# *The Analyst's Bookshelf*

## Joseph F. McCloskey, Editor

### REVIEWS

**CLAUDE BERGE, *Theorie des Graphes et ses Applications*, Dunod, Paris, 1958, 277 pages, 3400 fr**

THE TERM *graph*, the subject of this book, has not the common connotation of a plot or curve, but refers to an established but esoteric mathematical usage A graph is a set of points, certain pairs of which are connected by arcs These arcs may or may not be oriented, that is, have an associated definite direction from one endpoint to the other

Possibly a new name to dispel confusion would be apropos We offer *theory of linkages*

The geometric aspect of the above definition is of course not the heart of the matter, graphs can be symbolic diagrams of a rich variety of situations The points can represent almost any kind of object and the arcs almost any type of interrelation between them Thus we should expect the subject to abound in diverse applications, and Berge's book does

Thumbing through the volume, one is fascinated by the variegated array of illustrations, and the diverse examples attest to the very eclectic contents The research analyst will find much to charm as well as instruct him

Hitherto the standard work was Konig's *Theorie der endlichen und unendlichen Graphen* (Leipzig, 1936) One read here of a branch of classical mathematics that struck out in many directions but penetrated deeply in few, the minor efforts of many major mathematicians A purview of the subject can be had from a glance at a sampling of the more renowned problems

An *Euler line* is one that covers every arc of a graph and can be drawn without lifting the pencil or retracing It stems from the famous problem of the seven bridges of Königsberg as recounted in most histories of mathematics and is popularly described as being the starting point of topology

A *Hamilton line* is nearly a dual concept it need not cover *all* the arcs but must pass over each *vertex* once and only once

In both cases the problem is to ascertain conditions under which it is possible to draw the appropriate line The Euler problem is rather easy, but the Hamilton problem is still unsolved

One of the most famous of all unsolved problems is that of map coloring to show that four colors suffice to color any planar map so that adjacent countries are always of distinct hue It becomes a question of graph theory if we let each vertex represent a country, connecting them by an arc when the corresponding countries have a mutual border line

The cream of such older problems is elegantly treated in Berge's book But along with them are the current advances in the subject The disciple of operations research will recognize much material and many names, a good proportion is American

For example, there is a chapter on transportation networks It includes the Ford-Fulkerson algorithms and the theorems of Hoffman and Gale As usual, the applications are astonishingly diverse Besides questions of optimizing transport and routing, the same techniques are fitted to the problem of minimal covering, some combinatorial teasers, problems in set theory, and linear programming The methods continue to solve problems in some subsequent chapters, in one on *couplages* we find a problem typical of the applied scope of this theory

In what are called *simple* graphs, the vertices are divided into two sets such that all arcs connect only members of one set with points of the other A *couplage* is a subset of these arcs, with no two having a common endpoint The problem to find a maximum couplage

What is an application? Let one of the two sets of vertices above represent a set of workers, and the other, jobs to be done A connecting arc is drawn when a worker is capable of fulfilling that job Then a maximum couplage corresponds to a maximal scheme of assigning workers to suitable jobs

A second, but more trivial application, deserves to be quoted on less technical grounds

> Dans un collège mixte americain, toute jeune fille a $m$ "boy-friends," et tout garçon a $m$ "girl-friends", est-il possible de faire danser simultanement chaque jeune fille avec un de ses boy-friends et chaque garçon avec une de ses girl-friends?

There is a chapter on game theory (the author has done a separate monograph on this subject), others deal with matrices and trees There is an appendix on electrical circuit theory and some allied nonelectrical problems

Always there is stimulating diversity In the chapter entitled *Facteurs*, for instance, three consecutive examples run Voyage around the World (Hamilton), The Knight's Tour of the Chessboard (Euler), and—a quick plunge to modern times and queuing problems—The Bookbinding Problem (S Johnson)

The operations analyst will benefit from this work (aside from enjoying himself) in acquiring an arsenal of new and imaginative techniques Even should he never have occasion to use them, his wits cannot be but sharpened owing to the remarkable way seemingly disparate concepts are linked by common underlying ideas

The French should be no incubus The usual high school fare is enough, one amuses oneself with vernaculars special to such things as chess, bridge, and circuit theory

There are diagrams in the book that are literally unnecessary, but whose purpose is to aid the reader in the ratiocination necessary in following a proof It is a device too few authors adopt

We detected three errors Rado's Theorem (p 18) is incorrectly proved, but little used subsequently, likewise Theorem 4 (p 32), but the defect is easily remedied The nonvanishing of a determinant (p 244) is untrue

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