This article was downloaded by: [216.73.216.82] On: 24 July 2026, At: 16:31
Publisher: Institute for Operations Research and the Management Sciences (INFORMS)
INFORMS is located in Maryland, USA

Operations Research

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

The Plant Location Problem: New Models and Research Prospects

Charles S. Revelle, Gilbert Laporte,

To cite this article:
Charles S. Revelle, Gilbert Laporte, (1996) The Plant Location Problem: New Models and Research Prospects. Operations Research 44(6):864-874. https://doi.org/10.1287/opre.44.6.864

Full terms and conditions of use: https://pubsonline.informs.org/Publications/Librarians-Portal/PubsOnLine-Terms-and-Conditions

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact permissions@informs.org.

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

© 1996 INFORMS

Please scroll down for article—it is on subsequent pages

With 12,500 members from nearly 90 countries, INFORMS is the largest international association of operations research (O.R.) and analytics professionals and students. INFORMS provides unique networking and learning opportunities for individual professionals, and organizations of all types and sizes, to better understand and use O.R. and analytics tools and methods to transform strategic visions and achieve better outcomes.
For more information on INFORMS, its publications, membership, or meetings visit http://www.informs.org

Downloaded from informs.org by [216.73.216.82] on 24 July 2026, at 16:31 . For personal use only, all rights reserved.

THE PLANT LOCATION PROBLEM: NEW MODELS AND RESEARCH PROSPECTS

CHARLES S. REVELLE

The Johns Hopkins University, Baltimore, Maryland

GILBERT LAPORTE

Université de Montréal, Montréal, Canada

(Received August 1993; revisions received June 1995, April 1996; accepted April 1996)

The plant location problem has been studied for many years. Yet, a number of important real world issues and variants have not been investigated or resolved and merit further attention and research. This paper describes new statements of the problem (1) with new and different objectives, (2) with multiple products and multiple machines in which new models of production are considered, and (3) with spatial interactions.

The classic plant location problem has been very well studied over the years. The plant location problem is rooted in the seminal work of Weber (1909), but workable and realistic models and algorithms began to emerge only in the mid-1960s with the arrival of automatic computation capabilities. Significant results have been achieved by a variety of researchers in recent years. Some of this work has been reported and reviewed in books and surveys (Hansen et al. 1987, Brandeau and Chiu 1989, Mirchandani and Francis 1990, Francis et al. 1992, Labbé et al. 1995). Both the uncapacitated and the capacitated versions of the problem have received lavish attention, especially algorithmic attention. Nonetheless, a number of meaningful and practical issues in plant location remain unattended and unstudied. This article describes in detail the modelling issues associated with these outstanding problems and suggests new and realistic problem statements that have, to the authors' best knowledge, apparently never been considered in the literature. Problems are drawn from a recognition that certain features of existing models are not fully realistic, and from encounters with practical problems in industry. Realism seems to be achieved only with reformulation and complications in problem statement. As a result, existing solution methods are not likely to be adequate for the problems described here.

The contribution of this article is to suggest new problems and models, all having practical relevance. Although no concrete steps are taken here to resolve these unstudied issues, they are raised, lest someone think the field barren of challenges or ideas. In other words, our goal in this article is to describe new challenging problems, not solution techniques. We hope our work will stimulate needed research in the field of plant location. In recent years, powerful algorithms such as polyhedral branch and cut methods, genetic algorithms and tabu search have been suggested for a variety of hard combinatorial optimization problems. These methodologies are among those which could potentially be applied to the new problems described below.

The paper is organized as follows. In the immediately following section, we describe versions of the plant location problem in which the objectives differ significantly from those that have been explored in the past. We then develop and describe problems with multiple products and multiple machines. In the concluding section, we treat plant location problems with spatial interactions.

1. NEW OBJECTIVES IN PLANT LOCATION

1.1. The Capacitated Plant Fixed-charge Transport Location Problem

The tight formulation of the uncapacitated plant location problem due to Balinski (1965), forms the basis of several solution methods including the equivalent approaches of Bilde and Krarup (1977) and Erlenkotter (1978), as well as the LP approach of Morris (1978), and the algorithms of Galvão (1989), Körkel (1989), and Barceló et al. (1991). That formulation possesses a modelling advantage pertaining to shipment cost that is generally unrecognized, a modelling advantage that disappears when the uncapacitated version of the plant location problem gives way to the capacitated version. In the process of proceeding to

Subject classifications: Facility location: plant and machine location
Area of review. OR CHRONICLE.

0030-364X/96/4406-0864 $01.25
© 1996 INFORMS

Copyright © 2001 All Rights Reserved

Downloaded from informs.org by [216.73.216.82] on 24 July 2026, at 16:31 . For personal use only, all rights reserved.

the capacitated plant location problem, a more restrictive assumption on shipment costs becomes necessary. To see how this occurs and the modelling issue it creates, we need to review the Balinski formulation, modifying it to include linear processing or manufacturing costs:

$$\text{Minimize } z = \sum_{i\in I}\sum_{j\in J}(c_{ij} + e_i d_j)x_{ij} + \sum_{i\in I} f_i y_i ,$$

$$\text{subject to } \sum_{i\in I} x_{ij} = 1, \quad \forall j \in J ,$$

$$y_i \geqslant x_{ij}, \quad \forall i \in I, j \in J ,$$

$$x_{ij}, y_i \in \{0, 1\}, \quad \forall i \in I, j \in J, \text{ where}$$

- i, I = the index and set of eligible plant sites;
- j, J = index and set of demand points or customers;
- f_i = opening cost for a plant at i;
- e_i = expansion cost per unit of manufacture at i;
- c_{ij} = cost to deliver j's full demand from i;
- d_j = demand for product at j;
- y_i = 0, 1; it is 1 if a plant opens at i and 0 otherwise; and
- x_{ij} = 0, 1; it is 1 if i delivers j's full demand, and 0 otherwise.

As is usual in such models, all parameters are scaled so that they relate to the same planning horizon, e.g., a one-year period. The definition of the parameters is fairly standard, but one of them, c_{ij}, has an ambiguous origin. Its meaning is clear: it is the cost to transport all of j's demand, d_j, from the site i, but we are uncertain as to how c_{ij} was obtained. It could have been obtained by assuming the same transport cost for each additional unit transported from i to j, no matter the number of units; this is the linearity assumption. This assumption, however, is not necessary to arrive at a value of c_{ij}.

One assumption could be of a general concave function in quantity transported. Another assumption might be a fixed-charge cost function—a function that begins with an initial loading charge that occurs as soon as the first unit is loaded on board a transport container/vehicle. The fixed charge might be the cost to bring the transport container and loading equipment to the site of embarkation. The actual transport cost then might thereafter be linear in the amount of the item transported. The underlying function, however, does not matter in the uncapacitated plant location problem. All that matters is the cost to transport j's demand from i. This is because flows will probably never be split in an uncapacitated problem; only one plant will supply each demand point's need for goods.

In the capacitated plant location problem, though, the issue of the shape of the transport cost function takes on importance in the calculation of the actual tranpsort cost. The shape of the cost function becomes important because, if the capacity limits bind, flows from plants to demand points can be less than the full values of demand, resulting in some demand points receiving their supplies from more than one source. The transport cost of supplying demand point j in equal shares from two sources i and k would be represented as $(c_{ij} + c_{kj})/2$ using the convention established in the uncapacitated plant location problem. If the transport cost function were concave or a fixed charge function, however, the actual cost would be higher than the linear estimate because all the economies have not been captured with flows at half demand.

Thus, when plants are capacitated at levels that bind, i.e., they cause split flows, and when the transport cost function is not linear, a new and more difficult problem arises. If the transport cost has a fixed charge component, the new problem might be called the capacitated plant fixed-charge transport location problem. Its formulation might be represented using as a platform the structure suggested by Davis and Ray (1969).

$$\text{Minimize } z = \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J}(c_{ij} + e_i)x_{ij} + \sum_{i\in I}\sum_{j\in J} g_{ij} z_{ij} ,$$

$$\text{subject to } \sum_{i\in I} x_{ij} = d_j \quad \forall j \in J ,$$

$$\sum_{j\in J} x_{ij} \leqslant a_i \quad \forall i \in I ,$$

$$x_{ij} \leqslant \text{Min}(a_i, d_j)\cdot y_i \quad \forall i \in I, j \in J ,$$

$$x_{ij} \leqslant \text{Min}(a_i, d_j)\cdot z_{ij} \quad \forall i \in I, j \in J ,$$

$$x_{ij} \geqslant 0 \quad \forall i \in I, j \in J ,$$

$$y_i, z_{ij} \in \{0, 1\} \quad \forall i \in I, j \in J, \text{ where}$$

- x_{ij} = the amount supplied by i to j;
- a_i = the limit of supply from a plant at i;
- z_{ij} = 0, 1; it is 1 if $x_{ij} > 0$ and 0 otherwise;
- g_{ij} = the fixed charge component of the transport cost function; and
- c_{ij} = *unit* delivery cost (in this model).

All other parameters are as defined earlier. How this formulation might be exploited to yield a practical algorithm remains to be investigated. One possible approach would be to work along the lines suggested by Cabot and Erenguc (1984) for the fixed charge transportation problem.

1.2. The Maximum Return-on-Investment Plant Location Problem

As recently shown by Eiselt and Laporte (1995), a wide variety of objective functions have been used in location problems. However, the objective frequently used by industrial decision makers, has apparently not been explored. That is, no reference to any investigation that seeks plant locations and market area delineation under an objective of maximum return on investment has yet been found.

Meyer (1974) did explore the maximum return on investment (ROI) objective in the context of decisions on when to open plants that served predelineated regions. Meyer's problem statement did select which facilities to open and when to open them under the ROI objective, but the geographic regions that each plant would serve were

Copyright © 2001 All Rights Reserved

Downloaded from informs.org by [216.73.216.82] on 24 July 2026, at 16:31 . For personal use only, all rights reserved.

provided in advance by management judgement and experience. As a consequence, his was not a complete location problem in the sense that most analysts would define one.

In this article, we will formulate a location problem statement under the ROI objective. As expected, a demand area need not be served if the ROI is to be maximized. Return on investment is the annual return divided by the initial investment where annual return is the revenue minus cost of manufacturing and distribution. The return on investment criterion is equivalent to the objective of an individual who is choosing a bank or financial instrument into which to put his/her money, i.e., the individual would choose the bank or instrument with the highest effective interest rate (assuming equal risk of all the choices).

To make the structuring of the problem easier, we assume that the fixed and expansion costs of manufacturing (f_i and e_i) are financed from company reserves and that there is no resort to debt/amortization. The cost terms are only current manufacturing and distribution costs, and the investment shows only in the denominator of the ROI. The notation of the simple plant location problem structured earlier will be used again here, except, that we need also to define and redefine

x_{ij} = fraction of j's demand supplied by plant i;
p_j = price a unit is sold for at demand area j (a price-taker situation), and we need to distinguish the current cost of manufacture from the cost to expand the plant to build one more unit. That is, we now define
e_i = cost of expanding the physical plant per unit manufactured, and
g_i = current cost of producing a unit (including labor, materials, maintenance, repair).

The conventional assumption on c_{ij} is made as well.

The ROI plant location problem can now be structured as the optimization of a linear fractional functional subject to linear constraints (see Charnes and Cooper 1962):

$$\text{Maximize } z = \frac{\sum_{i\in I}\sum_{j\in J}(p_j d_j - c_{ij} - g_i d_j)x_{ij}}{\sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J} e_i d_j x_{ij}},$$

$$\text{subject to } \sum_{i\in I} x_{ij} \leqslant 1 \quad \forall j \in J,$$

$$y_i \geqslant x_{ij} \quad \forall i \in I, j \in J,$$

$$x_{ij} \geqslant 0 \quad \forall i \in I, j \in J,$$

$$y_i \in \{0, 1\} \quad \forall i \in I.$$

Note that a market need not be fully supplied, indeed, need not be supplied at all. As observed by Hansen (1993), there always exists an optimal solution to this model in which only the plant offering the highest return on investment is opened. It is thus meaningful to consider a second criterion and produce efficient solution sets. In this context, it makes sense to limit the size of the budget or to impose that a given proportion of the demand should be covered.

1.3. A Multiobjective Plant Location Problem

Several multiobjective plant location problems arise in practice. Here is one, practically structured, and apparently practical. A manufacturing company will be locating plants (without capacity limits) to serve a set of demands. The problem is one of locating plants to serve the demand efficiently. Efficient service, in the company president's mind, is reflected by two objectives. One is the minimum cost of transportation and manufacturing. The other is that the maximum demand be reachable by shipment within 24 hours. Such a problem arises in a number of mail or parcel delivery operations when both operating costs and service quality determine the shape of the solution. (Delorme et al. 1988). Another application recently arose in the siting of collection centers for household hazardous waste (Anex 1995). The goals are to minimize the sum of operating and disposal costs, and to maximize the population within a given time of the drop-off sites in order to increase participation in the collection program.

There are at least two ways to set this problem up. One way introduces only new sets, but no new variables or constraints. Another way, the more complicated, utilizes the notion of the maximal covering location problem and its special constraints and coverage variables (see Church and ReVelle 1974).

The problem can be cast as a bi-criterion problem, using the same notation as in the Balinski model, and some new notation:

t_{ij} = time in hours of between i and j;
$N_i = \{j : t_{ij} \leqslant 24\}$.

That is, N_i is the set of demand points j which can be served by i within 24 hours. The two objectives are:

$$\text{Minimize } z_1 = \sum_{i\in I}\sum_{j\in J}(c_{ij} + e_i d_i)x_{ij} + \sum_{i\in I} f_i y_i,$$

$$z_2 = -\sum_{i\in I}\sum_{j\in N_i} d_j x_{ij}.$$

The first objective is the usual minimum cost criterion. The second is the new objective, the maximum demand assigned to a plant capable of shipment within 24 hours. The constraint set remains the same as in the Balinski formulation shown earlier.

The second formulation, the more complicated, uses the notion of the maximal covering location problem. New sets and variables are:

$Q_j = \{i : t_{ij} \leqslant 24\}$ = the set of plants i that can serve demand node j within 24 hours;
u_j = 0, 1; it is 1 if demand j is covered by an open plant within Q_j.

The objectives are now:

$$\text{Minimize } z_1 = \text{the transport and manufacturing cost objective},$$

$$z_2 = -\sum_{j\in J} d_j u_j.$$

Copyright © 2001 All Rights Reserved

Downloaded from informs.org by [216.73.216.82] on 24 July 2026, at 16:31 . For personal use only, all rights reserved.

The first objective is the same as earlier. The second objective is the maximization of covered demand. Additional constraints are needed for this formulation:

$$u_j \leqslant \sum_{i \in Q_j} y_i \quad \forall j \in J.$$

While a superficial reading of these two formulations suggests that they both accomplish the same ends, a closer analysis reveals that they are, in fact, different and may even give different answers. The difference is subtle. The first formulation operates on an objective of maximizing demand assigned to a plant within the 24-hour coverage distance. The second formulation maximizes demand that is *covered* by an open plant anywhere within the 24-hour time standard; it does not require assignment for regular purposes to such a plant, only that a plant exists within that time standard for purposes of emergency shipments. It is possible for a demand node to be served on a regular basis by a plant outside the 24-hour time standard and be covered for the purpose of emergency shipment by a plant within the time standard. This could occur because the expansion costs, e_i, differ across plants, making it possible for a demand node to be assigned to a plant which is not its closest from a time or distance standpoint. Because of these circumstances, it may occur that the tradeoff curve for the first bi- objective formulation differs from the tradeoff curve for the second. The tradeoff curve for both formulations would be usefully traced out by application of the weighting method of multiobjective programming rather than the constraint method, because the weighting method leaves the integer friendly constraint set intact (ReVelle 1993).

As to the methodology for actual solution of the first formulation when approached by the weighting method, any of the now standard approaches to the plant location problem (Erlenkotter, Körkel) could probably be utilized because the objective and constraints will have precisely the same form as in the original Balinski formulation shown earlier in this paper. Note that since the underlying optimization problem is nonconvex, the weighting method may not identify all of the efficient points. However, each weighted solution is efficient (see Geoffrion 1968). For the second formulation, since this problem has a relatively integer friendly structure, linear programming plus branch and bound might prove a useful first step, but one cannot predict with confidence that such an approach would be useful in all cases.

2. MULTIPLE PRODUCT/MULTIPLE MACHINE PLANT LOCATION PROBLEMS

The following models arise from a practical problem investigated by the first author in the mid-eighties for a company producing checks. The company printed checks on one of several machines located in various plants scattered throughout the United States. Machines have different costs, specifications and capabilities, and the demand for checks varies across the country. This situation inspired the models described in this section. Here, we not only address the question of locating machines within plants, but we also ask where the plants should be located. We believe this family of problems covers fundamental issues that arise in the industrial sector, but which have not specifically been addressed in the plant location literature.

2.1. The Multiproduct Colocation Problem

The colocation problem is strongly related to the multicommodity or multiproduct plant location problem that was originally formulated by Warszawski (1973), and further analyzed by Karkazis and Boffey (1981). The original multicommodity location problem seeks the least cost sites for manufacture and distribution of a number of different products, where each site can accomodate the manufacture of at most one product. The multiproduct colocation problem also seeks the least cost sites for manufacture and distribution of many products, but allows, with additional costs, the manufacture of any number of these products at any site. A convenient way to think about the problem is that of siting dedicated machines in each of a number of plants, where neither plants nor machines have yet been positioned. The problem statement, notation and formulation follow.

A manufacturing firm makes a number of products indexed by r. Each product requires a different machine. That is, a particular product cannot be made on any but its particular machine. No two products require the same machine. Each machine requires a certain amount of floorspace. The machines can be located at different geographical locations or can be colocated at the same geographical position. In order for product r to be manufactured at site i, a plant must open at site i at a cost of f_i. Only then can an additional investment be made at i to produce each product. The plant opening cost includes the cost to purchase and prepare the site, install electricity, bring in water, provide parking, etc. The product opening cost is the cost to purchase, transport and install the machine or process to produce the item. Demands are located across the map; and each demand must be served. Let

i, I = the index and set of potential plant sites;
r, R = the index and set of products;
j, J = the index and set of demands;
d_j^r = demand for product r at location j;
c_{ij}^r = cost to ship all of the demand for product r at demand site j from plant i;
x_{ij}^r = 0, 1; it is 1 if plant i supplies j's demand for product r, and 0 otherwise;
f_i = cost to open a plant at i;
y_i = 0, 1; it is 1 if a plant is established at i, 0 otherwise;
g_i^r = cost to buy, transport and install a machine to make product r at site i;

Copyright © 2001 All Rights Reserved

v_i^r = 0, 1; it is 1 if product r is manufactured at i and 0 otherwise; and

e_i^r = cost per additional unit manufactured of product r at site i.

We need to structure a problem which sites plants and machines/processes to meet demands at the least possible cost. A "tight" formulation in the tradition of the Balisnki formulation of the plant location problem is suggested; i.e.,

$$\text{Minimize } z = \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{r \in R} g_i^r v_i^r + \sum_{r \in R} \sum_{i \in I} \sum_{j \in J} c_{ij}^r x_{ij}^r + \sum_{r \in R} \sum_{i \in I} e_i^r \sum_{j \in J} d_j^r x_{ij}^r,$$

$$\text{subject to } \sum_{i \in I} x_{ij}^r = 1 \quad \forall j \in J, r \in R,$$

$$v_i^r \geq x_{ij}^r \quad \forall i \in I, j \in J, r \in R,$$

$$y_i \geq v_i^r \quad \forall i \in I, r \in R,$$

$$x_{ij}^r, v_i^r, y_i \in \{0, 1\} \quad \forall i \in I, j \in J, r \in R.$$

The terms of the objective are in order (1) the total fixed costs of opening plants without regard to the products made at the plants, (2) the total fixed costs of providing machines/processes at the factories for the manufacture of specific products, (3) the cost of distribution of products, and (4) the cost of manufacture of the products at the factories. The first constraints type merely says that each demand area must receive its demand for each product. The second constraint type says that if any j received its demand for product r from site i that product r must be produced at site i. Both of these constraints correspond to constraints of the formulations of Warszawski and of Karkazis and Boffey. The third constraint says that the establishment of a factory at i must precede the manufacture of any product at i. As a consequence, a fixed opening cost for the factory itself must be incurred.

The problem could be approached as a relaxed LP because of the tight constraints utilized, but the problem could be exceptionally large making the approach problematic. The decomposition approach of Swain (1974), might also be utilized. Another possibility would be to use an approach such as that recently suggested by Holmberg (1994) to exploit the staircase cost structure of the problem. Two features of the problem could be realistically modified: the assumption of unlimited capacity of the machines, and the assumption that only one product can be manufactured on each machine. We first eliminate the assumption of unlimited capacity of a machine. However, as a step in the evolution of the problem, we reduce the problem to consider only a single product. We will then expand the problem once again to multiple products, first in a situation where no more than one machine for each product r can be installed at a site and then in a situation where multiple machines for product r can be installed at a site. Finally, we allow products to be made on more than one kind of machine.

2.2. The Single Product Capacitated Machine Siting Problem

The simple plant location problem presumes that the size of the plant in terms of its capability to manufacture product is unlimited. The machine siting problem, in contrast, places limited capacity machines for the manufacture of a single item type. Even though each machine has a finite capacity, there is no limit on the number of machines that can be placed at a site. Each machine placed, however, incurs a new fixed charge. Thus, the cost function for manufacture at site i is "lumpy." A manufacturing cost is still present because of the labor and materials needed for each additional unit of product. Here we offer a tentative formulation of the single product machine siting problem. New notation needed for the problem is:

k, K = index and set of the machines at a site;

y_i = 0, 1; it is 1 if a plant opens at i and 0 otherwise;

f_i = fixed charge opening cost at site i;

w_{ki} = 0, 1; it is 1 if k machines are placed at i and 0 otherwise;

g_i = the cost of purchasing, obtaining and installing each machine at site i;

Q = the capacity (units/month) of a single machine;

x_{ij} = the units of product delivered each month at j from i;

d_j = the monthly demand at j;

s_{ij} = the cost of delivering a single unit per month from i to j; and

e_i = the manufacturing cost at i per unit of product.

Also, x_{ij} is defined for this model as the *amount* shipped from i to j.

The objective is

$$\text{Minimize } z = \sum_{k \in K} \sum_{i \in I} g_i k w_{ki} + \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} (s_{ij} + e_i) x_{ij},$$

$$\text{subject to } \sum_{i \in I} x_{ij} = d_j \quad \forall j \in J,$$

$$\sum_{k \in K} w_{ki} \leq 1 \quad \forall i \in I,$$

$$y_i \geq w_{ki} \quad \forall i \in I, k \in K,$$

$$\sum_{j \in J} x_{ij} \leq Q \sum_{k \in K} k w_{ki} \quad \forall i \in I.$$

The first set of constraints stipulates that demand must be met at each site. The second set indicates that only one number of machines can be set up at site i, and the third set requires that plant i opens if any number of machines is sited at i. The fourth set limits production at i to Q times the number of machines installed at i.

Note that the opening cost differs from the cost of the machine and is incurred just once. Of course, if the machines are small enough, the problem essentially becomes the *Simple Plant Location Problem* because the cost function can be well approximated by a straight line. We have no experience with any algorithm for this problem.

Downloaded from informs.org by [216.73.216.82] on 24 July 2026, at 16:31 . For personal use only, all rights reserved.

Copyright © 2001 All Rights Reserved

Downloaded from informs.org by [216.73.216.82] on 24 July 2026, at 16:31 . For personal use only, all rights reserved.

2.3. The Multiple Product Machine Siting Problem (One Machine per Product per Site)

We will assume here that a machine is dedicated to the production of a single product and that only one machine for each product can be sited at the ith position and utilize the notation in Section 2.1. If the capacity of the machine installed to produce product r is limited to Q_r, we have a problem with capacities at each site. That is,

$$\sum_{j \in J} x_{ij}^r \leq Q_r v_i^r \quad \forall i \in I, r \in R .$$

The constraint says that the total of product r made at site i is less than the capacity of the machine which produces item r if the machine is installed at $i(v_i^r = 1)$ and zero if the machine is not installed at $i(v_i^r = 0)$. If it were argued that more than one machine could be installed at i, the limit Q_r could either be assumed away *or* a new variable would need to be introduced: a zero-one variable, equal to one if k machines capable of producing item r are installed at site i, where $k = 1, 2, \ldots, p$ and p is the largest conceivable number of such machines. The latter course is pursued here; multiple machines for product r are allowed at each site.

2.4. The Multiple Product Machine Siting Problem (Multiple Machines per Product per Site)

Again, we continue to assume that a machine is dedicated to the production of a single item. Since multiple products are being made we need to introduce:

Q_r = the capacity of a machine installed to produce item r;
w_{ki}^r = 0, 1; it is 1 if k machines producing item r are placed at site i, and 0 otherwise; and
s_{ij}^r = cost to supply one unit of product r from i to j.

Note that x_{ij}^r is still a flow variable rather than a zero-one variable. Except for these new variables and parameters, the problem can be structured using the notation of the previous multi-product model. The constraint on production r at site i becomes

$$\sum_{j \in J} x_{ij}^r \leq Q_r \sum_{k \in K} k w_{ki}^r \quad \forall i \in I, r \in R .$$

The remainder of the problem follows the structure of the multiproduct colocation problem. In summary, this problem is

$$\text{Minimize} \sum_{r \in R} \sum_{k \in K} \sum_{i \in I} g_i^r k w_{ki}^r + \sum_{i \in I} f_i y_i + \sum_{r \in R} \sum_{i \in I} \sum_{j \in J} s_{ij}^r x_{ij}^r + \sum_{r \in R} \sum_{i \in I} e_i^r \sum_{j \in J} x_{ij}^r ,$$

$$\text{subject to} \quad \sum_{k \in K} w_{ki}^r = 1 \quad \forall i \in I, r \in R ,$$

$$\sum_{i \in I} x_{ij}^r = d_j^r \quad \forall j \in J, r \in R ,$$

$$\sum_{j \in J} x_{ij}^r \leq Q_r \sum_{k \in K} k w_{ki}^r \quad \forall i \in I, r \in R ,$$

$$y_i \geq w_{ki}^r \quad \forall i \in I, k \in K, r \in R ,$$

$$y_i, w_{ki}^r \in \{0, 1\} \quad \forall i \in I, k \in K, r \in R .$$

Notation is defined in 2.1. except that x_{ij}^r is a flow variable. More challenging yet is relaxing the assumption that only a single product can be made on a particular machine.

2.5. The Multiple Product Machine Siting Plant Location Problem with Versatile Machines

2.5.1. The Activity-analysis Machine Purchasing Problem. This problem is a derivative of both the classic activity analysis problem and the classic machine loading problem of Charnes and Cooper (1963). It presumes that a number of different products can be made on each of a variety of different machines, but that the time for manufacture of a particular item is machine dependent. Time to adjust machines for the manufacture of different products is assumed to be negligible. Furthermore, a unit is assumed not to require more than one machine for its manufacture, and the item can be completed, if time is available, on at least one of the machines and possibly more. Finally, more than one of each machine type can be purchased. Clearly, this is a mixed integer programming problem because of machine purchase and installation, as well as an activity analysis problem.

We define

r, R = the index and set of different products;
k, K = the index and set of different machines;
x_{rk} = the monthly amount of product r made on machine type k;
t_{rk} = the time in hours to make one unit of product r on machine type k;
T_k = the hours in a month that machine type k is available for manufacture;
m_k = the number of machines of type k to be purchased;
g_k = the cost to purchase and install a single machine of type k amortized on a monthly basis;
c_{rk} = labor and energy cost to make a unit of product r on machine type k; and
d_r = the monthly demand for product r.

The time constraints are written as:

$$\sum_{r \in R} t_{rk} x_{rk} \leq T_k m_k \quad \forall k \in K .$$

The demand constraints are:

$$\sum_{k \in K} x_{rk} \geq d_r \quad \forall r \in R .$$

The objective of minimum cost to meet the required amounts of product is:

$$\text{Minimize } z = \sum_{r \in R} \sum_{k \in K} c_{rk} x_{rk} + \sum_{k \in K} g_k m_k .$$

This formulation has no spatial component; it presumes that all manufacture takes place at a single site. The lack of a spatial component can be relaxed in two steps. In the first step, manufacturing plants have already been established, and it remains to allocate machines to plants and production to the machines at specific sites. This is, in fact,

Copyright © 2001 All Rights Reserved

Downloaded from informs.org by [216.73.216.82] on 24 July 2026, at 16:31 . For personal use only, all rights reserved.

a problem that the first author encountered in a consulting situation. In the second step, plants have not been established, and the decisions to be made include plant location, machine siting and production allocation by machine and site.

2.5.2. The Activity Analysis Machine Siting Problem. The new variables, parameters and indices for this problem are:

I_E = the set of established plants, indexed by i;
J = the set of demand points, indexed by j;
m_{ki} = the number of type k machines purchased for site i;
x_{rki} = the monthly amount of product r to be made on a machine of type k at site i;
d_{rj} = the monthly requirement for product r at demand node j;
p_{rij} = the monthly amount of product r shipped from site i to demand node j;
s_{rij} = the cost to transport one unit of product r from site i to node j;
g_{ki} = the cost to purchase/install a new machine of type k at site i; and
c_{rki} = the energy and labor cost to manufacture one unit of product r on machine k at site i.

The constraints are now:

$$\sum_{r\in R} t_{rk} x_{rki} \leq T_k m_{ki} \quad \forall\, i \in I_E,\, k \in K,$$

where the above constraint limits the time utilized in making all products on machine type k to the time available on machines of type k at site i; and

$$\sum_{k\in K} x_{rki} = \sum_{j\in J} p_{rij} \quad \forall\, i \in I_E,\, r \in R,$$

where the above constraint equates the total amount of product r made on all machines at site i to the amount of product r shipped from i to all demand nodes j; and

$$\sum_{i\in I_E} p_{rij} \geq d_{rj} \quad \forall\, j \in J,\, r \in R,$$

where the above constraint requires that the amount of product r shipped from all plants i to demand node j equals or exceeds the requirement for product r at j. The minimum cost objective takes the form:

$$\text{Minimize } z = \sum_{r\in R}\sum_{i\in I_E}\sum_{j\in J} s_{rij} p_{rij} + \sum_{k\in K}\sum_{i\in I_E} g_{ki} m_{ki} + \sum_{r\in R}\sum_{k\in K}\sum_{i\in I_E} c_{rki} x_{rki},$$

where the first term is transport costs, the second is the purchase/installation of machines, and the third is the cost of manufacture exclusive of machine costs.

2.5.3. The Activity Analysis Machine Siting Plant Location Problem. In this final version of the problem both machines and plants need to be sited; manufacture needs to be allocated by product to machines and plants; and product distributed to the demand nodes. The only new notation needed is:

I, i = the set and index of eligible plant locations;
y_i = 0, 1; it is 1 if a plant is established at node i and 0 otherwise; and
f_i = the fixed charge of opening a plant at i.

All constraints in the previous formulation are needed again, and the set I_E is replaced by I. In addition, a machine cannot be sited at i unless a plant is opened at i and the upper limit of machines of type k at i is q. This constraint is written as

$$\sum_{k\in K} m_{ki} \leq q y_i .$$

As well, an additional term is needed in the objective, namely:

$$\sum_{i\in I} f_i y_i .$$

This completes our formulation of the problem of siting machines by type, siting manufacturing plants, allocating product by machine and plant site, and distributing product to demand nodes.

3. PLANT LOCATION PROBLEMS WITH SPATIAL INTERACTION

In the classical plant location problem, we are given a set I of potential plant sites, and a set J of customers or demand points. The problem is to locate plants or facilities at some of the sites and to allocate customers to these in order to minimize a function consisting of plant opening costs and of transportation costs between plants and customers. In this model, there are no interactions between plants, and all customers are served via single back and forth routes. There exist, however, several practical situations when either or both of these assumptions do not hold. These usually give rise to relatively complex problems, some of which have already been addressed in the Operations Research literature. Our aim is to describe three of these problems within a unified framework and to open up new avenues of research. These problems all belong to the class of two-layer *Location Routing Problems* described in Laporte (1988). These problems consist of simultaneously locating facilities and designing delivery or collection routes.

3.1. The Median Tour and Covering Tour Problems

We first describe the *Generalized Traveling Salesman Problem* (GTSP), a problem introduced in the late sixties. Here, the set of sites is partitioned into m subsets $I_1, \ldots, I_m$. The distance between two sites i and j is denoted by c_{ij} and the aim is to locate one facility in each subset in order to minimize the length of the Hamiltonian tour through the facilities (Figure 1). This problem has applications in the areas of vehicle routing, postal operations and computer file sequencing, for example (Laporte et al. 1987).

Copyright © 2001 All Rights Reserved

Downloaded from informs.org by [216.73.216.82] on 24 July 2026, at 16:31 . For personal use only, all rights reserved.

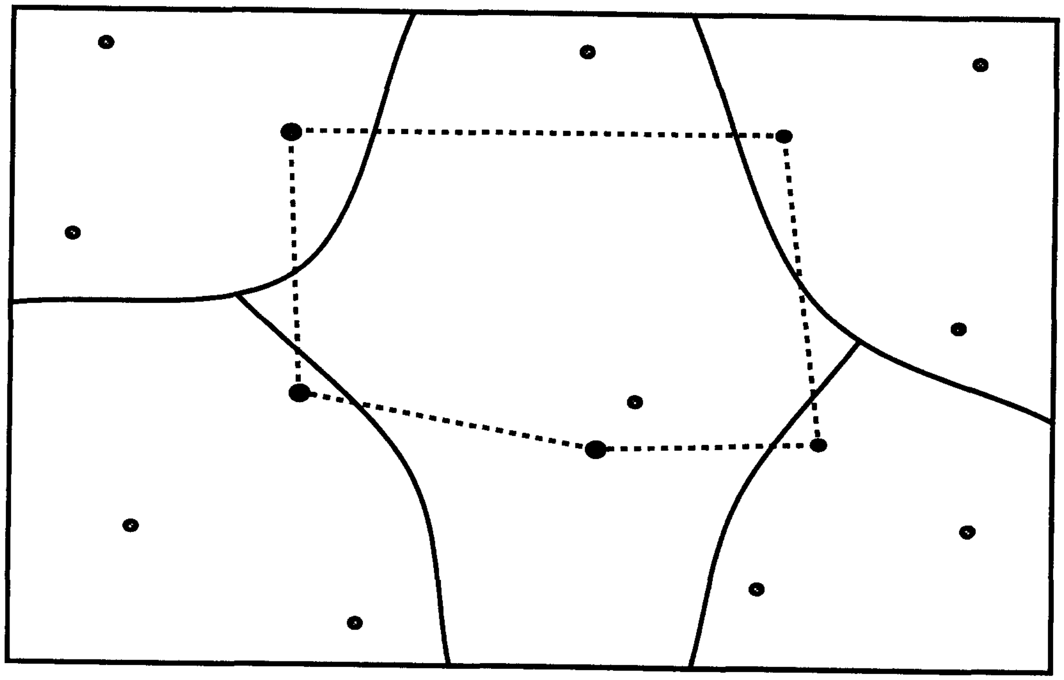

Figure 1. GTSP with $n = 15$ and $m = 5$. The Hamiltonian Tour is indicated by dotted lines.

Formulations and exact algorithms are described in Laporte and Nobert (1983) and Fischetti et al. (1992) for the case where the distance matrix (c_{ij}) is symmetric, and in Laporte et al. and Noon and Bean (1991) for the asymmetric case.

A natural extension of the GTSP is the Median Tour Problem (MTP), so called by Current and Schilling (1993). Here, the objective is to minimize the length of the Hamiltonian tour through the m facilities, plus the sum of radial distances from the remaining sites to their closest facility (Figure 2). This problem is sometimes referred to as the *Traveling Circus Problem*. Here, the sites may be thought of as villages. Over a given period, a circus will tour some of the villages, while people from the remaining villages will travel to the circus. It is assumed the circus will incur traveling and settling costs, while the profit derived from any village is a decreasing function of its distance from the tour. In some contexts, the MTP is truly a two-objective problem as the cost of the tour and that of

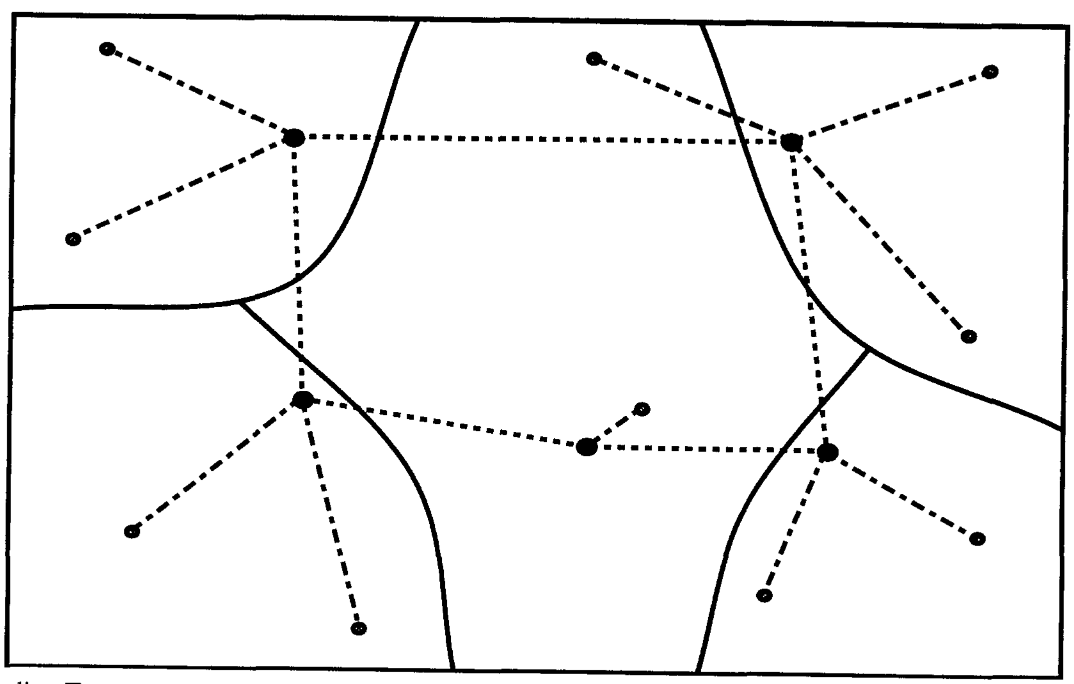

Figure 2. Median Tour problem with $n = 15$ and $m = 5$. The Median Tour is indicated by dotted lines and and the radial distances by dashed lines.

Copyright © 2001 All Rights Reserved

Downloaded from informs.org by [216.73.216.82] on 24 July 2026, at 16:31 . For personal use only, all rights reserved.

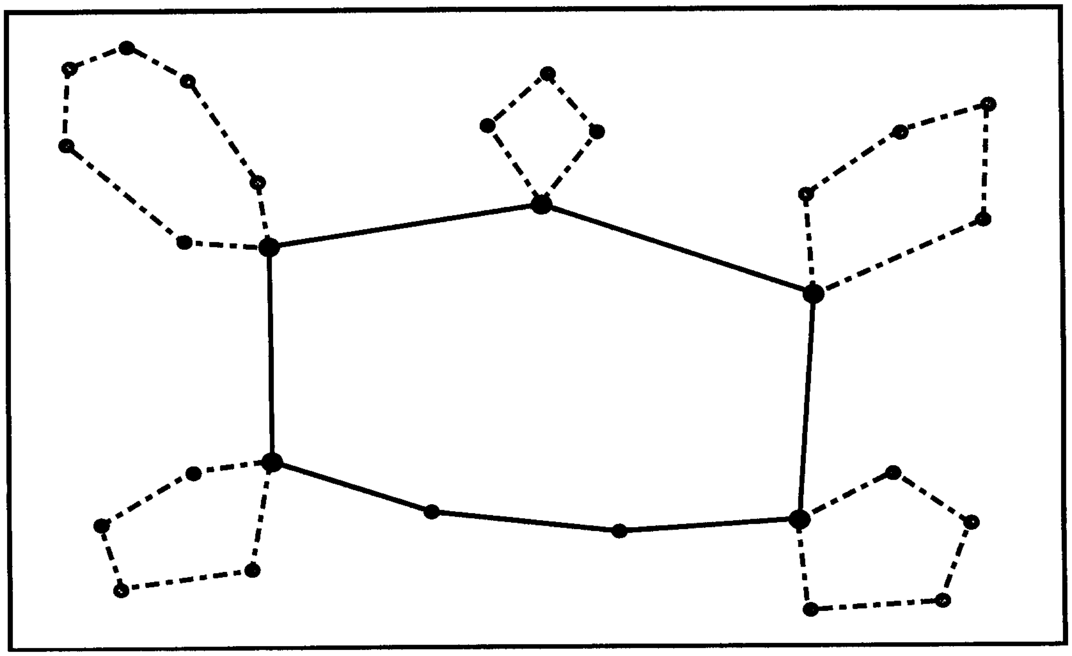

Figure 3. Primary route (full line) and secondary routes (dashed lines) in the NDP.

the radial distances are not borne by the same users. Current and Schilling (1992) propose a heuristic procedure to identify undominated solutions to the MTP. The MTP is also encountered in mailbox location. Here one wishes to locate mailboxes at potential sites in order to reduce the distance traveled by users, and the cost of emptying the mail boxes on a regular basis. The objective is a linear combination of the sum of radial distances between users and their closest mail box, and the length of a Hamiltonian tour through all mailboxes. This problem is formulated in Labbé and Laporte (1986). Finally, the MTP may be viewed as a hierarchical design problem in which the aim is to locate a main road (e.g., an expressway) through certain locations, and feeder routes to link it with the remaining locations (see Current et al. 1986).

A variant of the MTP is the *Covering Tour Problem* (CTP) in which one must design a tour through a subset of the sites, while ensuring that the distance between the tour and the the remaining sites does not exceed a given threshold (Current and Schilling 1994, Gendreau et al. 1996). This problem occurs, for example, in the planning of mobile health care facilities in developing countries (Oppong and Hodgson 1994). A variant of the mailbox location problem described above can also be modeled as a CTP. Here the distance between population centers and mailboxes is not explicitly taken into account in the objective function. Instead, this distance is controlled through a coverage constraint.

3.2. The Newspaper Delivery Problem

The *Newspaper Delivery Problem* (NDP) is derived from a study by Jacobsen and Madsen (1980). This problem is to design a primary tour through a subset of sites, and secondary tours linking the remaining sites to the primary tours (Figure 3). The total length of all tours is to be minimized. The primary tour may correspond to a newspaper delivery route comprising the printing plant and a number of transfer points (coinciding with customers, and also used as the anchor points for secondary delivery routes). The reason for this setup is that it enables all customers to be reached within a reasonable time limit. It is therefore natural to impose a maximum length constraint on the length of each route or a time window on the arrival time at each location. In the Jacobsen and Madsen study, the problem is even more complicated as there are 21 primary routes corresponding to as many printing plants.

Location-routing problems are often viewed as hierarchical decision problems in which location is a costly and often irreversible strategic decision, while routing is an operational problem. This need not always be the case. For example, in the NDP, facilities are inexpensive—all that is required is a parking space. Watson-Gandy and Dohrn (1973) describe a similar situation with "depots which effectively consisted of a parking lot with a telephone attached" (p. 324).

3.3. Multiple Tour Plant Location Problem

The *Multiple Tour Plant Location Problem* (MTPLP) is the final problem we present in the category of plant location with spatial interactions. It has aspects in common to the NDP discussed above—in the sense that tours originate from each of the facilities that are sited (Figure 4). It differs from the NDP in the sense that no tour links the facilities and the establishment of each facility incurs a fixed charge.

This is the plant location problem as it has been studied by Balinski, Körkel, and others, with the important exception that the demands are fulfilled by a multi-stop tour

Copyright © 2001 All Rights Reserved

Downloaded from informs.org by [216.73.216.82] on 24 July 2026, at 16:31 . For personal use only, all rights reserved.

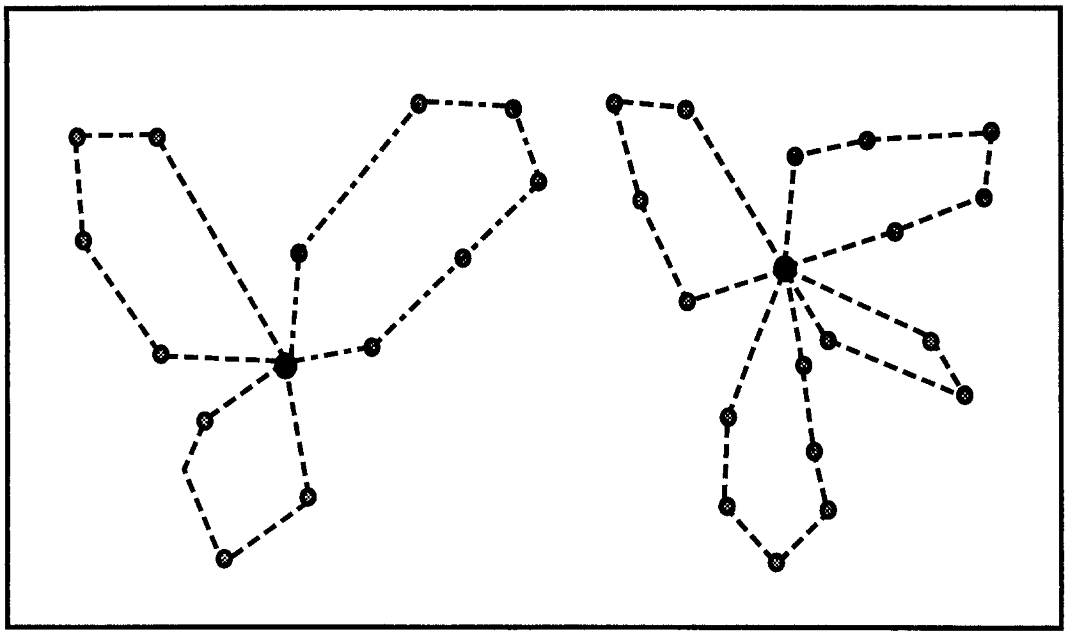

Figure 4. MTPLP with two depots.

rather than direct shipment. Kroll (1988), has studied many of the aspects of this challenging problem in the context of the delivery of medical services in a developing nation. Although a number of special cases of the MTPLP have been studied (see, e.g., Laporte 1988, and Laporte et al. 1988), there is still scope for research for this difficult problem.

3.4. Summary of Spatial Interaction Plant Location

We have considered four classes of location problems with interactions between the plants. These problems arise in a variety of practical situations and have so far received little or no attention. As a rule, these problems are more difficult than Traveling Salesman Problems of the same size, and even finding compact and workable formulations is sometimes in itself difficult. It is therefore doubtful whether exact algorithms capable of handling large size problems can be developed. Promising research avenues include the study of special structures such as tree networks, for example. Another potentially rich research area is the development of powerful heuristics. For example, recent results obtained by Gendreau et al. (1994) using tabu search methods for the Vehicle Routing Problem indicate that such methods may hold potential for the problems just described.

4. CONCLUSION

Lest some consider the field of plant location narrowing to algorithms only, we have presented a number of challenging plant location problems of different characters that have been little attempted or never before posited in the literature. The exercise is aimed at enriching the spectrum of problems for researchers to consider and creating new and more realistic decision tools for plant location.

ACKNOWLEDGEMENT

This work is partially supported by the Canadian Natural Sciences and Engineering Research Council under grant OGP 0038682. A less comprehensive version of this article appeared under the title "New Directions in Plant Location," *Studies in Locational Analysis* 5, 31–58, 1993. Permission to publish this expanded paper is gratefully acknowledged. Thanks are due to an anonymous referee for valuable comments.

REFERENCES

Anex, R. 1995. Private Communication. Davis, CA.

Balinski, M. L. 1965. Integer Programming: Methods, Uses, and Computations. *Mgmt. Sci.* **12,** 253–313.

Barceló, J., E. Fernandez, and K. O. Jörnsten. 1991. Computational Results from a New Lagrangean Relaxation Algorithm for the Capacitated Plant Location Problem. *European J. Opnl. Res.* **53,** 38–45.

Bilde, O., and J. Krarup. 1977. Sharp Lower Bounds and Efficient Algorithms for the Simple Plant Location Problem. *Ann. Discrete Math.* **1,** 79–97.

Brandeau, M. L., and S. S. Chiu. 1989. An Overview of Representative Problems in Location Research. *Mgmt. Sci.* **35,** 645–674.

Cabot, A. V., and S. S. Erenguc. 1984. Some Branch-and-Bound Procedures for Fixed Cost Transportation Problems. *Naval Res. Logist.* **31,** 145–154.

Charnes, A., and W. W. Cooper. 1963. *Management Models and Industrial Applications of Linear Programming*. Wiley, New York.

Charnes, A., and W. W. Cooper. 1962. Programming with Linear Fractional Functionals. *Naval Res. Logist.* **9,** 181–186.

Church, R. L., and C. S. ReVelle. 1974. The Maximal Covering Location Problem. *Papers of the Regional Science Association,* **32,** 101–118.

Copyright © 2001 All Rights Reserved

Current, J. R., C. S. ReVelle, and J. L. Cohon. 1986. The Hierarchical Network Design Problem. *European J. Opnl. Res.* **27,** 57–66.

Current, J. R., and D. A. Schilling. 1994. The Median Tour and Maximal Covering Tour Problems. *European J. Opnl. Res.* **73,** 114–126.

Davis, P. S., and T. L. Ray. 1969. A Branch-and-Bound Algorithm for the Capacitated Facilities Location Problem. *Naval Res. Logist.* **16,** 331–344.

Delorme, L., J. Roy, and J.-M. Rousseau. 1988. Motor-Carriers Operations Planning Models: A State of the Art. In L. Bianco and A. LaBella (eds.), *Freight Transport Planning and Logistics*. Lecture Notes in Economics and Mathematical Systems, Springer-Verlag, Berlin.

Eiselt, H. A., and G. Laporte. 1995. Objective in Location Problems. In Z. Drezner (ed.), *Facility Location: A Survey of Applications and Methods*. Springer-Verlag, Berlin.

Erlenkotter, D. 1978. A Dual-Based Procedure for Uncapacitated Facility Location. *Opns. Res.* **26,** 992–1009.

Fischetti, M., J. Salazar, and P. Toth. 1992. A Branch and Cut Algorithm for the Symmetric Generalized Traveling Salesman Problem. Presented at the ORSA/TIMS Meeting, San Francisco.

Francis, R. L., L. F. McGinnis, and J. A. White. 1992. *Facility Layout and Location: An Analytical Approach*. Prentice Hall, Englewood Cliffs, NJ.

Gendreau, M., A. Hertz, and G. Laporte. 1994. A Tabu Search Heuristic for the Vehicle Routing Problem. *Mgmt. Sci.* **40,** 1276–1290.

Gendreau, M., G. Laporte, and F. Semet. 1996. The Covering Tour Problem. *Opns. Res.* Forthcoming.

Geoffrion, A. M. 1968. Proper Efficiency and the Theory of Vector Maximization. *J. Math. Analyses and Application,* **22,** 618–630.

Hansen, P. 1993. Private communication.

Hansen, P., M. Labbé, D. Peeters, and J.-F. Thisse. 1987. Single Facility Location on Networks. *Ann. Discrete Math.* **31,** 113–146.

Hodder, J. E. 1984. Financial Market Approaches to Facility Location under Uncertainty. *Opns. Res.* **32,** 1374–1380.

Holmberg, K. 1994. Solving the Staircase Cost Facility Location Problem with Decomposition and Piecewise Linearization. *European J. Opnl. Res.* **75,** 41–61.

Jacobsen, S. K., and O. B. G. Madsen. 1980. A Comparative Study of Heuristics for a Two-level Location-Routing Problem. *European J. Opnl. Res.* **5,** 378–387.

Karkazis, J., and T. B. Boffey. 1981. The Multi-Commodity Facilities Location Problem. *J. Opnl. Res. Soc.* **32,** 803–814.

Körkel, M. 1989. Exact Solution of Large-Scale Simple Plant Location Problems. *European J. Opnl. Res.* **39,** 157–173.

Kroll, P. 1988. The Multiple-Depot, Multiple-Tour, Multiple-Stop Delivery Problem. Doctoral Thesis, The Johns Hopkins University, Baltimore, MD.

Labbé, M., and G. Laporte. 1986. Maximizing User Convenience and Postal Efficiency in Post Box Location. *Belgian J. Opns. Res. Statistic and Computer Sci.* **26,** 21–35.

Labbé, M., D. Peeters, and J.-F. Thisse. 1995. Location on Networks, In M. O. Ball, T. L. Magnanti, C. L. Monma, G. L. Nemhauser (eds.), *Network Models*, Handbooks in Operations Research and Management Science, North-Holland, Amsterdam.

Laporte, G. 1988. Location-Routing Problems. In B. L. Golden and A. A. Assad (eds.), *Vehicle Routing: Methods and Studies*, North-Holland, Amsterdam.

Laporte, G., H. Mercure, and Y. Nobert. 1987. Generalized Traveling Salesman Problem through *n* Sets of Nodes: The Asymmetrical Case. *Discrete Appl. Math.* **18,** 185–197.

Laporte, G., and Y. Nobert. 1983. Generalized Traveling Salesman through *n* Sets of Nodes: An Integer Programming Approach. *INFOR* **21,** 61–75.

Laporte, G., Y. Nobert, and D. Arpin. 1986. An Exact Algorithm for Solving a Capacitated Location-Routing Problem. In J. P. Osleeb and S. J. Ratick (eds.), *Location Theory: Methodology and Applications*, Baltzer, Basel, Switzerland.

Laporte, G., Y. Nobert, and S. Taillefer. 1988. Solving a Family of Multi-Depot Vehicle routing and Location-Routing Problems. *Transp. Sci.* **22,** 161–172.

Meyer, C. F. 1974. A Long-Range Selection and Timing Analysis System for Facility Location: Theory. *Opnl. Res. Quarterly* **25,** 457–464.

Mirchandani, P. B., and R. L. Francis (eds.). 1990. *Discrete Location Theory*, Wiley, New York.

Morris, J. G. 1978. On the Extent to which Certain Fixed-charge Depot Location Problems Can Be Solved by LP. *J. Opnl. Res. Soc.* **29,** 71–76.

Noon, C. E., and J. C. Bean. 1991. A Lagrangian Based Approach for the Asymmetric Generalized Traveling Salesman Problem. *Opns. Res.* **39,** 623–632.

Oppong, J. R., and M. J. Hodgson. 1994. Spatial accessibility to Health Care Facilities in Suhum District, Ghana. *Professional Geographer,* **46,** 199–209.

ReVelle, C. S. 1993. Facility Siting and Integer Friendly Programming. *European J. Opnl. Res.* **65,** 147–158.

Swain, R. W. 1974. A Parametric Decomposition Approach for Solution of Uncapacitated Location Problems. *Mgmt. Sci.* **21,** 189–198.

Warszawski, A. 1973. A Multi-Dimensional Location Problems. *Opnl. Res. Quarterly* **24,** 165–179.

Weber, A. 1909. *Über den Standort der Industrien*. J. C. B. Mohr, Tübigen.

Watson-Gandy, C. D. T., and P. J. Dohrn. 1973. Depot Location with Van Salesman—A Practical Approach. *Omega,* **1,** 321–329.

Downloaded from informs.org by [216.73.216.82] on 24 July 2026, at 16:31 . For personal use only, all rights reserved.

Copyright © 2001 All Rights Reserved