Appendix. Extension to Multiple Enterprises

In this appendix, we extend the model from two enterprises to any finite number, M, of enterprises, where $M > 2$. To simplify, we assume that in a resource-sharing platform, M enterprises are directly interconnected with each other, in which enterprise 1 is the only high-type enterprise and other $M-1$ enterprises are the low-type enterprises. This captures the practices that some resource-sharing platforms have one leading enterprise and several upstream and downstream related enterprises.

Consistent with the basic model, the two types of enterprises are heterogeneous in enterprise value, cost efficiency, and investment timing. We use subscript M to denote the multiple cases of this extension. We first test whether the above results still hold and analyze how security decisions are affected by the number of enterprises.

1 Validating the Propositions

In the benchmark case, the expected cost of M enterprises is characterized as follows:

$$\omega^B = [1-\theta_1\prod_{i=2}^{M}(1-q(1-\theta_i))]nL + \frac{1}{2}\varphi c\theta_1^2 + \sum_{i=2}^{M}\{[1-\theta_i\prod_{j=1, j\neq i}^{M}(1-q(1-\theta_j))]L + \frac{1}{2}c\theta_i^2\}$$

In the simultaneous game, the expected costs for the two types of enterprises are demonstrated as follow:

$$\omega_1^N = [1-\theta_1\prod_{i=2}^{M}(1-q(1-\theta_i))]nL + \frac{1}{2}\varphi c\theta_1^2$$

$$\omega_i^N = [1-\theta_i\prod_{j=1, j\neq i}^{M}(1-q(1-\theta_j))]L + \frac{1}{2}c\theta_i^2, i = 2...M$$

In the sequential game, we first derive the equilibrium solution of the sequential game where enterprise 1 acts as the leader and other $M-1$ enterprises as the followers.

We derive that the responding functions of the low-type enterprises are such that equations $\theta_i = \frac{\prod_{j=1, j\neq i}^{M}(1-q(1-\theta_j))L}{c}$. Hence, with backward induction, enterprise 1's expected cost is $\omega_1^{SL} = [1-\theta_1 \prod_{i=2}^{M}(1-q(1-\frac{\prod_{j=1, j\neq i}^{M}(1-q(1-\theta_j))L}{c}))]nL + \frac{1}{2}\varphi c \theta_1^2$.

Similarly, in the case that the low-type enterprise i acts as the leader and the high-type enterprise and other $M-2$ enterprises act as the followers, the responding functions of the enterprises are such that $\theta_1 = \frac{[\prod_{i=2}^{M}(1-q(1-\theta_i))]nL}{\varphi c}$ and $\theta_j = \frac{\prod_{k=1, k\neq j}^{M}(1-q(1-\theta_k))L}{c}, j \neq i$. Enterprise i's expected cost is $\omega_i^{SL} = [1-\theta_i(1-q(1-\frac{[\prod_{i=2}^{M}(1-q(1-\theta_i))]nL}{\varphi c}))\prod_{j=2, j\neq i}^{M}(1-q(1-\frac{\prod_{k=1, k\neq j}^{M}(1-q(1-\theta_k))L}{c}))]L + \frac{1}{2}c\theta_i^2$.

Given that the theoretical analysis of multiple cases proves to be analytically challenging and intractable, we employ numerical methods as an alternative approach to exploration. We use the following parameters for the numerical analysis: $L = 0.25$, $c = 5$, $\varphi = 0.3$, $n = 6$, $q = 0.3$ and $M = 3$.

Figure 4 shows that when multiple enterprises are interconnected with each other, both the security effort of the high-type enterprise and low-type enterprises decreases with the system vulnerability, and the high-type enterprise would always possess a superior security quality than that of the low-type enterprise. This shows that Proposition 1 and 3 still hold under multiple cases.

Figure 5 shows that as the ratio of cost inefficiency increases, the security quality of the high-type enterprise would suffer a larger decrease than that of the low-type enterprises, narrowing the security quality gap between the two enterprises. This shows that Proposition 2 still holds under multiple cases. Meanwhile, Figure 5 also validates Proposition 1, which states that for both types of enterprises, the information security quality always decreases with the ratio of cost inefficiency.

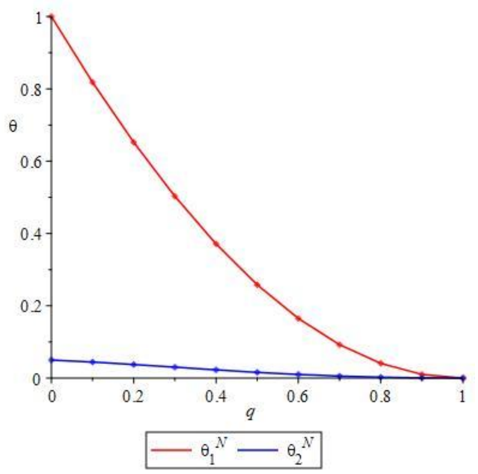


Fig.4 The security quality of enterprises decreases with q

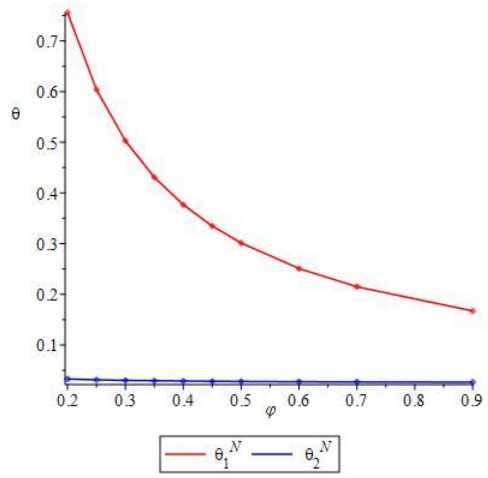


Fig.5 The security quality of enterprises decreases with φ

Figure 6(a) and 6(b) show that under multiple enterprises case, the socially optimal benchmark creates the highest security quality under all four situations, enterprises would always exert more effort in the sequential game than in the simultaneous game, and the security quality in the following mode is always lower than that in the leading mode. Moreover, Figure 7 shows that enterprises would achieve a lower expected cost in the sequential game than in the simultaneous game, and the expected cost in the following mode is always lower than that in the leading mode. Thus, Figures 6 and 7 together validate that Proposition 4 still holds under multiple enterprises case.

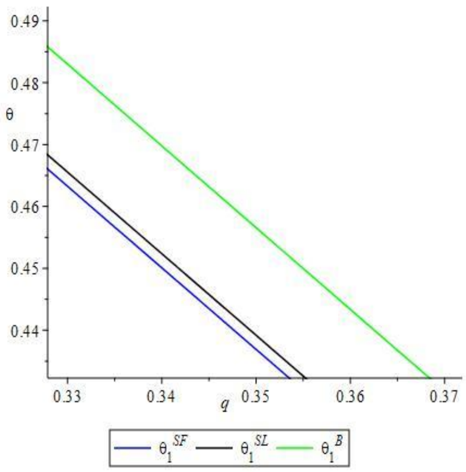


Fig.6(a) The security quality of the high-type enterprise decreases with q

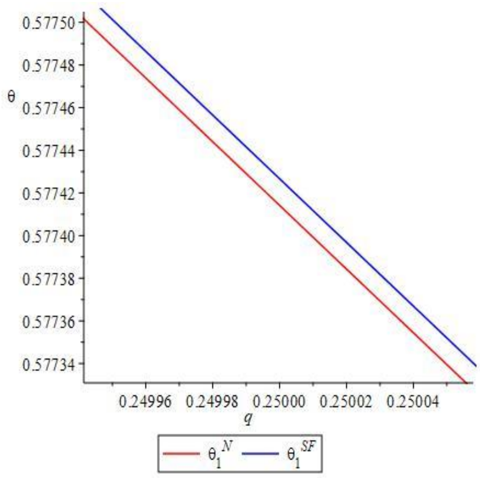


Fig.6(b) The security quality of the low-type enterprise decreases with q

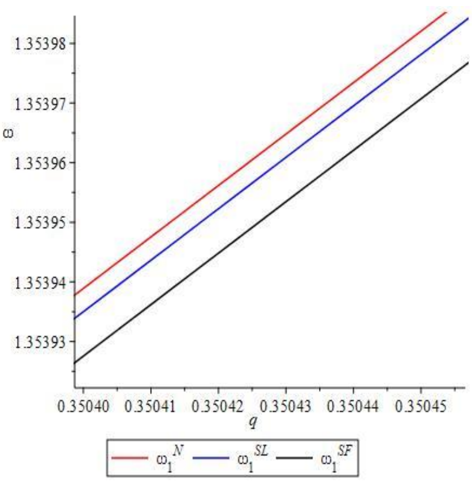


Fig.7 The expected cost of the high-type enterprise increases with q

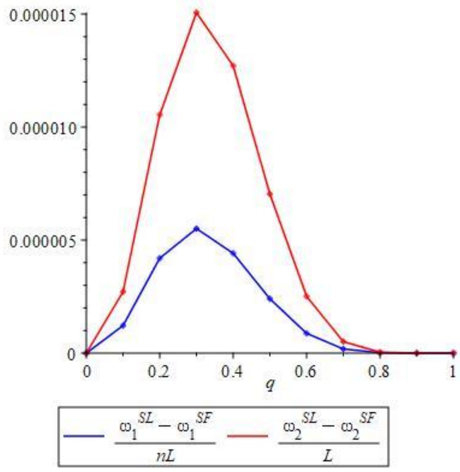


Fig.8 The expected cost of the high-type enterprise first increases and then decreases with q

Figure 8 shows that under multiple enterprises case, the relative loss of the high-type enterprise always exceeds that of the low-type enterprises, which indicates that the low-type enterprises could benefit more from acting as followers. This also shows that Proposition 5 still holds under multiple enterprises case.

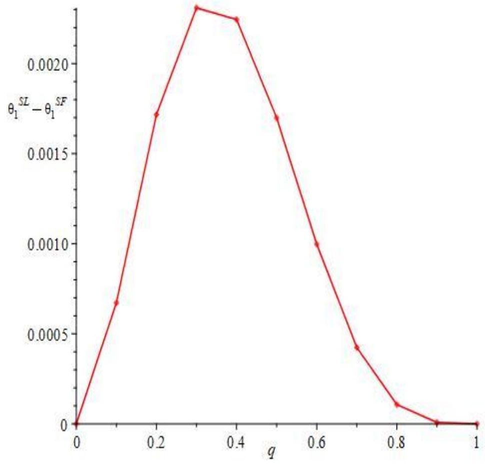


Fig.9(a) The security quality of the high-type enterprise first increases then decreases with q

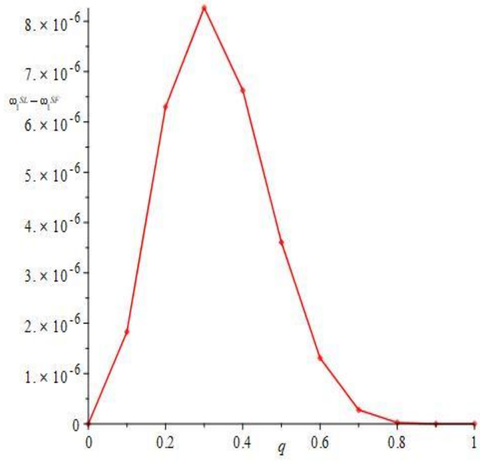


Fig.9(b) The expected cost of the high-type enterprise first increases then decreases with q

Figures 9(a) and 9(b) show that under multiple enterprises case, when the system vulnerability increases, the gap between the equilibrium security quality and the expected cost in the leading mode and that in the following mode first increases and then decreases. This also shows that Proposition 6 still holds under multiple enterprises case.

We now explore how the number of enterprises M affects the enterprises' security quality. We also use numerical analysis for the remainder of our exploration. We use the following parameters for the numerical analysis: $L=0.1$, $c=0.9$, $\varphi=0.8$, $n=2$, $q=0.15$.

Observation 3: Both the security quality of the high-type enterprise (θ_1) and that of the low-type enterprises (θ_i) decrease with the number of enterprises (M).

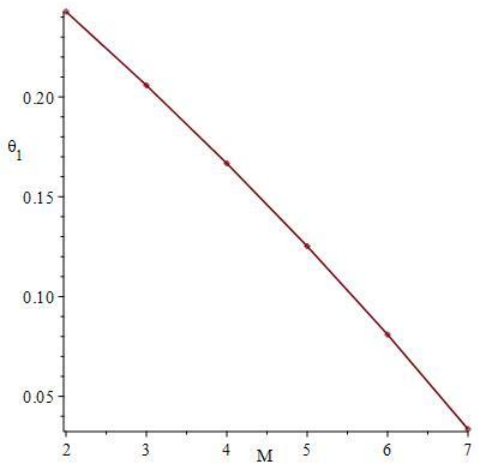


Fig.10(a)The security quality of the high-type enterprise decreases with M

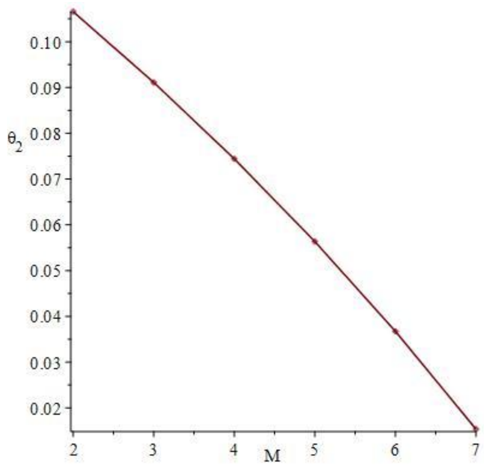


Fig.10(b) The security quality of the low-type enterprise decreases with M

As the M th enterprise joins the resource-sharing platform, the indirect security risk for other $M-1$ enterprises increase. This leads to a more severe free-riding problem, that is, $M-1$ enterprises expect the M th enterprise to exert security effort so that they can cut back on their security effort to save their security costs. Thus, as the number of enterprises increases, both types of enterprises tend to decrease their security effort.

2. Validating the Mechanism

Next, we validate the liability-based mechanism under multiple enterprises case. In the case of interconnection among multiple enterprises, indirect attacks through the system vulnerability can occur many times. For the sake of simplicity, we only consider the first-order indirect attack. That is, if enterprise 1 suffers an indirect attack, then we ignore subsequent indirect attacks through enterprise 1. This assumption is reasonable when q is small. Moreover, more than one security breach in a day is not likely to be very common. Thus, we also assume the loss is unchanged irrespective of the number of breaches. That is, the loss of another indirect attack can be ignored if the enterprise has already suffered an indirect attack. Without loss of generality, we assume that enterprise 1 is the high-type enterprise and others $M-1$ enterprises are the low-type enterprises.

Similar to the situation of liability with two enterprises case, enterprise 1 can suffer security losses in two ways. First, attackers directly breach enterprise 1 and then breach one or $M-1$ enterprises, enterprise 1 should take on all breached enterprises' losses; thus, enterprise 1's expected loss is $\sum_{i=2}^{M}[(1-\theta_1)q\theta_i(\prod_{j=2}^{i-1}(1-q\theta_j))(n+(1+\sum_{j=i+1}^{M}q\theta_j)\lambda_1)L]+(1-\theta_1)(\prod_{i=2}^{M}(1-q\theta_i))nL$. Second, when the hacker penetrates enterprise 1 indirectly through a successful direct attack against enterprise *i*, enterprise 1 would receive $\sum_{i=2}^{M}[(1-\theta_i)q\theta_1\lambda_i nL]$ from enterprise i, and thus enterprise 1 has the expected loss in this scenario such that $\sum_{i=2}^{M}[(1-\theta_i)q\theta_1(1-\lambda_i)nL]$. To conclude, we could obtain the expected cost function of the two types of enterprises under the liability-based mechanism as follows:

$$\begin{aligned}\overline{\omega_1}=&\sum_{i=2}^{M}[(1-\theta_1)q\theta_i(\prod_{j=2}^{i-1}(1-q\theta_j))(n+(1+\sum_{j=i+1}^{M}q\theta_j)\lambda_1)L]\\&+(1-\theta_1)(\prod_{i=2}^{M}(1-q\theta_i))nL+\sum_{i=2}^{M}[(1-\theta_i)q\theta_1(1-\lambda_i)nL]+\frac{1}{2}\varphi c\theta_1^2\end{aligned}$$

$$\begin{aligned}\overline{\omega_i}=&(1-\theta_i)q\theta_1(1+(n+\sum_{j=2,j\neq i}^{M}q\theta_j)\lambda_i)L\\&+\sum_{j=2,j\neq i}^{M}[(1-\theta_i)q\theta_j(\prod_{k=1,k\neq i}^{j-1}(1-q\theta_k))(1+(1+\sum_{m=j+1}^{M}q\theta_m)\lambda_i)L]\\&+(1-\theta_i)(\prod_{j=1,j\neq i}^{M}(1-q\theta_j))L+\sum_{j=1,j\neq i}^{M}[(1-\theta_j)q\theta_i(1-\lambda_j)L]+\frac{1}{2}c\theta_i^2\end{aligned}$$

Thus, under the simultaneous game and sequential game, M enterprises would exert socially optimal security effort when the liability levels are set properly. Let the above security quality equal to the socially optimal security quality under multiple enterprises case, we can obtain the liability levels.

Given that the theoretical analysis of multiple enterprises cases proves to be analytically challenging and intractable, we also employ numerical methods as an alternative approach to exploration. We use the following parameters for the numerical analysis: $L = 0.1$, $c = 0.9$, $\varphi = 0.8$, $n = 2$, $q = 0.1$, $M = 10$.

Figure 11(a) and 11(b) show that under multiple enterprises cases, the system vulnerability q always decreases the ratio of liability level of the high-type enterprise λ_1 while first increases then decreases that of the low-type enterprises λ_2. This also shows that Observation 1 still holds under multiple enterprises case.

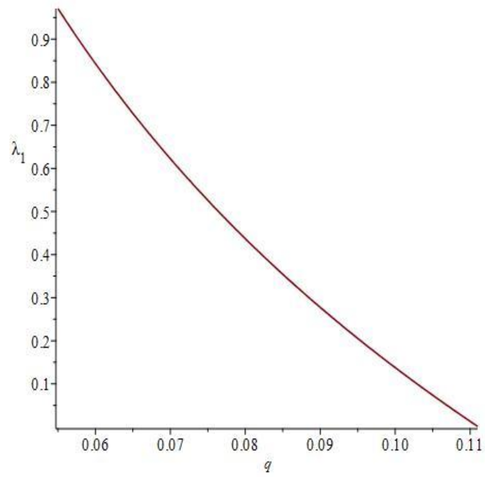


Fig.11(a)The liability level of the high-type enterprise decreases with q

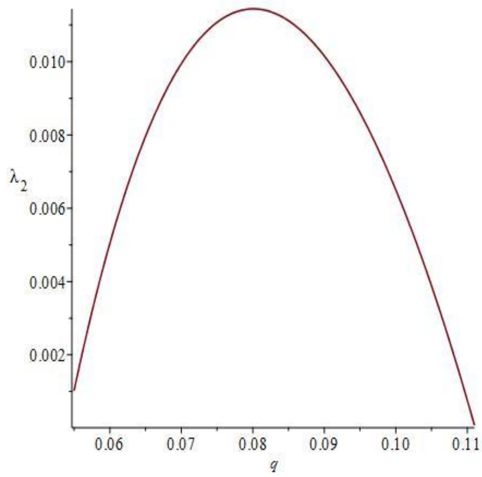
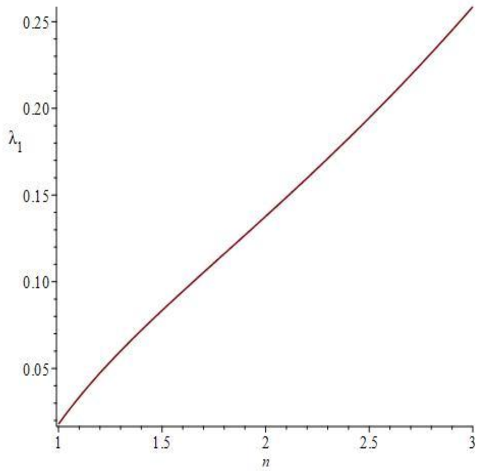


Fig.11(b) The liability level of the low-type enterprise first increases then decreases with q

Figure 12(a) and 12(b) show that under multiple enterprises cases, the ratio of security loss n always increases the ratio of liability level of the high-type enterprise λ_1 while decreases that of the low-type enterprises λ_i. This also shows that Observation 2 still holds multiple enterprises case.



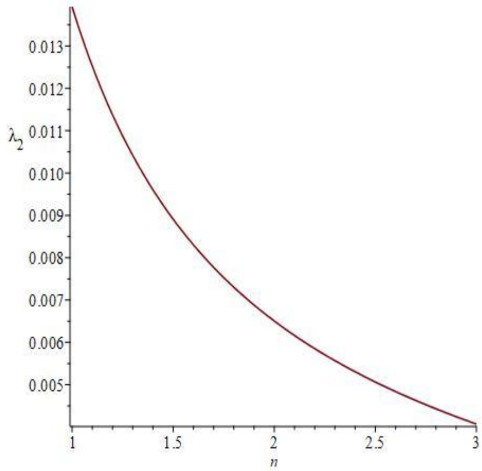
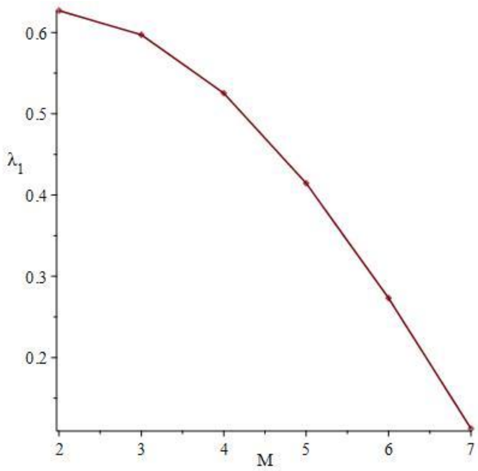


Fig.12(a)The liability level of the high-type enterprise increases with n

Fig.12(b) The liability level of the low-type enterprise decreases with n

On this basis, we next analyze how the number of enterprises M affects the ratio of liability level. We use the following parameters for the numerical analysis: $L = 0.1$, $c = 0.9$, $\varphi = 0.8$, $n = 2$, $q = 0.15$.

Observation 4: The ratio of liability level of the high-type enterprise (λ_1) and that of the low-type enterprises (λ_i) decrease with the number of enterprises (M).



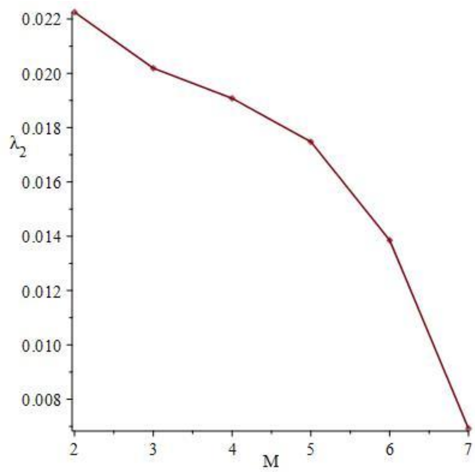


Fig.13(a)The liability level of the high-type enterprise decreases with M

Fig.13(b) The liability level of the low-type enterprise decreases with M

As the M th enterprise joins the resource-sharing platforms, other $M-1$ enterprises tend to decrease their security effort, as stated in Observation 3. Enterprises choose the liability-based mechanism in order to incentivize each other to achieve the socially optimal security effort to solve the free-riding problem. Thus, the ratio of liability level also decreases with the number of enterprises. These findings highlight the importance of adequate assessment of the number of enterprises in the resource-sharing platforms when implementing the liability-based mechanism.