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Letter to the Editor—On the “Logarithmic Law” of Attrition and its Application to Tank Combat

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ON THE 'LOGARITHMIC LAW' OF ATTRITION AND ITS APPLICATION TO TANK COMBAT

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In a recent article appearing in this JOURNAL, H. K. WEISS indicates that in extremely large scale engagements during the Civil War attrition tended to follow the logarithmic law and refers to the use of this law by the author in a study of tank combat. In this study it was found that in the 1-vs-n tank engagements in North West Europe during World War II the first kill of an engagement did seem to follow the logarithmic law, but that the second kill came closer to following the square law. Data on subsequent kills became too sparse to be statistically meaningful.

IN A NUMBER of recent papers on gross models of combat, use of the Lanchester 'square law' and 'linear law' has been augmented by reference to the 'logarithmic law' of attrition. In one such paper WEISS[1] indicates that, in the Civil War, this law seemed to hold for battles involving 15,000 combatants or more on a side. Because the initial conditions cited by Weiss are so much in contrast with those encountered by the writer in trying to ferret out a model of tank combat based on World War II data, it might be of passing interest to review quickly the factors that lead to the consideration of the logarithmic law in this endeavor and the extent to which it has any applicability.

We can read qualitative accounts of the great hub-to-hub movement of massed armor on the Eastern Front and the sea-like tank battles on the African desert. However, in spite of this it would seem that there is a dearth of data on this phase of the war of the type amenable to statistical analysis. After-actions reports become sparse and sketchy in a fast moving fluid situation. Data of the type required for proper modeling is more prevalent but still meager on the small unit actions of North West Europe.

Data based on these small unit actions include the initial strengths (m_0, n_0) and survivors (m_s, n_s). Moreover, it was found that the initial strengths seldom involved more than roughly a platoon of tanks. Such low numbers of combatants in an engagement might be rationalized by saying that, although a battle might be made up of a number of such small unit actions, terrain tended to compartmentalize the actual number of tanks that could have any appreciable influence. It was decided to use the probabilistic approach in analyzing these data.

It will be recalled[2] that the transition probability from the state (m, n) to the state $(m, n-1)$ for the square law,

$$dm/dt=-an, \qquad dn/dt=-bm, \tag{1}$$

is

$$p_1=[1+(a/b)(n/m)]^{-1}; \tag{2}$$

and, for the linear law

$$dm/dt=-a, \qquad dn/dt=-b, \tag{3}$$

it is

$$p_0=[1+a/b]^{-1}. \tag{4}$$

If the logarithmic law is written in the simple form

$$dm/dt=-am, \qquad dn/dt=-bn, \tag{5}$$

then its transition probability can be written

$$p_{-1}=[1+(a/b)(n/m)^{-1}]^{-1}. \tag{6}$$

It will be noted that all three transition probabilities can be written in the form

$$p_k=[1+(a/b)(n/m)^k]^{-1}, \tag{7}$$

with the parameter k taking on the values $+1$, 0 and -1 for the square, linear, and logarithmic laws respectively. Moreover, equation (7) can be transformed into the form

$$\log[p_k/(1-p_k)]=\log\,(a/b)+k\log(n/m), \tag{8}$$

where now k appears as a linear parameter. Thus, if p_k is known for a number of values of (n/m), k can be readily determined by a linear least-squares fit.

In the 1-vs-n engagement it is possible to infer the transition probabilities directly from the initial and terminal states. The lone combatant either inflicts the first kill or is killed. If he survives the first, he either inflicts the second kill or is killed, and so on. Without going into details on the numbers we can say that on the first kill k was found to be -0.95 in good agreement with the logarithmic law. However, on the second kill, k was found to be more nearly $+1$ indicating a rapid change in the nature of the engagement. Data on subsequent kills became too sparse to be statistically meaningful.

This is the extent to which the logarithmic law was associated with tank combat. Initial contact of tank units, reaction time, deployment, and gun-armor relations interplay in a complex manner.

The association of the writer's name with the logarithmic law made by Weiss is appreciated.

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