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Letters to the Editor

LETTERS TO THE EDITOR

Dear Sir:

In a recent paper in *Management Science*, Vol. 2, No. 2, January 1956, M. Beckman and R. Muth derive an elegant model for inventory policies in the case of lagged delivery. It is the intent of this comment to point out that some of the previous, simpler models yield a numerical answer very close to that of the new model. The allegedly significant differences are due to (a) a numerical error and (b) differences in terminology. With appropriate adjustments for these differences, it may well be argued that the additional complexities of the new model may be of dubious practical value.

The correctly calculated economic lot size in the article is approximately 30 units, rather than the 78 obtained by the authors, using their own numbers. Thus, the difference between 78 and 33, which they emphasize, is reduced to the difference between 30 and 33, *at most*. This is a high estimate of the difference because the interaction between lot size and reorder point quantity has not yet been considered.

Before bringing in this consideration, consider a simple model without interaction. Using 30 as economic lot size, one may obtain from a balancing of the costs and benefits of stocking a marginal unit. Rather than the assumption that stock-out costs are \$1.80 per day regardless of number of stock-outs, it is assumed here that stock-outs cost \$1.80 per unit demanded but not in stock[1]. Since average demand is one unit per day, the assumptions should not lead to substantially different results. The annual cost of adding the marginal unit to stock is $\$.002 \times 360$. The expected gain through avoidance of depletion is

$$\text{no. orders/year} \times \text{prob. that lead time demand} \geq \text{reorder point level} \times \text{depletion cost.}$$

Attaching numbers we have the result that it pays to add the marginal unit to stock if

$$12 \times \text{prob. (demand} \geq \text{reorder point level)} \times 1.80 > .002 \times 360.$$

That is, if

$$\text{prob. (demand} \geq \text{reorder point level)} > \frac{.002 \times 360}{12 \times 1.80} = \frac{1}{30}.$$

A reorder point level of 108 is the highest that satisfies the above inequality. The figure is extremely close to the 107 obtained by Beckman and Muth. Furthermore, consideration of the interaction between lot size and reorder point gives a lot size 32 and reorder point quantity of 107, leaving only one unit difference in lot size.

Simple graphical adjustments for goods on order (see p. 54 *Theory of In-*

[1] This assumption was used in various models of this commenter (see *Theory of Inventory Management*, Princeton 1953, pp. 56–62).

ventory Management) indicate that an *on hand* reorder point of 43 units is similar to a reorder point *on hand and on order* of 107 units. Expressing the ordering point in terms of the *sum of stocks and all orders outstanding* (Beckman and Muth, *op. cit.*, p. 155) is, therefore, not different from the older methods.

Sincerely,

T. M. WHITIN
U. S Atomic Energy Commission
Washington 25, D. C.

Dear Sir:

This is a reply to Prof. Whitin's letter concerning our paper. In the application of formula (2.3)—or rather the approximation four lines below—a numerical error was made and the correct solution is indeed 30. From this the optimal order size calculated under consideration of lags, which is 33, does indeed not differ very much. The second equation, which as we point out (p. 152 second paragraph) is of the type "marginal cost equals marginal returns", may then indeed be solved separately. Simple approximations will work.

It would be rash, however, to conclude that this is always so, i.e., that consideration of lags never makes any practical difference. For instance, when lead time demand is very large relative to the order size, a substantial effect of the delivery lags on the optimal order size may be expected.

Apart from all questions of practicality, the study of models involving delivery lags would still seem useful, for instance, as a means of determining how well the simpler models work in approximating the best solution.

Sincerely,

M. BECKMAN
R. MUTH
U. S. Atomic Energy Commission