Operations Research

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

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To cite this article:
George Weiss, (1963) Letter to the Editor—A Note on Highway Transparency. Operations Research 11(3):460-461.
https://doi.org/10.1287/opre.11.3.460

Letters to the Editor

A NOTE ON HIGHWAY TRANSPARENCY

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(Received September 21, 1962)

IN TWO recent papers studying the delay to a single vehicle waiting at a stop sign for a sufficiently large gap in a main stream of traffic,[1 2], the concept of highway transparency was introduced The highway transparency, R, was defined to be the percentage of time that a waiting driver would claim that it was safe to cross the highway with a given stream of stationary statistical properties We stated in reference 1 that R was *not* equal to $\overline{\alpha_0}$, the probability that a driver arriving at the stop sign at a random instant of time would choose to cross the road PROFESSOR WILLIAM JEWELL has called to our attention the fact that R is in fact equal to $\overline{\alpha_0}$, as can be verified by an integration by parts of equation (78) of reference 2, which is the expression for R in terms of given functions

An independent derivation of the result $R=\overline{\alpha_0}$ is also possible Suppose that we have a stream of traffic on a main highway and the distribution of headway gaps (measured in time) is described by a probability density $\varphi(t)$ Suppose that the waiting driver's characteristics are described by a gap acceptance function $\alpha(t)$ so that if a given gap has duration t the waiting driver will choose to cross the road with probability $\alpha(t)$ Then each gap is of duration $t=t_b+t_n$ where t_b is the blocked time and t_n is the nonblocked time The transparency is then

$$R=E(t_n)/E(t)=E(t_n)/\mu,$$

where E is the expectation operator We have only to calculate $E(t_n)$ The probability that the nonblocking interval is between x and $x+dx$ units long, given that the total gap is of duration t is $\alpha'(t-x)\,dx$ as has been shown in reference 2 Hence we have

$$E(t_n)=\int_0^\infty \varphi(t)\,dt\int_0^t x\alpha'(t-x)\,dx$$

But an integration by parts shows that

$$\int_0^t x\alpha'(t-x)\,dx=\int_0^t \alpha(x)\,dx$$

so that

$$E(t_n)=\int_0^\infty \varphi(t)\,dt\int_0^t \alpha(x)\,dx=\int_0^\infty \alpha(t)\,\Phi(t)\,dt,$$

where $\Phi(t)=\int_t^\infty \varphi(x)\,dx$ But a well-known renewal-theoretical result[3] shows that

$\Phi(t)/\mu$ is just the equilibrium probability density for the duration of a gap measured from a random point in time Hence

$$R=\frac{1}{\mu}\int_0^\infty \alpha(t)\,\Phi(t)\,dt=\int_0^\infty \alpha(t)\,\varphi_0(t)\,dt=\overline{\alpha_0},$$

as asserted

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A SIMPLE PROOF OF A THEOREM IN EXPONENTIAL SMOOTHING

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(Received February 7, 1963)

A simple proof is given of a theorem in exponential smoothing that is sometimes referred to as "the fundamental theorem" This theorem states that a polynomial, of degree N at most, minimizes the exponentially-discounted sum of squares of the errors if and only if it satisfies the set of $N+1$ equations obtained by equating the ith iteration of the exponentially-discounted average of the sequence to the corresponding average of the polynomial, $i=1, 2, \ldots, N+1$

D'ESOPO[1] proves a theorem on exponential smoothing, presumably the one referred to by Brown and Meyer[2] as "the fundamental theorem" in the title of their paper The proof given by D'Esopo uses the terminology of, and a result from, the theory of vector spaces Since the theorem in effect states that two sets of linear equations have the same solution, a simple direct proof should exist The proof given below might be of interest

Let α be a constant, such that $0<\alpha<1$ Let $\beta=1-\alpha$ For any sequence $x_T, x_{T-1}, x_{T-2}, \ldots$ of values of a variable, define the Tth member of a new sequence by

$$S_T^1(x)=\alpha\sum_{i=0}^{i=\infty}\beta^i x_{T-i} \tag{1}$$

This is the exponentially-weighted average of the values of the (x) sequence up to time T, since α is the reciprocal of the sum of the weights For $k\geqq 2$ define $S_T^k(x)$ by the equation

$$S_T^k(x)=\alpha\sum_{i=0}^{i=\infty}\beta^i S_{T-i}^{k-1}(x), \qquad (k=2, 3 \ldots) \tag{2}$$