary market, and a mixed brand-form market.

3.1 *A Brand-Primary Market*

All brands compete directly with one another in a brand-primary partitioning structure regardless of the form of the brand. Switching is proportional to share for all of the brands and a single switching constant applies for the entire market. A graphic display of such a partitioning structure and the marketing strategy implications are summarized in Table 1.

3.2 *A Form-Primary Market*

Switching between brands in a form-primary market within the same form partition (i.e., aerosol vs. nonaerosol anti-perspirants) is much higher among alternatives within the same partition relative to share than is switching across partitions. In other words, the switching constant will vary according to whether switching is across or within partitions. A graphic display of this type of market structure and some of the marketing strategy implications are shown in Table 1.

3.3 *A Mixed Brand-Form Market*

Kalwani and Morrison indicate that there can be either a "form-primary" or "brand-primary" partitioning structure. However, even if "brand" is primary, structured preferences can still exist below the brand level, using the Hendry partitioning model. A mixed brand-form is one in which consumer brand choice takes place primarily on the basis of brand image, but where form is a secondary consideration. This case implies that although consumers develop their preferences primarily on the basis of brand image, switching is disproportionately high among alternatives in the same set. Hendry calls such a market a "brand-primary with-MX-on form below brand" market. A market with structure below brand vs. the alternative hypothesis that form is primary will have subtle differences in switching patterns, but the alternative structures imply a completely different set of marketing opportunities and strategies. Selected marketing strategy implications for this type of market are shown in Table 1.

4. Hendry Partitioning Theory

If a market is partitioned as brand-primary there will be a single switching constant. An empirical estimate of this constant is given by (2). Hendry theory provides a theoretical value of Total Switching conditional upon a particular partitioning structure. Thus if the market is thought to be brand-primary, the theoretical value of Total Switching may be substituted into (2) and the resulting theoretical switching constant compared with the empirical switching constant as a basis for evaluating market structure. The theoretical value of Total Switching is called by Hendry the "entropy" of the market.[3] For a brand-primary market the Hendry theoretical Total Switching is:

$$\sum_{i=1}^{g} \frac{-\theta_i^2 \ln(\theta_i)}{1-\theta_i \ln(\theta_i)} . \tag{3}$$

We show in the appendix how (3) is derived.

[3] Herniter [6], [7] has derived a method for estimating switching using only market share information based upon "entropy." His notion of entropy is conceptually different from what Hendry calls "entropy" (see Wilson [10]). However, it doesn't matter whether the Hendry theoretical value for Total Switching is entropy or not. The point is that it is a theoretical value regardless of its name.

If the market structure is form-primary then according to Hendry theory the total switching between forms will be given by (3) where form share will be substituted for market share. Similarly, total switching within a form will be given by (3) where each brand's form share will be substituted for market share.

If the market structure is a mixed brand-form structure, Hendry theory posits that switching across forms will be given by the theoretical value for a brand-primary market and the within-form switching will be given by the theoretical values of a form-primary market.

It should be noted that theoretical switching constants are intended to apply, given the partitioning structure, under equilibrium conditions. The properties of equilibrium are: (1) no change in preference and (2) no disturbance of preference (e.g., by promotions). Therefore, if actual switching is generally somewhat higher than theoretical equilibrium switching this may only be a result of the departure of the market from equilibrium. Also, if actual switching is lower than the theoretical values, then Hendry would infer that some competitive brands (or forms) wrongly have not been included in defining the competing set. This conclusion could not be drawn from employing the approach suggested by Kalwani and Morrison.

The essential steps of both the Hendry partitioning algorithm and Kalwani and Morrison's purely empirical approach are compared graphically in Figure 1.

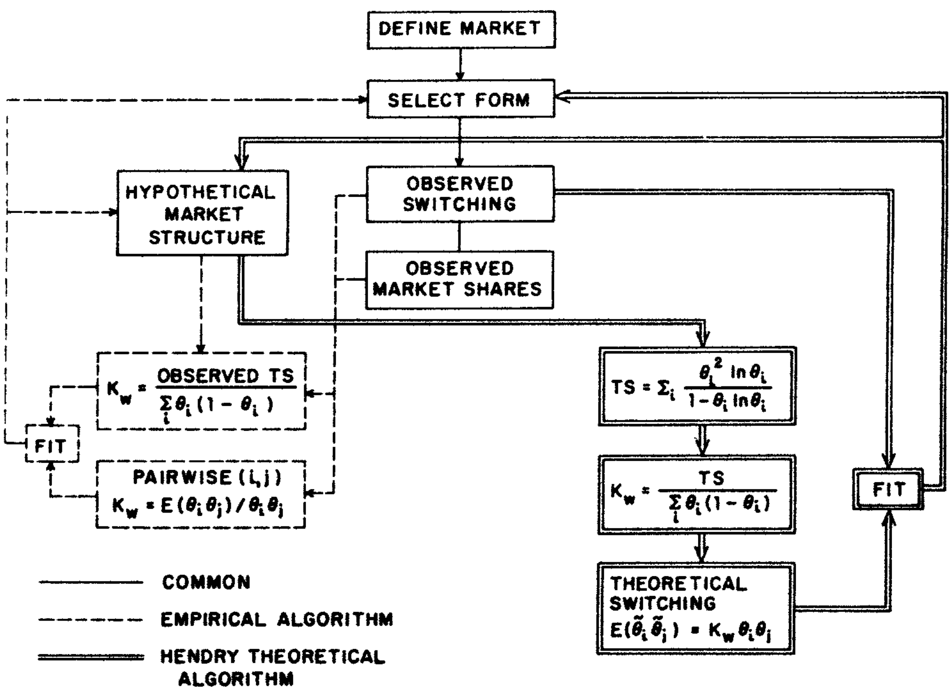


FIGURE 1. Comparison of Hendry and Pure Empirical Partitioning Algorithm.

5. Example

Both the Hendry partitioning methodology and the empirical approach are briefly illustrated for the market of facial tissues. The switching data were obtained through a mail survey and reflect purchase recall over two consecutive purchase occasions. The different attributes or forms considered were: size, number of plies, perfumed versus not, patterned versus plain, box design, price paid and color. Three individual family names were identified: Puffs®, Kleenex® and Scotties®. All others were aggregated into a fourth group, called "Others." An extensive description of the data is given in [9]. First, we shall focus on the Hendry procedure. Table 2 contains empirical and theoretical results for the size dimension. Note that in terms of total switching, the observed pattern is partially captured by the form-primary values. The switching magnitude across forms and within the small size partition is slightly overpredicted. Switching within the regular size category is underpredicted. No actual switching was observed in the large size category where the model predicted substantial switching. The theoretical and empirical proportionality factors are quite different and no specific relationship is apparent. Table 3 summarizes the Hendry theoretical results for the various attributes. Besides goodness-of-fit, it also contains observed total switching, form switching and the sample sizes. The χ^2-values and significance levels exhibit good fits for a variety of forms and structures. The different structures within a single attribute tend to produce similar fits. Some part of these remarkable results must, however, be attributed to the sample sizes which were relatively small compared to the dimensionality of the switching matrix. The best fit was obtained for a form-primary structure on size. This is illustrated in Table 4. Accordingly, brands within each form partition would compete at a much higher level than brands belonging to different form partitions.

The empirical switching constants for each pair of brands using (1) were computed following Kalwani and Morrison's approach. The results for the size dimension are contained in Table 5. Comparing these values with the K_w's obtained through (2) (see Table 2), it may be seen that the pattern closely resembles the brand-primary

TABLE 2

Partitioning Results for Facial Tissues—Size Attribute

TOTAL SWITCHING

		Hendry Theoretical	
	Actual	Form-Primary	Brand-Primary
Across Forms	0.0492	0.0653	0.2383
Within Form 1[a]	0.2222	0.2455	0.2383
Within Form 2[b]	0.3353	0.2536	0.2383
Within Form 3[c]	0.0000	0.2568	0.2383

SWITCHING CONSTANTS

	Empirical		Hendry Theoretical		
	Brand-Primary	Form-Primary	Form-Primary	Brand-Primary	Mixed
Across Forms	0.3808	0.3808	0.5056	0.3210	0.3210
Within Form 1	> 1	0.3653	0.4035	0.3210	0.4035
Within Form 2	0.4966	0.4759	0.3598	0.3210	0.3598
Within Form 3	0.0000	0.0000	0.5149	0.3210	0.5149

[a] Small.
[b] Regular.
[c] Large.

TABLE 3

Partitioning Results for Facial Tissues

Attribute and Structure	Value of χ^2-statistic	Significance Level	Total Switching	Form Switching	Sample Size
Size					
Form-Primary	47.022	0.0000	0.3607	0.0492	549
Brand-Primary	60.489	0.0000			
Mixed	50.733	0.0000			
Number of Plies					
Form-Primary	37.223	0.004	0.3466	0.0189	528
Brand-Primary	42.663	0.023			
Mixed	37.644	0.005			
Perfumed or Not					
Form-Primary	75.143	0.859	0.3596	0.1011	534
Brand-Primary	77.495	0.897			
Mixed	53.980	0.216			
Patterned or Plain					
Form-Primary	74.852	0.854	0.3914	0.1105	534
Brand-Primary	96.028	0.995			
Mixed	77.610	0.898			
Box Design					
Form-Primary	152.217	1.000	0.3948	0.1844	537
Brand-Primary	116.595	1.000			
Mixed	93.858	0.993			
Price Paid					
Form-Primary	116.841	0.055	0.4050	0.1996	521
Brand-Primary	135.791	0.344			
Mixed	115.578	0.047			
Color					
Form-Primary	261.984	0.000	0.5018	0.3456	544
Brand-Primary	358.176	0.000			
Mixed	265.366	0.000			

structure. For instance, for the small size brands, brand-primary suggests a $K_w > 1$ where the form-primary value is 0.3653.[4] Apart from the zero entries capturing non-switching, all values in Table 5 within the small size are indeed larger than one. Within both the regular and large size brands the empirical K_w's are close to those for a brand-primary structure shown in Table 2. Accordingly, the empirical algorithm identifies the brand-primary structure as the one underlying the market. This would suggest that all brands compete directly with one another, no matter what form they are. Hence, Kleenex® regular size tissues would compete with Kleenex® small size tissues at the same level as it does with Puffs® regular size tissues. This structure and its implications are quite different from the hierarchy identified by the Hendry methodology. These results do not argue that the Hendry approach is the superior one, only that it is different, as the example illustrates.

[4] K_w's > 1 were the result of the large switching relative to the small brand shares and can be attributed to small sample sizes.

TABLE 4

Joint Probability Matrix for Facial Tissues—Actual versus Predicted—Size Attribute

	Puffs®			Kleenex®			Scotties®			Others			Market
	Small 1	Regular 2	Large 3	Small 4	Regular 5	Large 6	Small 7	Regular 8	Large 9	Small 10	Regular 11	Large 12	Share at T
1	0.0091[a]	0.0036	0.0000	0.0026	0.0018	0.0000	0.0000	0.0036	0.0000	0.0018	0.0000	0.0000	0.0237
	0.0099[b]	0.0031	0.0000	0.0002	0.0024	0.0000	0.0000	0.0010	0.0000	0.0000	0.0013	0.0000	0.0178
2	0.0000	0.2714	0.0036	0.0000	0.0455	0.0036	0.0000	0.0146	0.0000	0.0000	0.0182	0.0000	0.3570
	0.0000	0.2859	0.0019	0.0000	0.0378	0.0017	0.0000	0.0167	0.0000	0.0000	0.0202	0.0000	0.3643
3	0.0000	0.0000	0.0073	0.0000	0.0000	0.0000	0.0000	0.0018	0.0000	0.0000	0.0000	0.0000	0.0091
	0.0000	0.0000	0.0049	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0051
4	0.0000	0.0055	0.0000	0.0128	0.0073	0.0000	0.0018	0.0018	0.0000	0.0000	0.0018	0.0000	0.0310
	0.0002	0.0048	0.0000	0.0155	0.0037	0.0000	0.0000	0.0016	0.0000	0.0000	0.0020	0.0000	0.0277
5	0.0000	0.0510	0.0000	0.0000	0.1767	0.0018	0.0018	0.0200	0.0000	0.0000	0.0237	0.0000	0.2750
	0.0000	0.0378	0.0014	0.0000	0.2110	0.0018	0.0000	0.0129	0.0000	0.0000	0.0156	0.0000	0.0047
6	0.0000	0.0000	0.0000	0.0000	0.0018	0.0036	0.0000	0.0000	0.0000	0.0000	0.0018	0.0000	0.0073
	0.0000	0.0000	0.0000	0.0001	0.0000	0.0045	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0047
7	0.0000	0.0000	0.0000	0.0000	0.0018	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0005	0.0000	0.0000	0.0004	0.0000	0.0016	0.0002	0.0000	0.0000	0.0002	0.0000	0.0030
8	0.0000	0.0346	0.0000	0.0018	0.0255	0.0018	0.0000	0.0619	0.0000	0.0000	0.0128	0.0000	0.1384
	0.0000	0.0167	0.0006	0.0000	0.0129	0.0006	0.0000	0.0861	0.0000	0.0000	0.0069	0.0000	0.1238
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0036	0.0000	0.0000	0.0036
	0.0000	0.0009	0.0000	0.0000	0.0007	0.0000	0.0000	0.0003	0.0000	0.0027	0.0004	0.0000	0.0050
11	0.0000	0.0164	0.0000	0.0018	0.0328	0.0000	0.0000	0.0091	0.0000	0.0000	0.0929	0.0000	0.0530
	0.0000	0.0202	0.0008	0.0000	0.0156	0.0007	0.0000	0.0069	0.0000	0.0000	0.1057	0.0000	0.1498
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Market	0.0091	0.3825	0.0109	0.0200	0.2933	0.0109	0.0036	0.1129	0.0000	0.0055	0.1512	0.0000	
Share $T+1$	0.0101	0.3699	0.0097	0.0160	0.2842	0.0088	0.0017	0.1257	0.0000	0.0029	0.1521	0.0000	

[a] Actual.
[b] Predicted by Form-Primary Structure.

TABLE 5

Pairwise Empirical K_w's

	SMALL				REGULAR				LARGE			
	Puffs® 1	Kleenex® 2	Scotties® 3	Others 4	Puffs® 5	Kleenex® 6	Scotties® 7	Others 8	Puffs® 9	Kleenex® 10	Scotties® 11	Others 12
1	—	6.217	0	23.860	0.5936	0.3862	1.746	0	0	0	0	0
2	0	—	39.216	0	0.5833	1.007	0.5616	0.6914	0	0	0	0
3	0	0	—	0	0	3.519	0	0	0	0	0	0
4	0	0	0	—	0	0	0	0	0	0	0	0
5	0	0	0	0	—	0.4329	0.3141	0.4820	0.9735	1.070	0	0
6	0	0	3.519	0	0.4853	—	0.5598	0.8168	0	0.6960	0	0
7	0	0.5616	0	0	0.7443	0.7138	—	0.9974	0	1.574	0	0
8	0	0.6914	0	0	0.4344	1.130	0.7091	—	0	0	0	0
9	0	0	0	0	0	0	1.432	0	—	0	0	0
10	0	0	0	0	0	0.6960	0	1.937	0	—	0	0
11	0	0	0	0	0	0	0	0	0	0	—	0
12	0	0	0	0	0	0	0	0	0	0	0	—

6. Conclusion

The Hendry theory provides a theoretical basis for discriminating between hypothesized market structures. Can a strictly empirical approach consistently reproduce the Hendry conclusions about structure? Perhaps. But in that case the empirical algorithm will be consistent with the Hendry theory. The contention that the Hendry System represents a "description" and not a theory of consumer choice is questionable. If a theory is a set of premises leading to conclusions then Hendry is based upon theory. Whether or not it is a good theory is another matter. The major purpose of a theory is to allow the analyst to generalize beyond his specific experience. Again, the Hendry System meets the test of being a theory when judged on this criterion. As shown in Table 1, irrespective of the product class, different marketing strategies become optimal depending on the type of market structure that exists. Although performing well in general, Hendry partitioning theory has not always successfully predicted all levels of switching observed between alternatives in a product class. In such cases, the usefulness in applying the theory is diminished since one must go outside of it in order to judge whether the hypothesized structure is incorrect or whether the poor predictions are explained by other factors.

The fact that the Hendry System has been rather widely applied suggests that it is important enough for management scientists to learn more about it and to debate its strengths and weaknesses. Hopefully, this note taken in conjunction with the Kalwani and Morrison discussion will contribute to these ends.[5]

Appendix

This appendix presents a derivation of equation (3). Although it does not use the same language or, in certain respects, the same mathematics as Hendry in the derivation, it does attempt to catch the spirit of their argument by placing it in a probability context. The mathematical development is given in two steps. First, a market is considered in which preferences of consumers for each of the g alternatives available do not exist. This is done because the fundamental assumption of Hendry about a market in which preferences do exist is an assumption about the way the existence of preference transforms a no-preference market. The number of ways in which a population of N elements can be partitioned into g subpopulations of which the first contains n_1 elements, the second n_2 elements, etc., is

$$N!/n_1!\,n_2!\cdots n_g!$$

Since preferences do not exist, it might be argued that each of these ways is equally likely.[6] Hence, in a market without preferences

$$P_r(n_1, n_2, \ldots, n_g) = 1/N!/n_1!\,n_2!\cdots n_g! \tag{4}$$

[5] The authors wish to acknowledge the comments of David Butler of the Hendry Corporation concerning this note. They are also thankful to Kimberly-Clark ("Kleenex®"), Proctor and Gamble ("Puffs®") and Scott Paper Co. ("Scotties®") for permission to use their trademark names in the example.

[6] It could also be argued that each consumer would choose between each brand with the same probability, $1/g$ (i.e., multinomial). In this case,

$$P_r(n_1, n_2, \ldots, n_g) = \left(N!/\prod_{i=1}^{g} n_i!\right)(1/g)^N.$$

However, "absence" of preference is not interpreted in this way.

where $P_r(n_1, n_2, \ldots, n_g)$ denotes the probability that a specific distribution of N over g, $\{n_i\}$, will occur. If all we know is $\{n_i\}$ or n_1 consumers choose brand 1, n_2 choose brand 2, etc., we are uncertain about the individual choice of each of the N consumers. The more ways there are for N consumers to distribute themselves over g, given $\{n_i\}$, the higher this uncertainty. Moreover, $N!/\prod_{i=1}^{g} n_i!$ is directly related to this uncertainty and so is any monotone transformation, including the logarithmic one. In thermodynamics, $\ln(N!/\prod_{i=1}^{g} n_i!)$ is called entropy (see Gibbs [4]). As shown in (4), its magnitude is also related to the probability that a specific $\{n_i\}$ will occur given each possible distribution is equally likely. Therefore, $\ln(N!/\prod_{i=1}^{g} n_i!)$ is the probability concept of entropy (see Wilson [10]). Using Stirling's approximation,

$$N!/n_1!\,n_2! \cdots n_g! = \left[\sqrt{2\pi} \Big/ \prod_{i=1}^{g} \sqrt{2\pi}\right]\left[N^{N+1/2} \Big/ \prod_{i=1}^{g} n_i^{n_i+1/2}\right]\left[e^{-N} \Big/ \prod_{i=1}^{g} e^{-n_i}\right].$$

This, in turn, may be written as

$$(1/2\pi)^{(g-1)/2}\left[N^{N+1/2} \Big/ \prod_{i=1}^{g} n_i^{n_i+1/2}\right].$$

The share of the buyers in $\theta_i = n_i/N$ and $\prod_{i=1}^{g} N^{n_i+1/2} = N^{N+g/2}$ and therefore

$$N!/n_1!\,n_2! \cdots n_g! = (1/2\pi N)^{(g-1)/2} \prod_{i=1}^{g} \theta_i^{-(n_i+1/2)}.$$

Hence, the probability that n_1 consumers will choose brand 1, n_2 will choose brand 2, and so on is

$$P_r(n_1, n_2, \ldots, n_g) = \prod_{i=1}^{g} (2\pi N)^{(g-1/2g)} \theta_i^{(n_i+1/2)} \tag{5}$$

or

$$P_r(n_1, n_2, \ldots, n_g) = \prod_{i=1}^{g} 1/W_i$$

where[7]

$$W_i = (2\pi N)^{(1-g)/2g} \theta_i^{-(n_i+1/2)}.$$

If N people can distribute themselves in $N!/n_1!\,n_2! \cdots n_g!$ ways then an "average" consumer (in a geometric mean sense) can distribute his purchases in $(N!/n_1!\,n_2! \cdots n_g!)^{1/N}$ ways. Therefore,

$$\left[P_r(n_1, n_2, \ldots, n_g)\right]^{1/N} = (1/2\pi N)^{(1-g)/2N} \prod_{i=1}^{g} \theta_i^{(\theta_i+1/2N)}.$$

As $N \to \infty$, then the "average" consumer's probability,

$$\bar{P}_r(n_1, n_2, \ldots, n_g) \to \prod_{i=1}^{g} P_i(\theta_i) = \prod_{i=1}^{g} \theta_i^{\theta_i}.$$

[7] Note that $N!/\prod_{i=1}^{g} n_i!$ after using Stirling's approximation can be expressed as $\prod_{i=1}^{g} W_i$.

Furthermore, since $\ln P_i = \theta_i \ln(\theta_i)$,

$$P_i(\theta_i) = \exp(\theta_i \ln(\theta_i)), \qquad i = 1, 2, \ldots, g,$$

where $P_i(\theta_i)$ denotes the probability that $\theta_i = n_i/N$ is an "average" consumer's probability for selecting alternative i.

The fundamental assumption in describing a market with preferences is made that brand preferences will restrict the number of ways the N consumers can distribute themselves over the choice alternatives, given a specific market share structure. This constraint could be expressed in a variety of ways. Hendry postulates that in a market in which preferences and buying habits do exist, the number of ways N consumers can distribute themselves over a set of g alternatives

$$\prod_{i=1}^{g} W_i^{v_i}$$

where W_i is defined as above and v_i represents the preference component of the probability of choosing brand i, θ_i. Hence,

$$\theta_i = u_i + v_i.$$

Moreover, $(v_1, v_2, \ldots, v_g)$ represents the vector of macro preferences and buying habits, where $(u_1, u_2, \ldots, u_g)$ captures the uncertainty or disorder.

For the average consumer $P_i^{-N} = W_i$ and $\ln W_i = -N \ln P_i$. In the limit, $\ln W_i = -N\theta_i \ln \theta_i$, or $\ln W_i = N \ln \theta_i^{-\theta_i}$. Hence, if entropy is defined as above as the logarithm of the number of ways consumers can distribute themselves it can be expressed as

$$\ln \prod_{i=1}^{g} W_i^{v_i} = \sum_{i=1}^{g} v_i \ln W_i = -N \sum_{i=1}^{g} v_i \theta_i \ln \theta_i$$

and

$$(1/N) \ln \prod_{i=1}^{g} W_i^{v_i} = -\sum_{i=1}^{g} v_i \theta_i \ln \theta_i.$$

Alternatively, since $-\sum_{i=1}^{g} v_i \theta_i \ln \theta_i$ measures total uncertainty or disorder, it is by definition equal to the sum of macro uncertainties or $\sum_{i=1}^{g} u_i$; hence

$$\sum_{i=1}^{g} u_i = -\sum_{i=1}^{g} v_i \theta_i \ln \theta_i \tag{6}$$

or

$$u_i = -v_i \theta_i \ln \theta_i, \qquad i = 1, 2, \ldots, g.$$

Since $u_i = \theta_i - v_i$, it can easily be shown that

$$v_i = \theta_i / (1 - \theta_i \ln \theta_i). \tag{7}$$

Substituting (7) into (6) yields equation (3). It is claimed by Hendry that switching in equilibrium is a measure of the uncertainty in the minds of consumers. Thus it is argued that in equilibrium (3) provides the best theoretical estimate of switching between the alternatives.

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