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David Khalifa, Michael Hottenstein, Sumer Aggarwal,

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Cannibalization Policies for Multistate Systems

DAVID KHALIFA
Bloomsburg State College, Bloomsburg, Pennsylvania
MICHAEL HOTTENSTEIN and SUMER AGGARWAL
Pennsylvania State University, University Park, Pennsylvania

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When spare parts are not available from conventional sources, cannibalization is often the only source of replacement. The interchangeability of components, however, is restricted by part-type restrictions and by the cannibalization policy that is being followed. We investigate and compare the performance of six cannibalization policies, from no cannibalization to unrestricted cannibalization. Each policy is applied via computer simulation to fourteen distinctly defined machine configurations in which we manipulate the variable of the number of part-types.

COMPLEX SYSTEMS often operate under conditions in which cannibalization is the only source of replacements, since the inventory of spare parts is depleted and immediate replenishments are not available. The number of possible cannibalization policies, however, can be quite large, and the decision maker may need guidance in selecting the policy that best suits his situation. We attempt to provide a better understanding of the performance of different cannibalization policies by applying them, via a computer simulation, to a variety of distinctly defined systems.

1. THE PROBLEM

In this section we first describe the five characteristics of a system and the manner in which they were manipulated in the study. Then we define the measures of performance and the various cannibalization policies.

A system is described in sufficient detail by specifying five system characteristics, which from the decision-maker's point of view are *independent variables*:

1. System Size. A multistate system of size M consists of $M>1$ units, called machines [2], which can operate independently of each other. A machine is assumed to be either operable or failed, and the system can therefore be in one of $M+1$ states. In this study system size is held constant at $M=8$. All the machines are of identical design and can operate independently.

2. The Number of Locations. The number of locations in a system, n, is the "order" of the system. A location is defined as a member of the set $\Lambda=\{\lambda_1, \cdots, \lambda_n\}$ and is associated with one and only one component [3]. The ith location, $i=1, \cdots, n$, is said to be "up" if the component it contains is operable, and "down" if the component is defective. In this study each machine consists of 12 locations, and the order of the system is 96.

3. Component Reliability. The reliability of the ith component is the probability that the component installed in the ith location will perform when the activating signal is sent. We assume that a component can be in one of two states only, operable or defective. Component reliability is held constant at $r_i=0.90$, $i=1, \cdots n$, and is assumed to be independent of time. It is also assumed that (i) component lifetimes are independent of each other, (ii) the reliability of the ith component is independent of its location, and (iii) cannibalization does not affect component reliability.

TABLE I

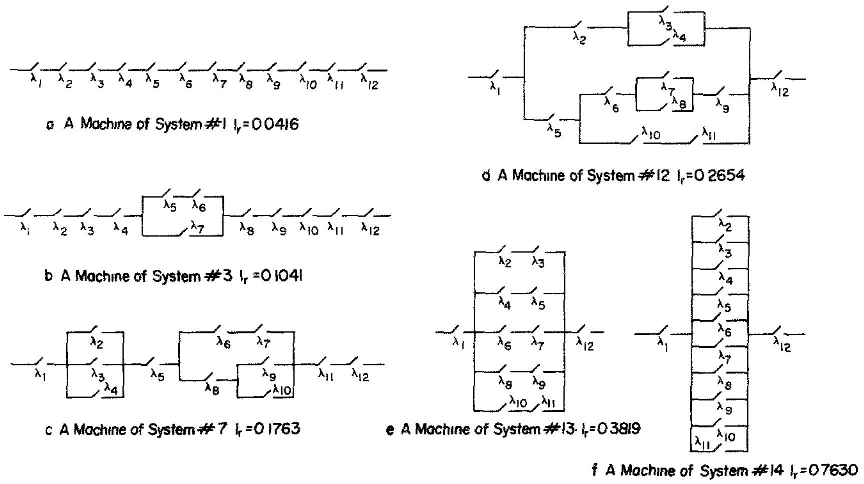
PART-TYPE—LOCATION ASSIGNMENTS

No. of part-types	Location											
	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	1	1	1	1	1	1	1	1
3	2	3	1	2	2	3	3	3	1	1	3	1
5	3	1	5	1	4	3	2	5	2	1	2	4

4. The Number of Part-Types. The n components in the system can be grouped into ω part-types, $1\leqq\omega\leqq n$, where components are interchangeable only if they are of the same part-type. Two components are said to be of the same part-type if (i) they are capable of operating in the same set of locations, and (ii) when installed in a given location, the contribution of each component to the performance of the system is the same [3]. With this definition, imperfect substitutions are also possible. Since very little is known about the interaction between ω and system performance, we manipulate it over three values, $\omega=1, 3, 5$. In the last two cases it was necessary to assign a specific part-type to each location. These associations, determined at random, are shown in Table I.

5. Machine Configuration. A machine with n locations can be represented by a switching circuit with n switches (see Figure 1), where the ith switch represents the ith location. The ith switch "closes" only if the ith location is up, and the machine is operable only if the circuit is closed. A machine with two terminals is dichotomous [1], in that it can be either operable or failed. In real life many systems consist of machines that are or can be reduced to this type.

Since machine configuration is a crucial factor of system reliability, we manipulate it here over a fairly wide range. However, from the experimental point of view this variable poses a problem of measurement since there are no established ways of representing it on a cardinal or ordinal scale. We resolved this problem by translating machine configuration into redundancy and developed a measure of system redundancy that we call the *index of redundancy*, I_r. The index is a general measure of the built-in redundancy of a system, and in a separate study [5] was found to correlate positively with system reliability.


a A Machine of System #1 $I_r=0.0416$

b A Machine of System #3 $I_r=0.1041$

c A Machine of System #7 $I_r=0.1763$

d A Machine of System #12 $I_r=0.2654$

e A Machine of System #13 $I_r=0.3819$

f A Machine of System #14 $I_r=0.7630$

Figure 1. Machine configurations.

The index of redundancy is computed by taking into account three factors: the number of locations n, the number of paths p, and the number of critical locations c. (A location is critical if the system fails when that location is down.) The first step is to compute the *index of flexibility*, $I_f=p+p(1-c/n)$ or

$$I_f=p(2-c/n). \tag{1}$$

Then
$$I_r=I_f/2n. \tag{2}$$

Example. The configuration shown in Figure 1(b) has two paths and nine critical locations. Thus $I_f=2(2-9/12)=2.5000$ and $I_r=2.5000/24=0.1041$.

In more complex configurations (see, for example, Figure 1(d)) the computational procedure is complicated by the need to partition the system into modules before computing I_f. The complete procedure is described in [5].

In this study we manipulate machine configuration over 14 values of I_r, ranging from 0.0416 to 0.7639. Some of the configurations are shown in Figure 1.

When spare parts are available only through cannibalization, as we assume here, the *decision variable* is the cannibalization policy to be followed. In the literature on cannibalization a policy is defined as a "restriction mapping" $\mu(\lambda)$ [6], which specifies all the locations from which it is permissible to cannibalize into a given location. A more meaningful, and practical, way of defining a cannibalization policy is to specify broad segments of the system from which it is permissible to cannibalize into a failed machine. Naturally, the part-type restrictions must be followed. In this study we define six cannibalization policies in that manner.

Policy 1. This is the policy of no cannibalization. Since spare parts are not available from other sources, the failed machines cannot be repaired.

Policy 2. In some cases it may be possible to repair a failed machine by cannibalizing from another location within that machine. Policy 2, then, prescribes that a component may be cannibalized only from the machine that is being repaired.

Policy 3. Policy 3 prescribes that a component may be cannibalized only from other inoperable machines.

Policy 4. This policy is a combination of policies 2 and 3.

Policy 5. Under this policy a component may be cannibalized from any other machine, operable or failed, but not from the machine that is being repaired. An obvious restriction on cannibalizing from an operable machine is that it should not result in the failure of that machine.

Policy 6. This is a combination of policies 2 and 5 and is the policy of unrestricted cannibalization.

We assume that detection and diagnosis of malfunctions are instantaneous and repair time is negligible. While these assumptions are not crucial to the analysis, they serve the purpose of isolating and clarifying the relationships of interest.

Previous studies of cannibalization [3, 4, 7] measure *system performance* by a single measure of reliability, namely, the expected system state, $E[\phi(t)]$, where t is a random instant of time, $t=1, 2, \cdots$. A general result in several studies [3] is

$$E[\phi(t)] = \textstyle\sum_{k=1}^{m} \prod_{i=1}^{\omega} \Pr\{W_i(t) \geqq n_i(k)\}, \tag{3}$$

where $W_i(t)$ is the number of operable parts of type i at time t and $n_i(k)$ is the number of operable parts of type i that will result in at least system state k (i.e., k machines available). The quantity $n_i(k)$, clearly, depends on the specific configuration of the machines in the system, and (3) is only an abstract representation of $E[\phi(t)]$ rather than a computational formula.

In this study we expand the concept of system performance and measure it by two reliability measures. For each reliability measure we also define a secondary measure of performance. The four measures are defined below.

1. The Expected System State. The expected system state, $E[\phi(t)]$, is a suitable measure of reliability for systems that operate continuously. Hirsch and Henisch [2] label such systems "k-missions systems," in that the system is "sent" on k missions, $k=1, 2, \cdots$. The first mission commences at time $t=0$, and it is assumed that at that time all the components in the system are operable.

2. Defectives per Failed Machine. This variable is the average number of defectives per failed machine, $D(t)$, and is associated with $E[\phi(t)]$ only. In terms of evaluating the resultant performance of a given policy, high $D(t)$ values are preferable, for they indicate that the defective components are concentrated in the failed machines, where they interfere the least with the reliability of the system.

3. The Mean Time to Complete Failure. The second reliability measure, MTTCF, is the average value of t when $E[\phi(t)]=0$, and is the time period at which the system reaches the state of complete failure.

4. Total Cannibalizations. This variable measures the average number of cannibalizations performed prior to system failure, C_t, and is the secondary measure of performance associated with the MTTCF.

The *experimental design* of the study consisted of a factorial design with 14 levels of system redundancy, 3 levels of the number of part-types, and 6 levels of the cannibalization policy, for a total of 252 cells. For each cell we conducted one run, consisting of 15 trials. Each trial started at time $t=0$, with all the locations being up. Then we simulated the system, under the prescribed policy, until all the machines were inoperable and beyond repair.

2. EXPERIMENTAL RESULTS AND CONCLUSIONS

As we pointed out earlier, the thrust of the study was to investigate and compare the performance of different cannibalization policies and to provide guidelines for selecting the best policy. The experimental results demonstrate that under emergency conditions cannibalization is a viable solution to the spare-parts problem, and it can be employed to maintain the system temporarily at a higher state than would be otherwise possible. The general results can be summarized as follows:

The Cannibalization Policies and Reliability. When system reliability is measured by the expected system state, higher level policies result in higher or equal reliability, as shown in Figure 2. When reliability is measured by the second measure, MTTCF, policies 3, 4, 5, and 6 result in identical

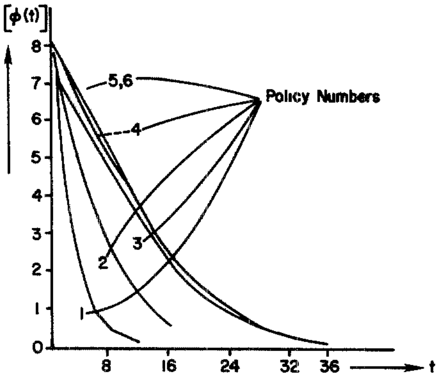


Figure 2. The cannibalization policies and the expected system state, $E[\phi(t)]$ as a function of time.

values, policy 2 is ranked second, and policy 1 has the lowest MTTCF.

The Number of Part-Types and Reliability. An increase in the number of part-types has a negative effect on $E[\phi(t)]$ (see Figure 3). The effect of increasing ω on MTTCF is also negative and follows a straight-line pattern.

System Redundancy and Reliability. System redundancy, as measured by I_r, is related positively to the two measures of reliability. Higher I_r values result in higher values of $E[\phi(t)]$ (see Figure 4) and in higher values of MTTCF (see Figure 5). In both cases, however, the upper limits of reliability are approached with relatively low values of I_r.

The Secondary Measures of Performance. The experimental results show that policies 3, 4, 5, and 6 result in the same number of cannibalizations prior to the complete failure of the system. In Figure 6 C_t is expressed as

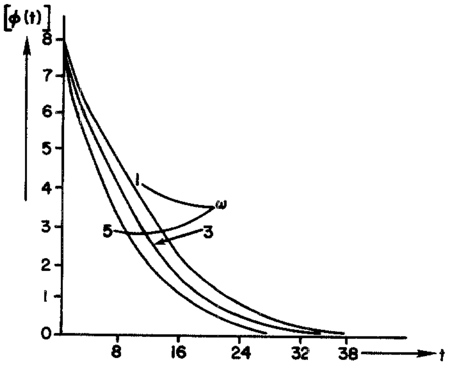


Figure 3. The number of part-types and the expected system state, $E[\phi(t)]$ as a function of time.

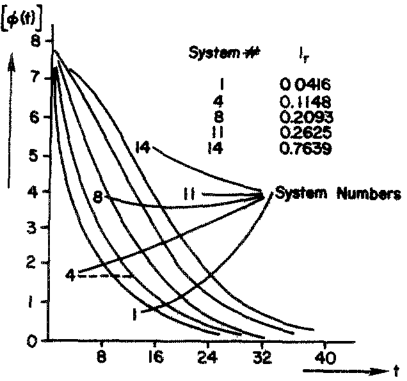


Figure 4. Selected values of system redundancy and the expected system state $E[\phi(t)]$ as a function of time.

a percentage of the total number of locations in the system (96) and is shown to increase from 0% for policy 1 to approximately 48% for policies 3–6. The other secondary measure, defectives per failed machine, was found to have the same terminal values for policies 3–6, to be lower when ω is higher, and to be higher when I_r is higher.

The results of the study provide useful insights into the reliability of multistate systems in which cannibalization is practiced, and should be

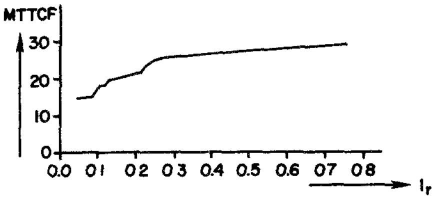


Figure 5. The relationship between system redundancy and MTTCF.

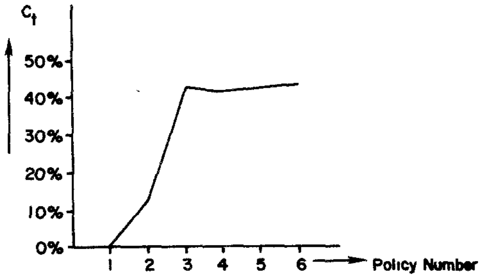


Figure 6. The relationship between the policy number and total cannibalizations, C_t.

useful in selecting the policy for a given system. The conclusions that we draw from the study are:

1. As compared to the case when cannibalization is not practiced, system reliability is improved substantially when cannibalization is employed. In general, the improvement is larger when components may be cannibalized from wider segments of the system.

2. The optimal cannibalization policy is not unique. Furthermore, policies that permit cannibalization from other machines result in the same MTTCF.

3. In most cases it is possible to trade off system reliability with the number of cannibalizations since higher reliability can be maintained only by performing a larger number of cannibalizations.

4. When cannibalization is not practiced, the number of part-types does not affect reliability. However, when cannibalization is practiced, the correlation between the two variables is negative. Since the relationship between the two variables is linear, it may be conjectured that the upper limits of reliability are reached when $\omega=1$, and the lower limits are reached when none of the components (in a single machine) are interchangeable.

5. In general, systems with higher redundancy can be maintained at higher states with fewer cannibalizations. However, the upper limits of reliability are reached with relatively small values of I_r, and increasing redundancy beyond these values does not result in further improvements in system reliability.

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