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Letter to the Editor—Comments on a Paper by Karush

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$P_r(t)$, for various values of k The case $n_0 \leqq k$ is not considered since in this case there is no queue To solve for the state probabilities when $n_0 > k$, it is necessary to solve the equations of detailed balance

$$dP_n(t)/dt + n\,\mu\,P_n(t) = (n+1)\,\mu\,P_{n+1}(t), \qquad (0<n<k) \quad (3)$$

$$dP_n(t)/dt + k\,\mu\,P_n(t) = k\,\mu\,P_{n+1}(t), \qquad (n \geqq k) \quad (4)$$

for the initial condition $P_i(0) = 0$ $(i \neq n_0)$, $P_{n_0}(0) = 1$

The solution to equation (4) is

$$P_n(t) = [(k\mu t)^{n_0-n}/(n_0-n)!]\,e^{-k\mu t} \qquad (n_0 \geqq n \geqq k) \quad (5)$$

Writing equation (3) for $n = k-1$ and substituting $P_k(t)$ from equation (5) we get

$$dP_{k-1}(t)/dt + (k-1)\,\mu\,P_{k-1}(t) = [\mu k\,(\mu k t)^{n_0-k}/(n_0-k)]\,e^{-k\mu t} \quad (6)$$

The solution for $k=1$ is immediately obtained by integrating equation (6) That is, the state probabilities for $k=1$ are given by

$$P_0(t) = 1 - \sum_{r=1}^{r=n_0} e^{-\mu t}\,(\mu t)^{n_0-r}/(n_0-r)!, \quad (7)$$

$$P_n(t) = [(\mu t)^{n_0-n}/(n_0-n)!]\,e^{-\mu t} \qquad (n_0 \geqq n \geqq 1) \quad (8)$$

The solution for $k = 2, 3,$ etc can be obtained by solving equation (6) for $P_{k-1}(t)$, inserting this value into equation (3), and then successively integrating equation (3)

Tables I–V show $\bar{n}$ (the average number of units either awaiting or undergoing repair) as a function of μt The initial number, n_0, takes on values of 4, 8, 12, 16, and 20 The number of service channels k is varied from 1 through 5 Use of these tables will permit a rapid evaluation of the readiness R defined in equation (1) For example, suppose that, of a total population of 25 units ($N=25$), 20 units needing service arrive at a service facility that contains 3 channels ($n_0=20$, $k=3$) It is desired to determine the percentage of the total population that will be 'ready' at a time μt measured from the time of arrival, $t=0$ Let μ be the mean service rate for each channel Let $\mu = 5$/hour and determine the readiness after 1 hour From Table 3, we find that for $k=3$, $n_0=20$, and $\mu t=5$, $\bar{n}$(1 hour) = 5 2996 Therefore, $R = (25-5\ 2996)/25 = 78\ 28$ per cent

COMMENTS ON A PAPER BY KARUSH

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IN THE PAPER, "A Queuing Model for an Inventory Problem," that appeared in OPERATIONS RESEARCH, volume 5, pages 693–703 (October 1957), WILLIAM KARUSH solved the following important inventory-control problem

A firm maintains a total inventory of n items of a certain article When an article is sold, a replenishment is immediately ordered The time of replenishment

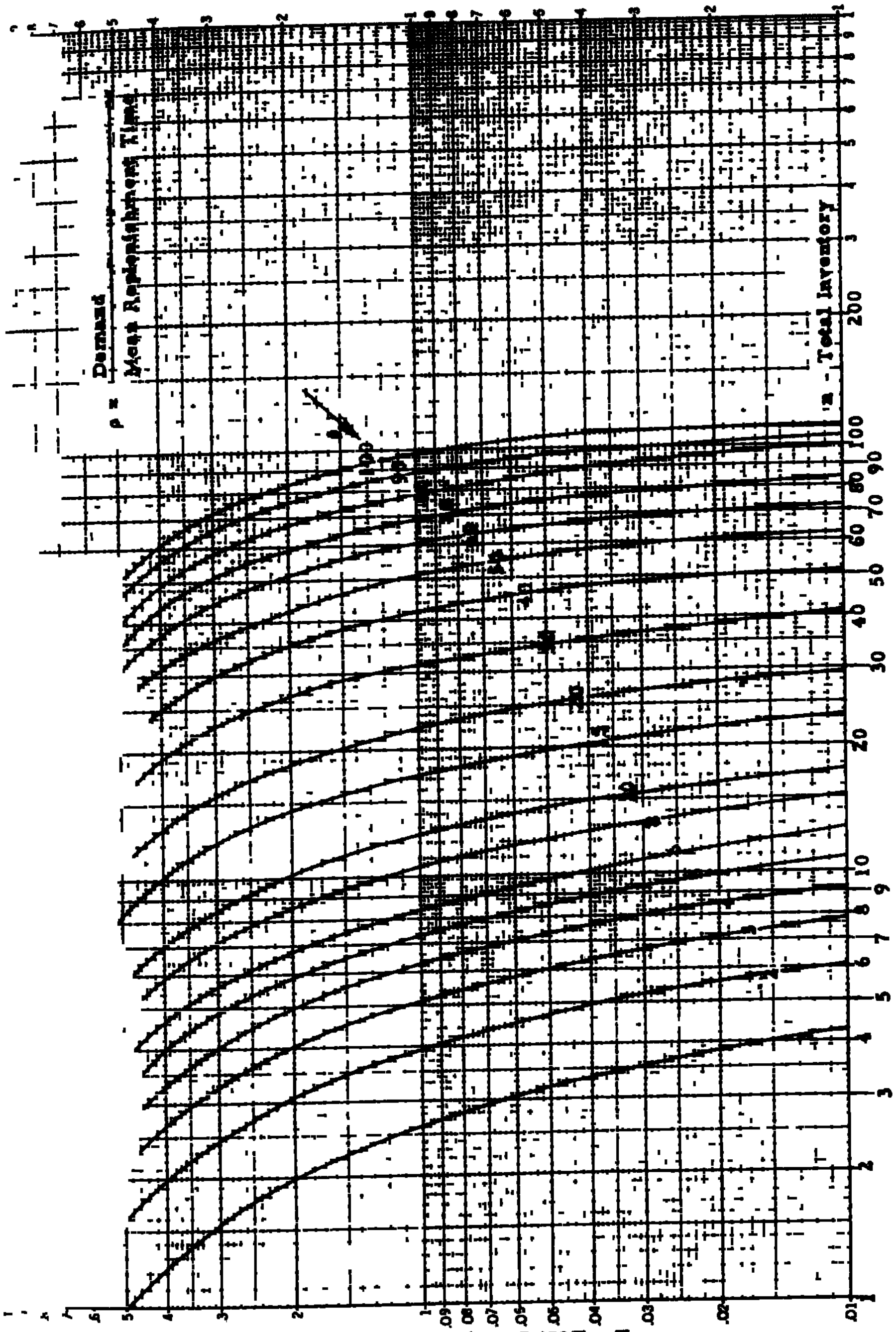

Fig 1. Lost sales versus inventory level

is a statistical variable with a known distribution and with a mean replenishment time of T [It is convenient to think of the n items as being divided into two classes (*a*) the items in stock or available for sale, (*b*) the items in the pipe line] The demand is Poisson with a mean of ρ items per mean replenishment time T When there is a demand for an article and there is no stock, then there is a lost sale The important measure of effectiveness for this system is ρ the ratio of lost sales to total demand (or the probability of shortage) Karush shows that *irrespective of the distribution function of the replenishment time*, p is given by

$$p=(\rho^n/n!)/(1+\rho/1!+\rho^2/2!+\quad+\rho^n/n!)$$

The purpose of this note is to present a chart, Fig 1, depicting in a graphical form the above equation As the evaluation is rather laborious, it is convenient to have such a chart Incidentally, the chart shows some interesting numerical values

Suppose the mean replenishment time is one month, and the mean demand ρ is 4 articles per the mean replenishment time of one month Say we keep a total of $n=8$ articles The ratio of lost sales to total demand is 0 03 or there are 3 per cent lost sales Say we increase n to 9 Lost sales will drop to 1 2 per cent! *A 12 per cent increase in inventory results in a 60 per cent decrease in lost sales*

Suppose demands double to $\rho=8$ Say we want to maintain a 3 per cent level of lost sales We need an inventory level of $n=13$ A 100 per cent increase in demand requires a 60 per cent increase in inventory, if the same level of lost sales is specified

NOTE ON A PAPER BY FIRSTMAN

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(Received February 16, 1960)

IN A PAPER, "A Vulnerability Model for Weapon Sites with Interdependent Elements," *Operations Research*, volume 7, pp 217–225, 533 (1959), SIDNEY I FIRSTMAN considers the transition matrix

$$P=\begin{Vmatrix} q & b & a & 1-(a+b+q) \\ 0 & b+q & 0 & 1-(b+q) \\ 0 & 0 & a+q & 1-(a+q) \\ 0 & 0 & 0 & 1 \end{Vmatrix}, \tag{1}$$

where the entries correspond to certain transition probabilities For an associated problem it is necessary to find P^n, $n=2, 3,$

The purpose of this note is to show that for any positive integer n we have

$$P^n=\begin{Vmatrix} q^n & (b+q)^n-q^n & (a+q)^n-q^n & 1-[(a+q)^n+(b+q)^n-q^n] \\ 0 & (b+q)^n & 0 & 1-(b+q)^n \\ 0 & 0 & (a+q)^n & 1-(a+q)^n \\ 0 & 0 & 0 & 1 \end{Vmatrix}, \tag{2}$$