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Letter to the Editor—Comments on “Queuing Systems with Transport Service Processes”

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Comments on "Queuing Systems with Transport Service Processes"

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In his paper, PEARCE presents a method for determining the stationary probabilities of a $G/M_t/1$ system, the service mechanism being such that it operates regardless of whether or not customers are present. This note is an attempt to correct his results.

In his paper, "Queuing Systems with Transport Service,"[1] Pearce establishes the following relation (omission of $i!$ in the denominator is an obvious misprint):

$$P_0(\tau_1, \tau_2, \cdots, \tau_{n+1}) - R_0(\tau_1, \tau_2, \cdots, \tau_{n+1}) + R_0(\tau_1, \tau_2, \cdots, \tau_n) \exp(-\mu\tau_{n+1}) = \sum_{i=0}^{i=\infty} P_i(\tau_1, \tau_2, \cdots, \tau_n) \exp(-\mu\tau_{n+1})(\mu\tau_{n+1})^i/i!. \quad (3)$$

The R.H.S. of (3) is the probability of the nth customer finding i customers waiting and of i service periods terminating before the arrival of the $(n+1)$th customer, summed over all possible values of i.

As the nth customer finds i customers in queue waiting for service, the queue size becomes $i + 1$ as a result of his arrival. Obviously, every termination of a service period reduces the queue size by one. Hence the probability given by the R.H.S. of (3) equals the sum of probabilities of the following mutually-exclusive events: (a) the $(n+1)$th customer finds a queue of one, with one customer being served (this event corresponds to $i \geqq 1$ in the summation); (b) the nth customer finds an empty station and no service period terminates before the arrival of the $(n+1)$th customer. Hence the L.H.S. of (3) should read:

$$P_1(\tau_1, \tau_2, \cdots, \tau_{n+1}) - R_1(\tau_1, \tau_2, \cdots, \tau_{n+1}) + R_0(\tau_1, \tau_2, \cdots, \tau_n) \exp(-\mu\tau_{n+1}). \quad \text{(L.H.S.3)}$$

The difference between (L.H.S.3) and the L.H.S. of (3) in reference 1 is that the subscript of P and R in the first two terms is one rather than zero. A relation involving R_0 only is desirable. Accordingly, we replace (3) with:

$$\textstyle\sum_{i=0}^{i=\infty} P_i(\tau_1, \cdots, \tau_n)(\mu\tau_{n+1})^{i+1} \exp(-\mu\tau_{n+1})/(i+1)! = P_0(\tau_1, \cdots, \tau_{n+1}) - R_0(\tau_1, \cdots, \tau_{n+1}). \tag{3a}$$

The L.H.S. of the proposed equation (3a) is the probability of the nth customer finding upon arrival i customers in the queue, and of the $i+1$ service periods terminating before the arrival of the $(n+1)$th customer. The R.H.S. of our (3a) equals the probability of the $(n+1)$th customer finding a single customer being served and a queue of zero size.

Applying a similar procedure as in reference 1 for stationary distributions, we have:

$$R_0 = \frac{1-T}{T}\psi(1). \tag{b}$$

(b) differs from the value of R_0 in reference 1. Assuming Poisson input, the probability of a customer finding an empty station (and any queue size) is $1 - \lambda/\mu$, whereas according to reference 1 it would be $[1 - (\lambda/\mu)^2]^2$.

REFERENCE

1. C. E. Pearce, "Queuing Systems with Transport Service Processes," *Trans. Sci.* **1,** 218–223 (1967).