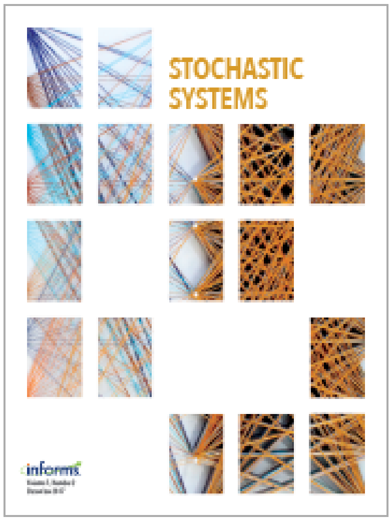

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Open Problem—Protocols for Resource-Sharing Networks with Locally Edge-Minimal Fluid Models

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Locally Edge-Minimal Resource-Sharing Networks

Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space on which all the random objects to follow are defined.

We consider a network with a finite number of resources (nodes), labeled by $j = 1, \ldots, J$, and a finite set of routes, labeled by $i = 1, \ldots, I$. Let $A = [a_{ji}]$ be the $J \times I$ incidence matrix in which $a_{ji} = 1$ if resource j is used by route i and $a_{ji} = 0$ otherwise. Let $\mathbf{J} = \{1, \ldots, J\}$, $\mathbf{I} = \{1, \ldots, I\}$. For $j \in \mathbf{J}$, let $\mathcal{F}(j) = \{i \in \mathbf{I} : a_{ji} = 1\}$. For convenience, we assume that all the resources have a unit service rate.

By a flow on route i, we mean a continuous transmission of a file through the resources used by this route. We assume that a flow takes simultaneous possession of all the resources on its route during the transmission. All the flows have soft deadlines for transmission completion. By the lead time of a flow, we mean the difference between its deadline and the current time.

For $i \in \mathbf{I}$, $t \geq 0$, and $s \in \mathbb{R}$, let $Y_i(t,s)$ be the cumulative idleness by time t with regard to transmission of flows on route i with lead times at time t less than or equal to s and let $Y(t,s) = (Y_i(t,s))_{i \in \mathbf{I}}$.

Definition 1 (Kruk 2017). Let $f, g : \mathbb{R} \to \mathbb{R}$ be such that, for some $a \in \mathbb{R}$, we have $f \equiv g$ on $(-\infty, a]$ and let $c = \sup\{a \in \mathbb{R} : f(x) = g(x)\ \forall x \in (-\infty, a]\}$. We write $f \ll g$ if either $c = \infty$ (i.e., $f \equiv g$ on $\mathbb{R}$) or $c < \infty$ and there exists $\epsilon > 0$ such that $f \leq g$ on $[c, c+\epsilon]$.

The relation "$\ll$" is a partial ordering in $\mathbb{R}^{\mathbb{R}}$.

Definition 2 (Kruk 2017). The state process $\mathcal{X}$ of a network is called locally edge minimal at a time $t_0 \geq 0$ if there exists a random variable $h > 0$ such that, for any other state process $\mathcal{X}'$ satisfying $\mathcal{X}'(t_0) = \mathcal{X}(t_0)$, we have $\sum_{i \in \mathbf{I}} Y_i(t, \cdot)(\omega) \ll \sum_{i \in \mathbf{I}} Y'_i(t, \cdot)(\omega)$ for every $\omega \in \Omega$, $t \in (t_0, t_0 + h(\omega))$. The state process $\mathcal{X}$ is called locally edge minimal if it is locally edge minimal at every $t_0 \geq 0$.

The intuition behind these notions is that a locally edge-minimal service protocol tries to transmit as many flows corresponding to the earliest deadlines as possible, and hence, its idleness with respect to such flows is as small as possible. Indeed, under mild distributional assumptions, the classes of locally edge-minimal disciplines and preemptive earliest deadline first (EDF) resource-sharing policies coincide. Moreover, without any distributional assumptions, a service policy is locally edge minimal if and only if it belongs to the class of so-called strong EDF protocols. (Roughly speaking, an EDF discipline for a resource sharing network is strong EDF if its tie-breaking rule allows for scheduling the transmission of flows with the smallest available lead times on as many routes as possible.) Consequently, local edge minimality is fairly close to being a characterization of the preemptive EDF policy for queueing systems with shared resources. See Kruk (2017) for more precise statements and proofs of these results.

Locally Edge-Minimal Fluid Models

A system $\overline{\mathfrak{X}}(t,s) = (\overline{Z}(t,s), \overline{D}(t,s), \overline{T}(t,s), \overline{Y}(t,s))$, $t \geq 0$, $s \in \mathbb{R}$, with continuous, nonnegative components, satisfying, for all $\tilde{t} \geq t \geq 0$, $s \in \mathbb{R}$,

$$\overline{Z}(t,s) = \overline{Z}(0, t+s) + \alpha(t + (s \wedge 0))^+ - \overline{D}(t,s),$$
$$\overline{D}_i(t,s) = \overline{T}_i(t,s)/m_i, \qquad \overline{T}_i(t,s) + \overline{Y}_i(t,s) = t, \qquad i \in \mathbf{I},$$
$$\sum_{i \in \mathcal{F}(j)} \left(\overline{T}_i(\tilde{t}, s-\tilde{t}) - \overline{T}_i(t, s-t)\right) \leq \tilde{t} - t, \qquad j \in \mathbf{J},$$

together with some natural monotonicity assumptions, is called a fluid model for the resource-sharing network under consideration. The components of the functions $\overline{Z}$, $\overline{D}$, $\overline{T}$, and $\overline{Y}$ are the fluid counterparts of the numbers of flows on each route, the departures, the transmission times, and the cumulative idleness, respectively, and α_i, m_i are the arrival rates and the mean service times. The following definition is an analog of Definition 2 in the fluid model setting.

Definition 3. A fluid model $\overline{\mathfrak{X}}$ for a resource-sharing network is called locally edge minimal at a time $t_0 \geq 0$ if there exists $h > 0$ such that, for any fluid model $\overline{\mathfrak{X}}'$ with the same data α, m_i, A and the same initial state $\overline{Z}'(0,\cdot) = \overline{Z}(0,\cdot)$, satisfying $\overline{\mathfrak{X}}'(t_0,\cdot) = \overline{\mathfrak{X}}(t_0,\cdot)$, we have $\sum_{i\in\mathbf{I}} \overline{Y}_i(t,\cdot) \ll \sum_{i\in\mathbf{I}} \overline{Y}'_i(t,\cdot)$ (equivalently, $\sum_{i\in\mathbf{I}} \overline{Z}_i(t,\cdot) \ll \sum_{i\in\mathbf{I}} \overline{Z}'_i(t,\cdot)$) for every $t \in (t_0, t_0 + h)$. The fluid model $\overline{\mathfrak{X}}$ is called locally edge minimal if it is locally edge minimal at every $t_0 \geq 0$.

The intuition here is that a locally edge-minimal fluid model tries to transmit as much “customer mass” corresponding to the earliest deadlines as possible. Consequently, such a model may be thought of as a macroscopic counterpart of a resource-sharing network working under the EDF protocol. It has been shown in Kruk (2019) that locally edge-minimal fluid models have very attractive features regardless of the underlying network topology. In particular, they are stable in the strictly subcritical case, and more generally, such subcritically loaded models converge to the invariant manifold in finite time. Unfortunately, fluid limits of locally edge minimal (i.e., EDF) networks are not necessarily locally edge minimal. In fact, in some resource-sharing network topologies, for example, in the ring C_6, implementing the EDF protocol may result in system instability.

Open Problem

Design a protocol (perhaps a sequence of protocols) for a real-time resource-sharing network such that, regardless of the network topology, the corresponding performance processes converge under fluid scaling to a locally edge-minimal fluid limit.

Solution of this problem would yield a service discipline with quality of service similar to that of the EDF (i.e., asymptotically consistent with the deadline ordering) but with much better asymptotic properties. It is plausible that using a proxy of a (strong) EDF protocol, in which we identify deadlines falling into a suitable time window, and rescaling the window size to be divergent but of the order $o(n)$ in the nth system, we may be able to get the required asymptotic behavior.

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