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Book Reviews

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BOOK REVIEWS

Studies in the Mathematical Theory of Inventory and Production. KENNETH ARROW, SAMUEL KARLIN AND HERBERT SCARF. Standard University Press, 1958. 340 pp. $8.75.

Two of the most interesting classes of dynamic programming processes, viewed from the vantage points of both analysis and application, are those of inventory control and production smoothing.

The typical inventory control process may be described in the following general terms. We possess various quantities of different items for which there is a demand of stochastic nature from time to time. Since there is a penalty of some type attached to not being able to satisfy this demand, at various stages additional quantities of these items are ordered, at costs dependent upon types and quantities ordered, the times of ordering, the rate of delivery desired, and other factors as well. The problem is to determine the ordering policies which are optimal with respect to preassigned measures of efficiency.

Production smoothing processes are of quite similar structure, with the additional feature that there may or may not be costs attached to varying rates of production and rates of ordering.

Both are representative of the kinds of multi-stage decision processes that arise in economic and industrial activities.

In order to assess the role of the present volume in the continuing research in these fields, let us briefly review the very recent history of the optimal inventory problem. In the first place, there is the recognition of this decision process as a problem worthy of attention for its own sake, apart from the larger activity within which it is always imbedded. For a history of this phase, see the book by T. Whitin, *The Theory of Inventory Management*, Princeton University Press, 1953, which also contains a discussion of various preliminary mathematical attacks upon particular aspects of the general problem.

The next phase involves the analytical formulation using the techniques of the modern approach to the analysis of stochastic processes. The classic paper of Arrow, Harris and Marschak, "Optimal Inventory Policy," *Econometrica*, vol. XIX, 1951, pp. 250–272, inaugurates the use of functional equations to describe the process.

From this point the road forks. To begin with, there are essential questions pertaining to the existence and uniqueness of solution of the nonlinear functional equations obtained in this fashion. These were first treated in the paper by Dvoretzky, Kiefer and Wolfowitz, "The Inventory Problem," *Econometrica*, vol. XX, 1952, pp. 187–222, 450–466. Only after these points are settled, can one be sure of a one-to-one correspondence between the optimal policies of the original process and the solutions of the defining equations.

Secondly, there is the basic problem of studying not only optimal policies, but also sub-optimal policies. Within a class of simple feasible policies, we wish to determine the most efficient. This road is followed by Arrow, Harris and

Marschak in the paper referred to above, and the inverse problem was studied by Dvoretzky, Kiefer and Wolfowitz.

Thirdly, the functional equation can be utilized to determine the structure of optimal policies as a function of the structure of the cost and penalty functions, and also of the probability distribution occurring. This study was initiated by Bellman, Glicksberg and Gross in their paper, "On the Optimal Inventory Equation," *Management Science*, vol. II, 1955, pp. 83–104; see also R. Bellman, *Dynamic Programming*, Princeton University Press, 1957, Chapter 5.

In the book under review we find a continuation of this effort, carried out quite elegantly and in great detail, and, in addition, a continuation of the description of the outcomes of various approximate policies. This last requires the use of the tools of recent renewal and queuing theory.

One of the most interesting chapters is that devoted to time-lag processes, which is to say those in which there is an appreciable delay between ordering and delivery. Under the assumption that backlogs occur, it is shown that the number of state variables required to describe the process may be drastically reduced. Consequently, the results obtained by Bellman, Glicksberg and Gross in the paper cited above may be extended to this case. This furnishes a ready approximation to the solution of more complicated processes.

Turning to the study of production processes over time, a pioneering work is that of Pierre Massé, "Les Reserves et la Régulation de L'avenir dans la vie Économique," Paris, Hermann, 1946, Volume I, Avenir Determinie, Volume II, Avenir Aleatoire. A number of interesting processes give rise via simplified mathematical models to variational problems of the following type: "Maximize a functional of the form $J(u) = \int_0^T g(u, u', t)\, dt$, subject to combinations of constraints of the type $a_1 \leqq u(t) \leqq a_2$, $a_3 \leqq u'(t) \leqq a_4$, $\int_0^T h(u, u')\, dt \leqq a_5$." The presence of the constraints renders some of these problems of far greater difficulty than might be supposed upon first observation. Let us note in passing that questions of this nature arise with great frequency in the study of current engineering control processes.

The present series of studies contains an analytic treatment of some production processes and of some related problems arising in the consideration of hydroelectric systems. References are given to other work by Hohn, Modigliani and Simon, by Koopmans, by Karush and Vaszonyi, and by Bellman, Glicksberg and Gross. A good deal of unpublished work on these problems was carried out at RAND from 1948 on by Paxson, Danskin, and Fleming, stimulated by the book of Massé referred to above. Further study of variational problems with constraints may be found in Bellman, Glicksberg and Gross, "On some Variational Problems Occurring in the Theory of Dynamic Programming," *Rendiconti del Circolo Matematico di Palermo*, Serie II, Tomo III, 1954, pp. 1–35, and in Bellman, Fleming and Widder, "Variational Problems with Constraints," *Annali di Matematica*, Serie IV, Tomo XLI, 1956, pp. 301–323.

The principal authors, as well as Beckmann, Gessford and Muth, have maintained a high mathematical level with no sacrifice of clarity. All of the studies are worthwhile starting points for further research in this fundamental domain,

and the introductory expository section is particularly valuable in enabling the reader to understand the origin of the following analysis.

There are, however, some ways in which the level of the book could be raised even higher. In the first place, it would have been worthwhile to have included some computational results indicating the dependence of optimal policies, minimum costs and maximum profits upon the different parameters entering into the mathematical models. Along these lines, it might also have been helpful to point out that the functional equation technique of dynamic programming furnishes computational solutions to the variational problems described in the preceding sections in a quite simple fashion without the aid of any assumptions concerning convexity, concavity and so forth; see Chapter 9 of the book referred to previously.

Secondly, it should be emphasized that one of the most important aspects of economic control processes of this type is that of predicting the future demand on the basis of the observed past demand. We thus encounter "learning" or "correlative" processes of the type that have come into prominence with the development of sequential analysis by Wald, and by Wolfowitz, Blackwell, Girshick, Karlin, and others. This fundamental problem was attacked in relation to inventory processes in the second of the papers by Dvoretzky, Kiefer and Wolfowitz referred to above.

Considering the size of the book and the amount of material it contains, it is understandable, however, that the authors felt that a line must be drawn somewhere. Perhaps, we can look forward to a second set of studies devoted exclusively to these deeper problems. In any case, the book is certainly to be recommended to anyone working in the fields of mathematical economics and operations research.

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Economic Models, an Exposition. E. F. BEACH. New York: John Wiley & Sons, Inc., 1957. 227 pp. $7.50.

Operations researchers have at least a twofold interest in economic models. First, economic models have been more intensively studied than any other models dealing with social phenomena. As such they can afford valuable insights into the problems of model-building in OR. Second, the need for predictions of economic variables in most OR problems results in a direct interest in economic models as a possible means to this end. A book devoted to a survey and exposition of economic models should, therefore, be of interest to many operations researchers.

Dr. Beach has undertaken to supply such a book. He states in the preface that *Economic Models* is directed towards economic theorists with limited knowledge of mathematics and towards students of economics. Since operations researchers usually have more mathematics than the minimum which Dr. Beach is assuming they should find this book a relatively easy introduction to economic models.

Part I of the book deals with mathematical models. There is one chapter devoted to each of four kinds of models: linear, nonlinear, continuous dynamic, and sequence. Sequence models use discrete intervals of time and, hence, are based on difference equations rather than differential equations. Under linear models are included discussions of a variety of single market models and the Liu and Chang, and Clark, models for the U. S. The chapter on nonlinear models includes discussions of Hicks' classical and general models and of the Modigliani models. The chapter on continuous dynamic models covers the models of Schultz, Domar (debt), Evans (price adjustment), Allen (speculative), Samuelson (investment), and Littaler. As examples of sequence models Dr. Beach chooses the cobweb model and models due to Harrod and Samuelson. The choice of models to exemplify the various types discussed is generally good and the presentation is cogent and perspicuous.

The remainder of the book, Part II, is devoted to econometric models. This portion of the book contains 88 pages. Of these 88 pages 68 are devoted to a presentation of some elementary statistical theory, including a chapter devoted to simple regression and another to multivariate regression. This leaves only 20 pages for a discussion of the basic work of Haavelmo, Koopmans, Klein, Christ, and many others in econometrics. This is grossly inadequate and one must raise a question concerning the apportionment of space. The statistical theory is readily available elsewhere and the space could have been devoted to a fuller discussion of econometric models.

The most glaring omission in the book is the lack of any discussion of input-output models. Leontief is referenced twice but there is no mention of his work in the text. This verges on the inexcusable in a book devoted to economic models. One can also wish that Dr. Beach could have found space for some treatment of programming models.

Various minor errors occur throughout the book. For example, the slope of the line discussed on page 40 is incorrect, there is an error in the equilibrium value given on page 50, and one of the equations in the footnote on page 134 is wrong. It might be noted that Dr. Beach's graphs reverse the price and quantity axes from that which is usual in economics books in this country. This probably shouldn't, but often does, cause difficulties for persons familiar with the other presentation.

We conclude that operations researchers will probably find the first part of the book of the most interest. This provides a valuable introduction to mathematical models in economics.

Finally, we must note our objection to the price set for this book. $7.50 for this book of 227 pages is exorbitant.

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