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Letters to the Editor

Traffic Delay at a Signalized Intersection with Binomial Arrivals

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It is shown that the formula derived by Dunne in a recent paper with the above title can be obtained by a simple direct calculation.

In a recent paper[1] with the above title, DUNNE derived delay formulas for an intersection problem in which the vehicle arrivals were assumed binomial. The method Dunne used was an extension of a discrete analog of that of DARROCH, ET AL.,[2] for random arrivals. It is the purpose of this letter to point out that the delay formulas for the binomial model can be easily obtained by a direct calculation.

With Dunne's notation (except that the subscripts will not be required) the vehicle arrivals in one arm of the intersection are assumed to be generated by a binomial process with q = probability of an arrival in a τ sec interval and $p = 1 - q$ = probability of no arrival or a gap. During a cycle of the traffic light, the queue builds up from zero length to a maximum during an assumed effective red period of $r\tau$ sec, and then discharges at a constant rate of one vehicle each τ interval during the effective green period.

Instead of plotting queue lengths as a function of time, it is useful to consider the cumulative arrival and discharge curves as illustrated in Fig. 1. The area between the graphs is the total delay to be determined, and it is easily evaluated by summing over horizontal strips.

The derivation of the formula for the generating function Δ_r of the delay X_r follows by first defining:

Y_1 = number of vehicles arriving before the first gap,

Y_j = number of vehicles arriving between the $j - 1$st and jth gap.

The Y_1 vehicles are delayed $r\tau$ sec, the Y_2 vehicles delayed $(r - 1)\tau$ sec, and so on until the queue is discharged on the arrival of the rth gap. The total delay is (apart from the factor τ)

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$$X_r = rY_1 + (r-1)Y_2 + \cdots + Y_r,$$

or

$$X_r = \Sigma_1^r j Y_{r-j+1}. \tag{1}$$

The Y_j are independent random variables identically distributed according to the negative binomial distribution with the generating function[3]

$$E(z^{Y_j}) = p/(1-qz). \tag{2}$$

The generating function for X_r follows immediately from (1):

$$\Delta_r = E(z^{X_r}) = \Pi_1^r p/(1-qz^j). \tag{3}$$

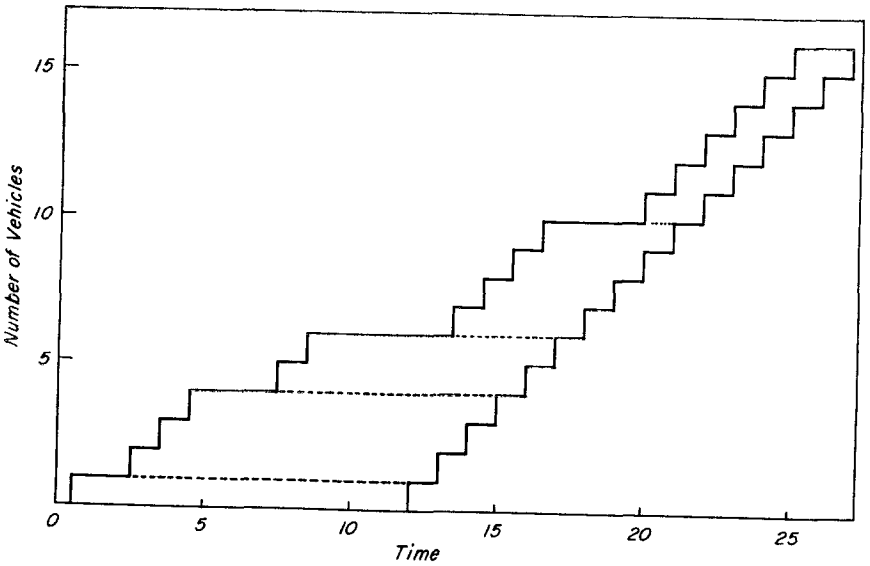

Fig. 1. Graph illustrating buildup and discharge of a traffic queue at an intersection throughout one cycle of the traffic light. The top graph is a plot of the cumulative number of arrivals. The bottom graph represents the cumulative number of vehicles discharging from the queue after an effective red period of 11τ. The total delay is represented by the area between the graphs; this may be calculated from the strips between the dotted lines. Time is measured in units of τ and arrivals and departures are assumed to occur, as indicated, at the midpoints of the τ intervals.

This is formula (16) of reference 1 (except for the exclusion of the first term in the product which is, in fact, equal to unity).

In the example illustrated in Fig. 1, the arriving traffic is represented by the sequence

1 0 1 1 1 0 0 1 1 0 0 0 0 1 1 1 1 0 0 1 1 1 1 1 1 0 0

so that the values of Y_j are

j	1	2	3	4	5	6	7	8	9	10	11
Y_j	1	3	0	2	0	0	0	4	0	6	0.

The first car discharges at the 12th τ interval after an effective red period $r = 11$. As can be verified from the figure, the area between the graphs, giving the delay, is

$$\Sigma_1^{11} jY_{12-j} = 85 \text{ units.} \tag{4}$$

The direct calculation of formula (3) for the generating function illustrates the simplicity of the binomial model and the usefulness of the cumulative arrival and discharge curves in delay problems.

REFERENCES

1. M. C. DUNNE, "Traffic Delay at a Signalized Intersection with Binomial Arrivals," *Trans. Sci* **1**, 24–31 (1967).
2. J. N. DARROCH, G. F. NEWELL, AND R. W. J. MORRIS, "Queues for a Vehicle-Actuated Traffic Light," *Opns. Res.* **12**, 882–895 (1964).
3. W. FELLER, *An Introduction to Probability and its Applications*, Vol. 1 (2nd Ed.), Wiley, New York, 1957.

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Errata

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I WISH to correct two errors in my review of the Australian Road Research Board Third Conference, Sydney, 1966, which appeared in *Transportation Science*, **1**, 32–46 (1967). In my comments on N. F. CLARK's paper, "Some Benefits and Costs of Road Works and Their Relevance for Priority Decisions," I incorrectly described his consumption benefits as social benefits. I am informed that his productive benefits are what economists know as social benefits. I have now found a mention of accident costs in the paper.

The A.R.R.B. Proceedings are now available in two parts at a price of $A15 (Australian dollars) including surface mail postage. The volumes may be purchased separately for $A8 each. The papers that I reviewed are in Part 1.

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