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Letter to the Editor

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The papers by Fulkerson and Kelley are considered important because of their success in adding to the body of knowledge on the widely used critical-path and PERT-type techniques The problem of completing a project at minimum cost within a required time or of obtaining the cost-time trade-off curve is solved in these papers and in a way that makes possible effective computation for the large networks that occur in practical problems

The Committee would like to express its great appreciation for the enormous amount of work done by Dr James H Griesmer of IBM Research in helping to organize and carry out the survey in a most efficient way and for his help during the Chairman's illness

LETTER TO THE EDITOR

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(Received April 30, 1962)

SOME years ago I sought to publish another version of the paper, *A Game Theory Model of Convoy Routing*, now published in this issue of Operations Research, under a disguise

Disguises are sometimes hard to come by The basic elements of this problem were (1) merchant ships, operating on various (1a) routes, (2) submarines, preying on the ships, and (3) escort vessels, decreasing the effectiveness of the submarines I sought an operation, in which certain (1) activities were going on at various (1a) places, (2) these were elements preying on those activities, and (3) those interested in the activities under (1) were devoting effort to decrease the effectiveness of the elements doing the preying I discarded disguise after disguise because they were too obvious

Then the one reported on here occurred to me I wrote it up, with the ships, subs, and escorts replaced by the agents, police, and bribes below, and the mathematics retained It seemed then too frivolous, and I never did anything with it Years later I found the proposal in my files Now a paper giving the real problem is published I can see no harm in now publishing the memo as originally written in 1954

PROPOSED INTRODUCTION TO A PAPER FOR ORSA

A Model of an Organized Bookmaking Operation

Let us suppose that a large entrepreneur of horserace betting has n establishments situated in an environment hostile to such establishments i e , he is making illegal book in n joints

The environment in question expresses its hostility by dispatching police forces to the various establishments The group interaction of the police and the agents of the entrepreneur results in the arrest of some of the agents The number of arrests is assumed equal to the product of the volume of bets made at the establishment raided and the number of police sent there, multiplied by a proportionality factor (called p_i) depending on the character of the neighborhood in question

The entrepreneur has at his disposal amounts of money with which he may effect the release of the arrested agents A given agent can only work at one establishment, and the entrepreneur's funds at any time are limited

The following questions arise Suppose that the entrepreneur has decided on a given total volume of business, which he will distribute among the various establishments He has the given total of release monies How shall he distribute these, and in turn, how shall the police distribute their forces?

We begin the analysis of this question by considering the effect of release money on the agents Suppose that there are x_i agents at the ith establishment—we consider the number of agents proportional to the volume of business, z_i police there, and y_i dollars of release money available This gives (y_i/x_i) dollars available for the release of each agent We assume that the expectation that an arrest occurs when this amount of money is offered for each agent is

$$\exp(-q_i y_i/x_i),$$

the q_i measuring the effectiveness of a given amount of release money per agent in the ith neighborhood (This quantity depends on numerous sociological factors, such as the juvenile delinquency rate, the number of churches, overcrowding, police pay scales, the number of shooflies in the neighborhood, and on group attitudes on the part of low-level policemen towards higher authority This sort of subject has been beaten to death elsewhere*) In this way the expected number of arrests is

$$p_i x_i z_i \exp(-q_i y_i/x_i)$$

As the operations of our entrepreneur are quite large, he uses B***ks armored cars to transport funds to the various establishments, and these are billed to him at a cost depending on the establishment and proportional to the amount of funds sent there for release We assume the cost per unit to the ith establishment is b_i, so that the cost of transporting y_i dollars to the ith establishment is $b_i y_i$

Finally, each establishment has a unit operating cost proportional to the number of agents, or alternatively, the amount of business transacted there Call this cost a_i for the ith establishment, it then costs $a_i x_i$ to operate the ith establishment

The total cost of operating the ith establishment is the sum of the direct cost of operating it, the cost of supplying it with release money, and the cost of fines for effectively arrested agents Thus with units adjusted we may take the cost for all the establishments to be

$$C = \sum a_i x_i + \sum b_i y_i + \sum p_i x_i z_i \exp(-q_i y_i/x_i)$$

The profits of the ith establishment are proportional to the net amount of agents left, i e ,

$$P = \sum x_i - \sum p_i x_i z_i \exp(-q_i y_i/x_i)$$

* *See*, e g , L TACKFORS, "Soziologische attituten og oekonomie," *Skandinavisk Tidskrift*, 29 Feb 1953, or *Izvestiya Ekonomiya*, Goztekhizdat, Moscow-Leningrad, 1937 Also recent [1954] articles on shooflies (*see* the last *New Yorker*) in New York publications

The profit per unit investment is then P/C The entrepreneur seeks to maximize this, and the police seek to minimize this (the higher echelons of police, that is)

It is striking to observe that this problem, which occurred to me as a fruitful problem for study in connection with some confidential private research work, has its analogy in work already studied in military connections As these military connections are classified, I am not at liberty to divulge them But using the results obtained for them I can state that the criteria for the entrepreneur should be roughly as follows

1 There is a critical direct operating cost a^0 such that all establishments with direct costs higher than a^0 should be abandoned, all those with costs lower than that level continued in operation

2. The bribe money should be sent to those places where the sociological factor q_i is the highest, that is, where social conditions are worst or cop bribability is the highest If social conditions are good, i e , q_i is low, then little money is sent, in fact, there may be establishments where conditions are so good there is no point in sending any money at all Just give up bribing

3 If the entrepreneur is behaving optimally, the number of arrests will be directly proportional (with an absolute proportionality constant) to the number of police sent, regardless of where they go

4 If a new establishment is under consideration, it should be added if its a_i is less than a^0, otherwise not

5 There is a constant b^0 depending on all the various other constants given If this comes out negative, abandon bribing

CRITICAL PATH PLANNING—PERT INTEGRATION

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THE Critical Path Programming System, even though restricted to project planning situations in which network structure and activity statistics are invariant during reduction of project duration, is proving to be a practical and useful technique In fact, the efficiency of the algorithm developed by KELLEY* for project time reduction permits one to adjust rapidly and inexpensively to changes in network structure or activity time-cost estimates as these changes develop However, the effects of random variations in activity time estimates on earliest event occurrence time and project duration are not evaluated in the basic model described by Kelley, and the incorporation of the PERT procedure provides additional information concerning these effects

The Kelley algorithm requires a linear graph specifying activity sequence and the assignment of a convex time-cost function to each link as basic input 'Char-

* JAMES E KELLEY, JR , "Critical-Path Planning and Scheduling Mathematical Basis," *Opns Res* **9**, 296–320 (1961)