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Technical Note—On an Algorithm of Ghare and Taylor

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On an Algorithm of Ghare and Taylor

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(Received December 4, 1972)

This note points out an example in which an algorithm reported by P. M. GHARE AND R. E. TAYLOR [*Opns. Res.* 17, 838–847 (1969)] for determining optimum redundancy in a series system does not produce an optimal solution. It presents the Ghare-Taylor solution along with a better feasible solution, and explains the necessary corrections to the Ghare-Taylor algorithm and the cause of the difficulty with it. The note thus questions the validity of the computer times reported by Ghare and Taylor and indicates a source yielding computer times for the corrected algorithm.

THE PURPOSE of this note is to question the computer times reported by P. M. GHARE AND R. E. TAYLOR in "Optimal Redundancy for Reliability in Series Systems," *Opns. Res.* **17,** 838–847(1969).

FORMULATION OF THE PROBLEM

RELIABILITY IS DEFINED as the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered. The reliability of a system of stages is the product of the individual-stage reliabilities if the failure of any stage causes failure of the system, the failure of any stage is independent of the failure of any other stage, and the system is to be operated for a fixed period of time.

The mathematical representation of the optimization problem with resource constraints and with backup units operating actively in parallel is: maximize $f(n)=\prod_{i=1}^{i=m} f_i(n_i)$, n_i integer,
subject to

$$\sum_{i=1}^{i=m} a_{ij} n_i \leqq d_j, \; j=1, \cdots, s,$$

where a_{ij} and d_j are real for all i and j, m and s are positive integers, n_i is the number of components of the ith type employed at the ith stage, $f_i(n_i)$ is the reliability of the ith stage, $f_i(n_i)=1-p_i^{n_i}$, $f(n)$ is the reliability of the system, a_{ij} is the jth type of cost of the ith type of component, d_j is the amount of the jth resource available, m is the number of stages, and s is the number of resource types.

The Ghare-Taylor Algorithm

Ghare and Taylor reformulated the problem as

$$\text{maximize } Z=\sum_{i=1}^{i=m} \sum_{k=1}^{k=\infty} c_{ik} x_{ik},$$

subject to

$$\sum_{i=1}^{i=m} \sum_{k=1}^{k=\infty} a_{ij} x_{ik} \leqq b_j, \qquad (j=1, \cdots, s)$$

$$x_{ik}=0 \text{ or } 1,$$

$$x_{ik}=0 \text{ implies } x_{ip}=0 \text{ if } p>k,$$

where $c_{ik}=\ln f_i(n_i)-\ln f_i(n_i-1)$, $b_j=d_j-\sum_{i=1}^{i=m} a_{ij}$, and k is the largest index such that $x_{ik}=1$.

Since $Z=h(k)$, where $h(k)=\ln f(n)$ and $n_i=k_i+1$, $i=1, \cdots, m$, maximizing Z is equivalent to maximizing $f(n)$.

In generating the enumeration tree for their implicit enumeration algorithm, Ghare and Taylor chose the following search procedure:

1. Select the variable x_{ik} for a branching decision that yields the highest upper bound on the $x_{ik}=1$, or 'inclusive', branch.
2. Always branch from the most recently created node associated with an inclusive branch.

EXPERIENCE WITH THE GHARE-TAYLOR ALGORITHM

THE COMPUTER TIMES reported in the Ghare and Taylor paper stand in question. After conversion to the CDC 6600 at Indiana University, the Ghare and Taylor algorithm was tested and found to be suboptimizing, as shown in the example of Table I. The listing of the algorithm was kindly supplied by P. M. Ghare. An explanation of the heuristic rule causing the lack of optimality suggests the modifications necessary to correct the problem.

The Heuristic Rule of Ghare and Taylor

The original Ghare-Taylor algorithm employs a heuristic which, while it removes any guarantee of optimality, does not prevent the algorithm from yielding optimal solutions on many small test problems. If a bound on an exclusive branch

TABLE I
The Example

d_1	d_2	d_3	d_4	d_5
205.3	141.4	155.8	77.8	235.8

i	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}
1	6.0330	7.4130	8.1290	5.4130	6.7510
2	8.1570	9.3660	3.0620	2.4240	8.7760
3	9.0510	4.3770	7.4020	8.5700	0.5620
4	1.8020	2.7000	3.6190	1.1020	9.2110
5	8.1660	7.5640	3.7580	4.6510	0.7947
6	7.5710	8.7060	7.9540	8.6510	7.8230
7	3.4300	5.5320	6.7870	7.2990	4.7430
8	9.6190	2.1210	1.0420	2.8400	9.5670
9	2.8660	4.6010	6.3490	2.3850	9.0930

p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
0.0153	0.0127	0.0408	0.0369	0.0182	0.0290	0.0473	0.0265	0.0226

The Ghare-Taylor solution

k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
0	1	1	3	1	0	1	2	1

Reliability is 0.95139.

A better feasible solution

k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
0	1	1	2	0	1	1	1	1

Utilization of resources:

Resource 1	Resource 2	Resource 3	Resource 4	Resource 5
100.9930	92.4830	87.9360	77.7080	116.3067

Reliability is 0.96078.

is found to be greater than the current best solution, a new solution is generated from the exclusive branch. If the new solution is not better than the current best, the branch is simply cut. Bounds on the exclusive branches created during the generation of a 'new solution' are disregarded so that the principles of branch and bound are violated.

Source of the Better Feasible Solution

Research carried out by the writer under the direction of JOHN F. MUTH of the Indiana University Graduate School of Business, *Optimizing Reliability with Backup Components* (D.B.A. dissertation, 1972), provides computer times for a revised version of the Ghare-Taylor algorithm.

The better feasible solution is, in fact, optimal. It was obtained by a version of the Ghare-Taylor algorithm that adhered strictly to branch-and-bound principles.

On a Paper by Simon

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(Received November 8, 1971)

In a paper on "Stationary Properties of a Two-Echelon Inventory Model for Low-Demand Items" [*Opns. Res.* 19, 761–773 (1971)], R. M. Simon presents an argument for finding the stationary distribution of inventory position that is not always valid. This note gives a corrected version of Simon's method.

THIS NOTE assumes the reader is familiar with R. M. SIMON's paper on "Stationary Properties of a Two-Echelon Inventory Model for Low-Demand Items."[1] A counterexample shows that Simon's derivation of the probability distribution of the base inventory position is not completely valid; however, it is correct for the two special cases in which nonbase-reparable failures are either all depot-reparable, or all nondepot-reparable.

Building on Simon's analysis, we derive more complex expression that is always valid. However, to obtain a solution using this expression, a large amount of enumeration is required. No attempt is made to assess the degree to which Simon's result could serve as a useful approximation to this new expression. Numerical approximations are possible.

Notation

We use Simon's basic definitions and notational scheme, which are repeated here with some additions for the reader's convenience.