Appendix

Lemma 1. For constant equilibrium solution c^0 and z^0, payoffs of the firm and the hacker are given by

$$\begin{cases} J_f = -\dfrac{1}{\rho_f}(1+f\lambda)z^0 + \dfrac{h\lambda}{\rho_f}c^0 - \dfrac{L}{\rho_f}\Delta(z^0,c^0) + \dfrac{L\alpha}{\rho_f} + \lambda y_0 \\ J_h = -\dfrac{f\mu}{\rho_h}z^0 - \dfrac{1-h\mu}{\rho_h}c^0 + \dfrac{G}{\rho_h}\Delta(z^0,c^0) + \dfrac{(1-\alpha)G}{\rho_h} + \mu y_0 \end{cases}. \tag{A1}$$

Proof. Because c^0 and z^0 are constant, the security breach probability rate $\Delta(z^0,c^0) = \varepsilon(z^0)^{-\beta}(c^0)^{\phi}$ is constant as well. The related linear state equation $\dot{y}(t) = ry(t) - fz^0 + hc^0$ can be solved as follows

$$y(t) = \left(y_0 - \frac{fz^0 - hc^0}{r}\right)e^{rt} + \frac{fz^0 - hc^0}{r}, \tag{A2}$$

which yields

$$\int_0^\infty y(t)e^{-\rho t}dt = \frac{\rho y_0 - fz^0 + hc^0}{\rho(\rho - r)}. \tag{A3}$$

Hence, one can finally obtain

$$\begin{aligned} J_f &= \int_0^\infty \left[\left(\alpha - \Delta(z^0,c^0) - ky\right)L - z^0\right] e^{-\rho_f t}dt \\ &= -kL\frac{\rho_f y_0 - fz^0 + hc^0}{\rho_f(\rho_f - r)} + \frac{L}{\rho_f}\left(\alpha - \Delta(z^0,c^0)\right) - \frac{1}{\rho_f}z^0 \\ &= -\frac{1}{\rho_f}\left(1 - \frac{kfL}{\rho_f - r}\right)z^0 - \frac{khL}{\rho_f(\rho_f - r)}c^0 - \frac{L}{\rho_f}\Delta(z^0,c^0) + \frac{L\alpha}{\rho_f} - \frac{kLy_0}{\rho_f - r} \\ &= -\frac{1}{\rho_f}(1+f\lambda)z^0 + \frac{h\lambda}{\rho_f}c^0 - \frac{L}{\rho_f}\Delta(z^0,c^0) + \frac{L\alpha}{\rho_f} + \lambda y_0 \end{aligned} \tag{A4}$$

and

$$\begin{aligned} J_h &= \int_0^\infty \left[\left(1 - \alpha + \Delta(z^0,c^0) + ky\right)G - c^0\right] e^{-\rho_h t}dt \\ &= \frac{1}{\rho_h}\left(1 - \alpha + \Delta(z^0,c^0)\right)G - \frac{1}{\rho_h}c^0 + kG\frac{\rho_h y_0 - fz^0 + hc^0}{\rho_h(\rho_h - r)} \\ &= -\frac{kfG}{\rho_h(\rho_h - r)}z^0 - \frac{1}{\rho_h}\left(1 - \frac{khG}{\rho_h - r}\right)c^0 + \frac{G}{\rho_h}\Delta(z^0,c^0) + \frac{(1-\alpha)G}{\rho_h} + \frac{kGy_0}{\rho_h - r} \\ &= -\frac{f\mu}{\rho_h}z^0 - \frac{1-h\mu}{\rho_h}c^0 + \frac{G}{\rho_h}\Delta(z^0,c^0) + \frac{(1-\alpha)G}{\rho_h} + \mu y_0 \end{aligned}. \tag{A5}$$

Proof of Proposition 1.

Because the costate variables λ and μ are constant, the equilibrium solution is constant. One can calculate directly by Lemma 1 that

$$J_{f\_si} = -\frac{1}{\rho_f}(1+f\lambda)\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{\frac{1-\phi}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}} + \frac{h\lambda}{\rho_f}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{1+\beta}{1-\phi+\beta}}$$

$$-\frac{L}{\rho_f}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}} + \frac{L\alpha}{\rho_f} + \lambda y_0$$

$$= -\frac{1}{\rho_f}(1+f\lambda)\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{\frac{1-\phi}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}} \quad \text{(A6)}$$

$$-\frac{1}{\rho_f}\left(L - h\lambda\frac{\phi G}{1-\mu h}\right)\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}} + \frac{L\alpha}{\rho_f} + \lambda y_0$$

$$= -\frac{L}{\rho_f}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left(1+\beta-\frac{h\lambda}{L}\frac{\phi G}{1-\mu h}\right) + \frac{L\alpha}{\rho_f} + \lambda y_0$$

and

$$J_{h\_si} = -\frac{f\mu}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{\frac{1-\phi}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}} - \frac{1-h\mu}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{1+\beta}{1-\phi+\beta}}$$

$$+\frac{G}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}} + \frac{(1-\alpha)G}{\rho_h} + \mu y_0$$

$$= -\frac{f\mu}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{\frac{1-\phi}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}} + \frac{G(1-\phi)}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}} \quad \text{(A7)}$$

$$+\frac{(1-\alpha)G}{\rho_h} + \mu y_0$$

$$= \frac{G}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left(1-\phi-\frac{f\mu}{G}\frac{\beta L}{1+\lambda f}\right) + \frac{(1-\alpha)G}{\rho_h} + \mu y_0$$

Proof of Proposition 2.

The equilibrium solution is constant in a similar fashion, which implies that by Lemma 1

$$
\begin{aligned}
J_{f\_fh} &= -\frac{1}{\rho_f}(1+f\lambda)\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left[\frac{\beta L}{(1-\phi)(1+\lambda f)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{\frac{1-\phi}{1-\phi+\beta}} \\
&+\frac{h\lambda}{\rho_f}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{1+\beta}{1-\phi+\beta}}\left[\frac{\beta L}{(1-\phi)(1+\lambda f)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{-\frac{\beta}{1-\phi+\beta}} \\
&-\frac{L}{\rho_f}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left[\frac{\beta L}{(1-\phi)(1+\lambda f)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{-\frac{\beta}{1-\phi+\beta}}+\frac{L\alpha}{\rho_f}+\lambda y_0 \qquad \text{(A8)} \\
&= -\frac{1}{\rho_f}(1+f\lambda)\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left[\frac{\beta L}{(1-\phi)(1+\lambda f)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{\frac{1-\phi}{1-\phi+\beta}} \\
&-\frac{1}{\rho_f}\left(L-\lambda h\frac{\phi G}{1-\mu h}\right)\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left[\frac{\beta L}{(1-\phi)(1+\lambda f)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{-\frac{\beta}{1-\phi+\beta}}+\frac{L\alpha}{\rho_f}+\lambda y_0 \\
&= -\varepsilon^{\frac{1}{1-\phi+\beta}}\frac{(1-\phi+\beta)L}{\rho_f(1-\phi)}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left[\frac{\beta L}{(1-\phi)(1+\lambda f)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{-\frac{\beta}{1-\phi+\beta}}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right) \\
&+\frac{L\alpha}{\rho_f}+\lambda y_0 \\
&= -\frac{L(1-\phi+\beta)\varepsilon^{\frac{1}{1-\phi+\beta}}(1-\phi)^{\frac{1-\phi}{1-\phi+\beta}}}{\rho_f}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)^{\frac{1-\phi}{1-\phi+\beta}}+\frac{L\alpha}{\rho_f}+\lambda y_0
\end{aligned}
$$

and

$$
\begin{aligned}
J_{h\_fh} &= -\frac{f\mu}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left[\frac{\beta L}{(1-\phi)(1+\lambda f)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{\frac{1-\phi}{1-\phi+\beta}} \\
&-\frac{1-h\mu}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{1+\beta}{1-\phi+\beta}}\left[\frac{\beta L}{(1-\phi)(1+\lambda f)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{-\frac{\beta}{1-\phi+\beta}} \\
&+\frac{G}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left[\frac{\beta L}{(1-\phi)(1+\lambda f)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{-\frac{\beta}{1-\phi+\beta}}+\frac{(1-\alpha)G}{\rho_h}+\mu y_0 \qquad \text{(A9)} \\
&= -\frac{f\mu}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left[\frac{\beta L}{(1-\phi)(1+\lambda f)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{\frac{1-\phi}{1-\phi+\beta}} \\
&+\frac{G(1-\phi)}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left[\frac{\beta L}{(1-\phi)(1+\lambda f)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{-\frac{\beta}{1-\phi+\beta}}+\frac{(1-\alpha)G}{\rho_h}+\mu y_0 \\
&= \varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left[\frac{1}{1-\phi}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{-\frac{\beta}{1-\phi+\beta}}\times \\
&\left[\frac{G(1-\phi)}{\rho_h}-\frac{f\mu}{\rho_h}\frac{\beta L}{(1-\phi)(1+\lambda f)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]+\frac{(1-\alpha)G}{\rho_h}+\mu y_0
\end{aligned}
$$

Proof of Proposition 3.

The equilibrium solution is constant as well and thus one can obtain by Lemma 1

$$
\begin{aligned}
J_{f\_hf} &= -\frac{1}{\rho_f}(1+f\lambda)\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{\frac{1-\phi}{1-\phi+\beta}}\left[\frac{\phi G}{(1+\beta)(1-\mu h)}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{\phi}{1-\phi+\beta}} \\
&\quad +\frac{h\lambda}{\rho_f}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left[\frac{\phi G}{(1+\beta)(1-\mu h)}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{1+\beta}{1-\phi+\beta}} \\
&\quad -\frac{L}{\rho_f}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left[\frac{\phi G}{(1+\beta)(1-\mu h)}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{\phi}{1-\phi+\beta}}+\frac{L\alpha}{\rho_f}+\lambda y_0 \\
&= \frac{h\lambda}{\rho_f}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left[\frac{\phi G}{(1+\beta)(1-\mu h)}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{1+\beta}{1-\phi+\beta}} \\
&\quad -\frac{L(1+\beta)}{\rho_f}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left[\frac{\phi G}{(1+\beta)(1-\mu h)}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{\phi}{1-\phi+\beta}}+\frac{L\alpha}{\rho_f}+\lambda y_0 \\
&= -\frac{1}{\rho_f}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left[\frac{1}{1+\beta}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{\phi}{1-\phi+\beta}}\times \\
&\quad \left[L(1+\beta)-\frac{h\lambda\phi G}{(1+\beta)(1-\mu h)}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]+\frac{L\alpha}{\rho_f}+\lambda y_0
\end{aligned}
\tag{A10}
$$

and

$$
\begin{aligned}
J_{h\_hf} &= -\frac{f\mu}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{\frac{1-\phi}{1-\phi+\beta}}\left[\frac{\phi G}{(1+\beta)(1-\mu h)}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{\phi}{1-\phi+\beta}} \\
&\quad -\frac{1-h\mu}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left[\frac{\phi G}{(1+\beta)(1-\mu h)}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{1+\beta}{1-\phi+\beta}} \\
&\quad +\frac{G}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left[\frac{\phi G}{(1+\beta)(1-\mu h)}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{\phi}{1-\phi+\beta}}+\frac{(1-\alpha)G}{\rho_h}+\mu y_0 \\
&= -\frac{1-h\mu}{\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left[\frac{\phi G}{(1+\beta)(1-\mu h)}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{1+\beta}{1-\phi+\beta}} \\
&\quad +\frac{(1+\beta)(1-\mu h)}{\phi\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left[\frac{\phi G}{(1+\beta)(1-\mu h)}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{1+\beta}{1-\phi+\beta}}+\frac{(1-\alpha)G}{\rho_h}+\mu y_0 \\
&= \frac{(1-\phi+\beta)(1-h\mu)}{\phi\rho_h}\varepsilon^{\frac{1}{1-\phi+\beta}}\left(\frac{\beta L}{1+\lambda f}\right)^{-\frac{\beta}{1-\phi+\beta}}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{1+\beta}{1-\phi+\beta}}\left[\frac{1}{1+\beta}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{1+\beta}{1-\phi+\beta}} \\
&\quad +\frac{(1-\alpha)G}{\rho_h}+\mu y_0
\end{aligned}
\tag{A11}
$$

Proof of Proposition 4.

By Proposition 1 and Proposition 2, we can get

$$\begin{cases} \dfrac{z_{fh}^*}{z^*} = \left[\dfrac{1}{1-\phi}\left(1-\dfrac{\lambda h}{L}\dfrac{\phi G}{1-\mu h}\right)\right]^{\frac{1-\phi}{1-\phi+\beta}} \\ \dfrac{c_{fh}^*}{c^*} = \left[\dfrac{1}{1-\phi}\left(1-\dfrac{\lambda h}{L}\dfrac{\phi G}{1-\mu h}\right)\right]^{-\frac{\beta}{1-\phi+\beta}} \\ \dfrac{\Delta(z_{fh}^*, c_{fh}^*)}{\Delta(z^*, c^*)} = \left[\dfrac{1}{1-\phi}\left(1-\dfrac{\lambda h}{L}\dfrac{\phi G}{1-\mu h}\right)\right]^{-\frac{\beta}{1-\phi+\beta}} \end{cases}, \tag{A12}$$

Noting that $\lambda < 0$ and $\mu > 0$, we have

$$\frac{1}{1-\phi}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right) > 1, \tag{A13}$$

which yields Proposition 4 (i).

Similarly, by Proposition 1 and Proposition 3, we have

$$\begin{cases} \dfrac{z_{hf}^*}{z^*} = \left[\dfrac{1}{1+\beta}\left(1-\dfrac{\mu f}{G}\dfrac{\beta L}{1+\lambda f}\right)\right]^{\frac{\phi}{1-\phi+\beta}} \\ \dfrac{c_{hf}^*}{c^*} = \left[\dfrac{1}{1+\beta}\left(1-\dfrac{\mu f}{G}\dfrac{\beta L}{1+\lambda f}\right)\right]^{\frac{1+\beta}{1-\phi+\beta}} \\ \dfrac{\Delta(z_{hf}^*, c_{hf}^*)}{\Delta(z^*, c^*)} = \left[\dfrac{1}{1+\beta}\left(1-\dfrac{\mu f}{G}\dfrac{\beta L}{1+\lambda f}\right)\right]^{\frac{\phi}{1-\phi+\beta}} \end{cases}, \tag{A14}$$

which implies Proposition 4 (ii) because

$$\frac{1}{1+\beta}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right) < 1. \tag{A15}$$

Proof of Proposition 5.

By proposition 1 and proposition 2, it is obvious that $J_{h\_fh} < J_{h\_si}$ if and only if

$$\left[\frac{1}{1-\phi}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{-\frac{\beta}{1-\phi+\beta}}\left[1-\phi-\frac{f\mu}{G}\frac{\beta L}{(1-\phi)(1+\lambda f)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right] < 1-\phi-\frac{f\mu}{G}\frac{\beta L}{1+\lambda f} \tag{A16}$$

namely,

$$\frac{1-\phi-\frac{f\mu}{G}\frac{\beta L}{(1+\lambda f)(1-\phi)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)}{1-\phi-\frac{f\mu}{G}\frac{\beta L}{1+\lambda f}}<\left[\frac{1}{1-\phi}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{\frac{\beta}{1-\phi+\beta}}, \tag{A17}$$

which holds because

$$\frac{1}{1-\phi}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)>1 \tag{A18}$$

and

$$\frac{1-\phi-\frac{f\mu}{G}\frac{\beta L}{(1+\lambda f)(1-\phi)}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)}{1-\phi-\frac{f\mu}{G}\frac{\beta L}{1+\lambda f}}<1. \tag{A19}$$

Furthermore, we are able to obtain that $J_{h\_hf} > J_{h\_si}$ if and only if

$$\frac{(1-\phi+\beta)(1-h\mu)}{\phi}\left(\frac{\phi G}{1-\mu h}\right)^{\frac{1+\beta}{1-\phi+\beta}}\left[\frac{1}{1+\beta}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{1+\beta}{1-\phi+\beta}} > G\left(\frac{\phi G}{1-\mu h}\right)^{\frac{\phi}{1-\phi+\beta}}\left(1-\phi-\frac{f\mu}{G}\frac{\beta L}{1+\lambda f}\right) \tag{A20}$$

namely,

$$\left[\frac{1}{1+\beta}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{1+\beta}{1-\phi+\beta}} > \frac{1}{1-\phi+\beta}\left(1-\phi-\frac{f\mu}{G}\frac{\beta L}{1+\lambda f}\right), \tag{A21}$$

which holds by the continuity of k provided that

$$\left(\frac{1}{1+\beta}\right)^{\frac{1+\beta}{1-\phi+\beta}} > \frac{1-\phi}{1-\phi+\beta}, \tag{A22}$$

namely

$$(1-\phi+\beta)\ln(1-\phi)-(1-\phi+\beta)\ln(1-\phi+\beta)+(1+\beta)\ln(1+\beta)<0, \tag{A23}$$

where $\ln(x)$ is natural logarithms of x. Denoting

$$\Lambda(\phi)=(1-\phi+\beta)\ln(1-\phi)-(1-\phi+\beta)\ln(1-\phi+\beta)+(1+\beta)\ln(1+\beta),$$

one can obtain

$$\Lambda'(\phi) = \ln\left(1+\frac{\beta}{1-\phi}\right) - \frac{\beta}{1-\phi} \triangleq \ln(1+\mathrm{K}) - \mathrm{K}, \quad \text{(A24)}$$

where $\mathrm{K} = \frac{\beta}{1-\phi} \in (1,\infty)$. The derivative of $\ln(1+\mathrm{K}) - \mathrm{K}$ with respect to K is $-\frac{\mathrm{K}}{1+\mathrm{K}} < 0$, which implies $\Lambda'(\phi) = \ln(1+\mathrm{K}) - \mathrm{K} < \ln(2) - 1 < 0$ and further $\Lambda(\phi) < \Lambda(0) = 0$. Therefore, condition (A23) holds.

By proposition 1 and proposition 2, one can derive that $J_{f\_fh} > J_{f\_si}$ if and only if

$$\left(1+\beta-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right) > (1-\phi+\beta)(1-\phi)^{-\frac{1-\phi}{1-\phi+\beta}}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)^{\frac{1-\phi}{1-\phi+\beta}}, \quad \text{(A25)}$$

namely,

$$\frac{1}{1+\beta-\phi}\left(1+\beta-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right) > \left[\frac{1}{1-\phi}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)\right]^{\frac{1-\phi}{1-\phi+\beta}}, \quad \text{(A26)}$$

which holds by the continuity of k provided that

$$\frac{1+\beta}{1+\beta-\phi} > \left(\frac{1}{1-\phi}\right)^{\frac{1-\phi}{1-\phi+\beta}} \quad \text{(A27)}$$

or

$$(1-\phi+\beta)\ln(1+\beta) - (1-\phi+\beta)\ln(1-\phi+\beta) + (1-\phi)\ln(1-\phi) \triangleq \Phi(\phi) > 0. \quad \text{(A28)}$$

One can obtain easily that

$$\Phi'(\phi) = \ln\frac{1-\phi+\beta}{(1-\phi)(1+\beta)} > 0, \quad \text{(A29)}$$

implying that $\Phi(\phi) > \Phi(0) = 0$, and therefore equation (A28) holds. Meanwhile, by proposition 2 and proposition 3, one can obtain that $J_{f\_hf} > J_{f\_fh}$ if and only if

$$(1-\phi+\beta)(1-\phi)^{-\frac{1-\phi}{1-\phi+\beta}}\left(1-\frac{\lambda h}{L}\frac{\phi G}{1-\mu h}\right)^{\frac{1-\phi}{1-\phi+\beta}} > \left[\frac{1}{1+\beta}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right]^{\frac{\phi}{1-\phi+\beta}}\left[(1+\beta) - \frac{h\lambda\phi G}{(1+\beta)L(1-\mu h)}\left(1-\frac{\mu f}{G}\frac{\beta L}{1+\lambda f}\right)\right], \quad \text{(A30)}$$

which holds by the continuity of k provided that

$$(1-\phi+\beta)(1-\phi)^{\frac{1-\phi}{1-\phi+\beta}} > (1+\beta)^{\frac{1-2\phi+\beta}{1-\phi+\beta}}, \tag{A31}$$

equivalently

$$(1-\phi+\beta)\ln(1-\phi+\beta)-(1-\phi)\ln(1-\phi) > (1-2\phi+\beta)\ln(1+\beta). \tag{A32}$$

It suffices to prove

$$(1-\phi+\beta)\ln(1-\phi+\beta) > (\beta-\phi)\ln(1+\beta)+(1-\phi)\ln[(1-\phi)(1+\beta)]. \tag{A33}$$

Denoting

$$\Theta(\beta) = (1-\phi+\beta)\ln(1-\phi+\beta)-(\beta-\phi)\ln(1+\beta)-(1-\phi)\ln[(1-\phi)(1+\beta)], \tag{A34}$$

one can easily obtain that

$$\Theta'(\beta) = \ln\frac{1-\phi+\beta}{1+\beta}+\frac{2\phi}{1+\beta} = \ln\left(1-\frac{\phi}{1+\beta}\right)+\frac{2\phi}{1+\beta} \triangleq \ln(1-\Delta)+2\Delta, \tag{A35}$$

where $\Delta = \dfrac{\phi}{1+\beta} \in \left(0,\dfrac{1}{2}\right)$.

The derivative of $\ln(1-\Delta)+2\Delta$ with respect to Δ takes the form:

$$\frac{1-2\Delta}{1-\Delta} > 0, \tag{A36}$$

which implies $\Theta'(\beta) = \ln(1-\Delta)+2\Delta > 0$. Hence,

$$\Theta(\beta) > \Theta(1) = (2-\phi)\ln(2-\phi)-(1-\phi)\ln[4(1-\phi)] \triangleq \Omega(\phi). \tag{A37}$$

Noting that

$$\Omega'(\phi) = \ln\frac{4(1-\phi)}{2-\phi}, \tag{A38}$$

one can get $\Omega'(\phi) > 0$ with $\phi \in \left(0,\dfrac{2}{3}\right)$ and $\Omega'(\phi) < 0$ with $\phi \in \left(\dfrac{2}{3},1\right)$, implying

$$\Theta(\beta) > \Omega(\phi) > \min\left(\Omega(0),\Omega(1)\right) = 0. \tag{A39}$$

Hence, equation (A31) holds.