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Letters to the Editor

A NOTE ON HIGHWAY TRANSPARENCY

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IN TWO recent papers studying the delay to a single vehicle waiting at a stop sign for a sufficiently large gap in a main stream of traffic,^[1, 2] the concept of highway transparency was introduced. The highway transparency, R , was defined to be the percentage of time that a waiting driver would claim that it was safe to cross the highway with a given stream of stationary statistical properties. We stated in reference 1 that R was *not* equal to $\bar{\alpha}_0$, the probability that a driver arriving at the stop sign at a random instant of time would choose to cross the road. PROFESSOR WILLIAM JEWELL has called to our attention the fact that R is in fact equal to $\bar{\alpha}_0$, as can be verified by an integration by parts of equation (78) of reference 2, which is the expression for R in terms of given functions.

An independent derivation of the result $R = \bar{\alpha}_0$ is also possible. Suppose that we have a stream of traffic on a main highway and the distribution of headway gaps (measured in time) is described by a probability density $\varphi(t)$. Suppose that the waiting driver's characteristics are described by a gap acceptance function $\alpha(t)$ so that if a given gap has duration t the waiting driver will choose to cross the road with probability $\alpha(t)$. Then each gap is of duration $t = t_b + t_n$ where t_b is the blocked time and t_n is the nonblocked time. The transparency is then

$$R = E(t_n) / E(t) = E(t_n) / \mu,$$

where E is the expectation operator. We have only to calculate $E(t_n)$. The probability that the nonblocking interval is between x and $x + dx$ units long, given that the total gap is of duration t is $\alpha'(t-x) dx$ as has been shown in reference 2. Hence we have

$$E(t_n) = \int_0^\infty \varphi(t) dt \int_0^t x \alpha'(t-x) dx$$

But an integration by parts shows that

$$\int_0^t x \alpha'(t-x) dx = \int_0^t \alpha(x) dx$$

so that

$$E(t_n) = \int_0^\infty \varphi(t) dt \int_0^t \alpha(x) dx = \int_0^\infty \alpha(t) \Phi(t) dt,$$

where $\Phi(t) = \int_0^\infty \varphi(x) dx$. But a well-known renewal-theoretical result^[3] shows that

$\Phi(t)/\mu$ is just the equilibrium probability density for the duration of a gap measured from a random point in time. Hence

$$R = \frac{1}{\mu} \int_0^{\infty} \alpha(t) \Phi(t) dt = \int_0^{\infty} \alpha(t) \varphi_0(t) dt = \bar{\alpha}_0,$$

as asserted

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- 2 G H WEISS, A A MARADUDIN, "Some Problems in Traffic Delay," *Opns Res* **10**, 74-104 (1962)
- 3 W L SMITH, "Renewal Theory and its Ramifications," *J Roy Stat Soc B* **20**, 243-302 (1958)

A SIMPLE PROOF OF A THEOREM IN EXPONENTIAL SMOOTHING

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(Received February 7, 1963)

A simple proof is given of a theorem in exponential smoothing that is sometimes referred to as "the fundamental theorem." This theorem states that a polynomial, of degree N at most, minimizes the exponentially-discounted sum of squares of the errors if and only if it satisfies the set of $N+1$ equations obtained by equating the i th iteration of the exponentially-discounted average of the sequence to the corresponding average of the polynomial, $i=1, 2, \dots, N+1$.

D'ESOPO^[1] proves a theorem on exponential smoothing, presumably the one referred to by BROWN AND MEYER^[2] as "the fundamental theorem" in the title of their paper. The proof given by D'Esopo uses the terminology of, and a result from, the theory of vector spaces. Since the theorem in effect states that two sets of linear equations have the same solution, a simple direct proof should exist. The proof given below might be of interest.

Let α be a constant, such that $0 < \alpha < 1$. Let $\beta = 1 - \alpha$. For any sequence $x_T, x_{T-1}, x_{T-2}, \dots$ of values of a variable, define the T th member of a new sequence by

$$S_T^1(x) = \alpha \sum_{i=0}^{T-1} \beta^i x_{T-i}, \quad (1)$$

This is the exponentially-weighted average of the values of the (x) sequence up to time T , since α is the reciprocal of the sum of the weights. For $k \geq 2$ define $S_T^k(x)$ by the equation

$$S_T^k(x) = \alpha \sum_{i=0}^{T-1} \beta^i S_{T-i}^{k-1}(x), \quad (k=2, 3, \dots) \quad (2)$$