Operations Research

The Analysts' Bookshelf

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PHILIP M. MORSE, *Queues, Inventories and Maintenance*, John Wiley and Sons, Inc , New York, 1958, 202 pp , $6.50

THIS IS No 1 of the series, *Publications in Operations Research*, sponsored by the OPERATIONS RESEARCH SOCIETY OF AMERICA In view of the great amount of interest in queuing problems by operations analysts, it is fitting that this book start the series

Although this monograph is meant to be an introduction to queuing theory and its applications, it should not be considered an elementary introduction The readers of this JOURNAL will recognize most of the titles in the Table of Contents

Introduction

1 Representation in Terms of Probabilities
  State Probabilities

2 Probabilistic Description of Arrivals and Service Times
  Service Time Distribution Arrival Distribution

3 Single Exponential Channel
  Equations of Detailed Balance Balance between Service Cost and Customers Lost Infinite Queues Balance between Mean Wait and Service Cost Effect of Customer Impatience

4 Multiple Exponential Channels
  Service Channels in Parallel Optimizing the Number of Channels Sequential Service Lines

5 Simulation of Non-Exponential Distributions
  Simulation by Series Arrangement Effects of Service-Time Distributions Effects of Arrival-Time Distributions Simulation by Parallel Arrangement Examples with Hyper-Exponential Distributions

6 General Considerations, Transient Solutions
  The General Equations of Detailed Balance An Example of Transient Behavior Transient Solution for Finite Queues

7 Single Channel, Infinite Queues
  The General Equations The Single Exponential Channel Effect of Service-Time Distribution Details of the $k=2$ Case Hyper-Exponential Service Non-Poisson Arrivals Details of the $l=2$ Case Both Service and Arrivals Non-Exponential Hyper-Poisson Arrivals Example of Service Line

8 Multiple Channels, Infinite Queues
  Exponential Channels Effects of Arrival-Time Distribution. Effects of Service-Time Distributions

This book covers only the analytical aspects of queuing theory It is expected that it will be a basic reference for all such future work and that it will stimulate research in this field

The only shortcoming this reviewer finds with the book is the obvious omission of a discussion of Monte Carlo techniques of computing results for queuing situations The author indicates that he is preparing additional publications to cover the computational methods and machine techniques used in getting numerical answers to queuing problems Since it is desirable to exploit all possible analytical applications before resorting to a Monte Carlo approach, the analytical aspects of queuing theory are certainly worthy of a volume by itself

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HAROLD O. DAVIDSON, *Biological Effects of Whole-Body Gamma Radiation on Human Beings*, The Johns Hopkins Press, Baltimore, Maryland, 1957, 101 pp., \$4.50

THE AUTHOR of this monograph attempts to find solutions in the very considerable problem of estimating the effect on humans of whole-body gamma radiation and applying these estimates to practical problems of military and civil-defense planning Those familiar with radiobiology will realize the paucity of data and the extent of extrapolation confronting the author The interpretation of

Blair's injury-recovery hypothesis, which forms the basis of the estimates of effect on humans, is ingenious and intuitively appealing, although data to establish its validity are lacking The graphic methods for solving practical problems appear sound and useful within the limits of the extrapolations and estimates on which they are based

Unfortunately, the work suffers from brevity and perhaps from an attempt to write to a non-technical audience In any event, the Appendices, which present more technical detail of selected aspects of the work, seemed to this reviewer to be more convincing and, in fact, more easily read than the main part Minor irritations, such as indiscriminate use of the terms "dose" and "dosage," also detract from the work

Used judiciously by one aware of the pitfalls and complexity of the problem, this book could be most useful, the author's estimate of ±50 per cent reliability of the various inputs indicates the degree of caution and judgment required

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T. W. ANDERSON, *Introduction to Multivariate Statistical Analysis*, John Wiley & Sons, Inc , New York, 1958, 374 pp , $12 50

S N ROY, *Some Aspects of Multivariate Analysis*, John Wiley & Sons, Inc , New York, 1958, 214 pp , $8 00

IN MANY experimental situations, each item is characterized by a number of measured quantities such as the output characteristics of a system or the physical characteristics of an individual Each set of measurements is a set of random variables, and where these measurements are to be analyzed simultaneously, the analytic procedures based upon *multivariate* theory are required The emphasis on multivariate techniques has increased steadily since World War II and considerable literature now exists in the journals Many texts allocate a chapter or section to these techniques, however, no comprehensive treatment has been available These two books are the first devoted exclusively to the exposition of multivariate techniques

In practice the analyst is *usually* confronted with a multivariate situation, however, until recently most of the solutions obtained were based upon univariate theory As our experimental techniques improve, as our knowledge increases, and as our experimental design and analysis become more sophisticated, multivariate techniques will become more and more prevalent In operations research, multivariate analysis is frequently needed

The Book by Anderson is designed for use as a text in multivariate analysis The prerequisites for an understanding of this text are a knowledge of the usual theory of univariate statistics and matrix algebra, as well as maturity in mathematics The more basic and important topics are treated, while recent advanced topics are given only brief treatment in the last chapter The theory presented assumes a multivariate normal (Gaussian) distribution The first two chapters serve as a general introduction covering the basic elements and properties of the multivariate normal distribution Then follow five chapters covering estimation and sampling distributions for the mean vector, covariance matrix, correlation coefficients, and generalized variance In the next three chapters, testing hypotheses are presented covering the general linear hypothesis, independence of sets of variables, and the equality of mean vectors and covariance matrices The remaining chapters cover the topics of principal components, canonical correlation, the distribution of certain characteristic roots and vectors, and recent advanced topics

For those individuals who have the mathematical background, Anderson's book presents in an organized and systematic manner the extensively developed multivariate methods based on the normal distribution

THE BOOK BY ROY is concerned with those particular developments in multivariate analysis that have been of special interest to the author There is no attempt to cover the entire field of multivariate analysis The major emphasis is on obtaining confidence bounds on certain parametric functions Except in the last chapter, the author is concerned exclusively with normal variates

In the first five chapters (25 pages), the basic elements of the multivariate normal distribution are presented along with the definition of certain classes of tests The next six chapters (48 pages) cover tests of the null hypothesis, operating characteristics, lower bounds on power functions, and the monotonic character of the lower bounds The last four chapters (61 pages) cover the topics of least squares, multivariate extensions of the analysis of variance and covariance, confidence bounds, and nonparametric generalizations The appendices (77 pages) discuss matrix theory, quadratic forms, transformations, Jacobians, and certain types of multiple integration

The subject of multivariate analysis is of considerable importance in operations research The reviewer feels that every operations-research analyst should have some comprehension of the topic However, Roy's book will be of little interest to such analysts unless they are also advanced students of theoretical statistics

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***Proceedings, Executive Conference on Organizing and Managing Information*, University of Chicago, Chicago, 1957, 64 pp , $2 00**

FOR THOSE members of management and librarians who are interested in knowing in what activities their Company libraries should be engaged, these *Proceedings* make worthwhile reading

The papers call attention to the importance of a centralized area for the receiving, dissemination, and retrieval of information, and give many facets of the library's function in making this information useful to the industrial organization

The authors of the several papers are qualified to write about their particular aspects of making the library an effective part of the industrial picture They are either practicing librarians, or executives who head up research and development departments It is their job to be cognizant of the part that libraries play in providing businesses with the answers to their many problems

The papers are short, concise, and well written For the Research Director and the Special Librarian, this publication can provide a half-hour of reading time well spent

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TWO EUROPEAN STUDIES

THE PROBLEM of location of items in a warehouse is discussed in an article by IVAR ORTLIEB published under the title "Wahrscheinlichkeitsrechnung" (probability theory) in the Swiss journal *Industrielle Organisation*, No 11, November 1957, pp 395–414 A warehouse ships orders normally composed of a number of items These items have to travel certain distances between their respective locations in the warehouse and the packing department The author considers the problem of minimizing total expected covered distance by optimally assigning warehouse locations to items By sampling the composition of orders, he is able to set up total expected covered distance as a function of the warehouse geometry alone From here on, he proceeds intuitively to find an optimal assignment of locations Safety stock levels for the various items are also taken into consideration

IN *Unternehmensforschung*, vol 1, pp 166–172 (1956–1957), ANNA KLINGST considers the problem of minimizing total annual costs of set-ups, inventory, and shortages for given seasonal demand A computer is employed to examine a large number of feasible inventory policies and select the optimum policy

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FIVE PAPERS BY CONNY PALM

TELE, English Edition No 1, 1957 (107 pages), Information from The Royal Board of Swedish Tele-Communications, edited by E MALMGREN, contains five papers by Conny Palm (1907–1951) prefaced by a short article, "Research on Telephone Traffic Carried by Full Availability Groups" It would be desirable to say more regarding these useful papers, but due to space limitations, we shall confine ourselves to selected aspects

THE FIRST PAPER, "Some Propositions Regarding Flat and Steep Distribution Functions," characterizes two classes of distribution functions arising in telephone traffic congestion The distribution $f(t)$ that is considered is equal to unity for $t=0$ and decreases to zero as $t\to\infty$, e g , the probability of finding a holding time of duration zero is unity The density function is then $-df(t)=-f'(t)\,dt$ If, for example, s denotes the mean holding time and σ^2 the variance, and if one defines the form factor of the distribution $f(t)$ by

$$\epsilon=\frac{2}{s^2}\int_0^\infty t\,f(t)\,dt, \tag{1}$$

then
$$\epsilon=1+\frac{\sigma^2}{s^2},\quad s=-\int_0^\infty t\,f'(t)\,dt,\quad \sigma^2=-\int_0^\infty (t-s)^2\,f'(t)\,dt, \tag{2}$$

and one has a steep distribution if $1\leq\epsilon<2$ and a flat distribution if $2<\epsilon$ Note that ϵ is always $\geqq 1$ The exponential distribution forms the boundary between the two types

The mean residual duration given by

$$\frac{1}{f(t)}\int_0^\infty f(t+x)\,dx \tag{3}$$

is the mean value of the remaining duration of all those holdings whose duration is at least a given time t It plays an important part in this characterization

Flat distributions are studied through the representative completely monotone functions which have the property that at each point their derivatives alternate in sign Thus, for each $t \geqq 0$ the pth derivative satisfies the relation

$$(-1)^p f^{(p)}(t) \geqq 0 \tag{4}$$

An example to be used for representing n different types of 'flat' holding times is

$$\sum_{i=1}^{i=n} A_i \exp[-t/s_i], \qquad \sum_{i=1}^{i=n} A_i = 1, \tag{5}$$

which, it is shown, can be used to represent any completely monotone function A description of how to generate (5) is also given A completely monotone function has the form

$$\int_0^\infty e^{-xt}\,dG(x), \tag{6}$$

where $G(x)$ is bounded and nondecreasing for $x \geqq 0$, $G(0)=0$ Conversely, every function that has the form (7) is completely monotone

For such functions, it is shown that the mean residual duration increases with time An illustration of the ideas is then given

The study of steep distributions, of which the constant-holding-time distribution with $\epsilon = 1$ is an example, is simplified by assuming that the holdings are progressively proceeding through different stages, each of which is regulated by an exponential distribution Let $f_i(t)$ $(i=1, \quad, n)$ be a set of holding-time distributions where, for each i, $f_i(t)$ gives the probability of an occupation not having gone over to the $(i+1)$st stage in time t after initiation, $i=n$ yielding the desired distribution As above, let the probability of having gone to the ith stage at $x<t$ and not having left it by time t be $-df_{i-1}(t)$ The probability that it does not leave the ith stage during $t-x$ owing to the exponentiality assumption is $\exp[-(t-x)/s_i]$ The product of the two probabilities integrated for $0 \leqq x \leqq t$ gives the probability of staying at the ith stage by time t, having gone into it prior to t

This argument finally gives

$$f_i(t) = f_{i-1}(t) - \int_0^t \exp[-(t-x)/s_i]\,df_{i-1}(t)$$

If $s_i \neq s_j$ for all i and j, one has

$$f_n(t) = \sum_{k=1}^{n} \frac{s_k^{n-1}\exp[-t/s_k]}{(s_k-s_1)(s_k-s_2)\cdot\ (s_k-s_n)},$$

where $f_1(t) = \exp[-t/s_1]$ If $s_i = s_j$ for some i and j, and if $f_1(t) = \exp[-t/s_0]$, then one can show that the nth distribution function is given by

$$f_n(t) = \left\{1 + \frac{t}{s_0} + \frac{1}{2!}\left(\frac{t}{s_0}\right)^2 + \quad + \frac{1}{(n-1)!}\left(\frac{t}{s_0}\right)^{n-1}\right\}\exp[-t/s_0] \tag{7}$$

The mean residual duration decreases for the steep distributions with increasing t, i e , its derivative is negative

IN THE SECOND PAPER, "Some Observations on the Erlang Formulae for Busy-Signal Systems," an attempt is made at a brief systematic investigation of the form and properties of the most important traffic functions, in addition to some numerical calculations and tabulation procedures The special formulae are reviewed for a limited source of input It is indicated graphically that Erlang's well-known loss formula in the case of c channels,

$$\frac{\rho^c/c!}{1+\rho+\rho^2/2!+\quad+\rho^c/c!}$$

cannot be approximated by the Poisson distribution $P_n=\rho^c e^{-r}/c!$ without committing large errors, using large values for the loss Here $\rho=s\ \lambda$, where s is the service rate and λ is the arrival rate

A discussion of limited input formulae then follows

THE THIRD PAPER is called, "Contributions to the Theory on Delay Systems" Unlike a busy-signal system in which a call that finds all channels occupied makes a new try at calling later, a delay system permits a call to wait to obtain service With exponential holding time it is easy to treat such a system mathematically If all c channels are occupied and q calls are waiting, the mean value per unit time of the total time in which waiting occurs is denoted by $[c, q]$ If ρ is defined as before, then

$$[c, 0]=(\rho/c)\ [c-1],$$
$$[c, q]=(\rho/c)\ [c, q-1]=(A^c/c!)(A/c)^q\ [0],$$
$$[p]=(\rho/p)\ [p-1],$$

where $[p]$ is the mean value per unit time of the total time that $p<c$ channels are occupied simultaneously These expressions are then used to obtain, for example, the average total delay per unit time, $\sum_{q=1}^{q=\infty} q\,[c, q]$, and the expected delay for a call which must wait,

$$\sum_{q=1}^{q=\infty} q\,[c, q]/y \sum_{q=0}^{q=\infty} [c, q]$$

Some of Crommelins' work is examined The author develops the interesting fact that the congestion condition in large groups (i e , for large c) will be independent of the distribution function of the holding times and is the same as in groups with exponential holding-time distributions The case of impatient customers is also examined

THE FOURTH PAPER is called "Waiting Times with Random Served Queue" Let calls arrive at random with a mean of λ calls per unit time before c service facilities and be served exponentially with identical distributions with mean s for all channels Suppose calls are randomly selected for service—unlike first-come, first-served queues, the waiting time of a call depends on later arrivals Let $f_s(t)$ be the probability that a call will not have obtained service after time t from its arrival Suppose at time zero there are q calls waiting, one has

$$f_q(t+dt) = \lambda\, f_{q+1}(t)\; dt + [(q-1)/q](c/s)\, f_{q-1}(t)\; dt + [1 - \lambda\; dt - (c/s)\; dt]\, f_q(t), \tag{8}$$

which, as usual, yields a differential difference equation One also has

$$f_1'(t) + [\lambda + (c/s)]\, f_1(t) = \lambda\, f_2(t) \tag{9}$$

Let $\rho = \lambda s/c$, then the differential equation for $q>1$ reads

$$\frac{1}{\lambda} f_q'(t) + \left(1 + \frac{1}{\rho}\right) f_q(t) = f_{q+1}(t) + \frac{q-1}{q}\,\frac{1}{\rho}\, f_{q-1}(t) \tag{10}$$

Here λ may be taken as the unit of time, i e , set $\lambda = 1$ Now define the generating function

$$\psi(x, t) = \sum_{q=1}^{q=\infty} f_q(t)\, x^q, \tag{11}$$

then the boundary conditions are

$$\psi(0, t) = 0, \qquad \psi(x, 0) = \sum_{q=1}^{q=\infty} x^q = x/(1-x) \tag{12}$$

Note that $t \geqq 0$, $0 \leqq f_q \leqq 1$, and ψ is bounded and regular for $|x| < 1$ Expand $\psi(x, t)$ in increasing powers of t, this gives

$$\psi(x, t) = \sum_{\mu=0}^{\mu=\infty} A_\mu(x)\, t^\mu/\mu! \tag{13}$$

where $A_\mu(x) = \psi^{(\mu)}(x, 0)$, according to McLaurin's series expansion, is the μth derivative of ψ at $t=0$ Thus

$$A_\mu(x) = \sum_{q=1}^{q=\infty} f_q^{(\mu)}(0)\, x^q, \tag{14}$$

and the coefficients $f_q^{(\mu)}(0)$ may be determined from (10) with $\lambda = 1$ by differentiating $\mu - 1$ times Since $f_q(0) = 1$, one has for $\mu = 1$ $f_q^{(1)}(0) = -1/(q-\rho)$ This can be used to determine $f_q^{(2)}(0)$, etc Thus, with $\psi(x, t)$ determined, $f_q(t)$ are obtained as the coefficients in the series expansion of ψ in terms of increasing powers of x The differential equation of the generating function is given by the hyperbolic partial differential equation

$$\frac{\partial^2 \psi(x, t)}{\partial x \partial t} + \frac{(1-x)(x-\rho)}{\rho x}\,\frac{\partial \psi(x, t)}{\partial x} + \frac{1}{x^2}\,\psi(x, t) = 0 \tag{15}$$

By introducing the series expansion (13) in (15) and equating the coefficient of each power of t to zero, one obtains

$$A_{\mu(\rho)} = -\lim_{x\to 0} \frac{A_{\mu-1}(x)}{x} - \frac{1}{\rho}\int_0^\rho A_{\mu-1}(x)\; dx, \tag{16}$$

which is too complicated for obtaining all the A's The moments of the generating function are given by

$$M_\mu = -\int_0^\infty t^\mu\, \frac{\partial \psi(x, t)}{\partial t}\; dt = \mu \int_0^\infty t^{\mu-1}\, \psi(x, t)\; dt$$

By integrating the differential equation in ψ with respect to t and using the second of the boundary conditions (12), one obtains

$$M_1(x) = [\rho/(2-\rho)]\; x\,(2-x)/(1-x)^2$$

Now the probability of waiting $\geqq t$ is given by $\sum_{q=1}^{q=\infty} [c, q-1]\, f_q(t) \big/ \sum_{q=1}^{q=\infty} [c, q-1]$

Hence, in the exponential holding time case, it is given by $[(1-\rho)/\rho \sum_{q=1}^{q=\infty} \rho^q f_q(t) = [(1-\rho)/\rho]\, \psi(\rho, t)$, for then $[c, q-1] = \rho^{q-1}[c, 0]$, and the mean waiting time is then obtained by integration as $[(1-\rho)/\rho]\, M_1(\rho)$

The second moment for the waiting time distribution is given by

$$[(1-\rho)/\rho]\, M_2(\rho) = [\rho/(1-\rho)]^2\, 4/(2-\rho)$$

By expanding $M_1(x)$ and $M_2(x)$ in series one obtains the first and second moments for the separate $f_q(t)$ They are the coefficients of x^q in each series respectively The form factor may now be determined Expressions for higher moments are given Interesting conclusions regarding telephone traffic are given at the end of this paper

THE FIFTH PAPER is entitled "Duration of Congestion States in Delay Systems" To avoid long delays it is desirable first to study the distribution function for the duration of the congestion states Usually, this is not the same as the waiting-time distribution Suppose calls arrive at random before c service facilities, where calls may be selected for service in any style, and the service times are identical exponential distributions Let $f_n(t)$ be the probability distribution expressing the probability that during time t, after a state of n waiting has occurred, the congestion has not ceased and thus the state $c-1$ has not occurred Thus $f_0(t)$ gives the probability that a congestion lasts at least the time t It is the desired distribution Assume that all c facilities are occupied and n are waiting

As before, one has without difficulty

$$f_n'(t) = -[(1+\rho)/\rho]\, f_n(t) + f_{n+1}(t) + (1/\rho)\, f_{n-1}(t), \qquad (n>0)$$
$$f_0'(t) = -[(1+\rho)/\rho]\, f_0(t) + f_1(t), \qquad (n=0)$$
$$f_n(0) = 1$$

To obtain the solution of this system one defines

$$\psi(x, t) = \sum_{n=0}^{n=\infty} f_n(t)\, x^n,$$

and multiplies both sides of the above two equations by x^n and sums over all n This gives

$$d\psi(x, t)/dt + [(1-x)(x-\rho)/\rho x]\, \psi(x, t) + f_0(t)/x = 0,$$

with $\quad \psi(0, t) = f_0(t), \qquad \psi(x, 0) = 1/(1-x)$

Note that since $0 \leqq f_n(t) \leqq 1$, the series defining ψ is convergent

Here again ψ is used to obtain expressions for the moments of the functions $f_n(t)$ The functions are determined by a series expansion about the origin The coefficients of the expansion are the derivatives $f_0^{(1)}(0)$ $f_n^{(1)}(0)$, $f_0^{(2)}(0)$ · $f_n^{(2)}(0)$, , $f_0^{(n+1)}(0)$ $f_n^{(n+1)}(0)$, , evaluated at the origin and obtained from the initial equations given above The probability that during time t after a moment selected at random, with congestion prevailing, the congestion state does not cease, is given by $(1-\rho)\, \psi(\rho, t)$, which is computed in a similar fashion to that in the fourth paper Graphs and tables are included.

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