Online Supplement for "Stockout Compensation: Joint Inventory and Price Optimization in Electronic Retailing"

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A Stockout Compensation Policies in Practice

We provide the following 5 examples from online retailing.

- VERGE Music offers a 5% discount on out of stock items:

  Because we will go out of stock of products from time to time, we will occasionally not be able to complete an order. If you choose the backorder option and your order is over $50 before postage we will give you a 5% discount. Backorders will take between 2-3 weeks for North American based labels up to 5-7 weeks for some labels overseas.

  http://www.vergemusic.com/new/howtoorder.cfm

- MAP LINK offers a 2% discount on out of stock items

  http://catalog.maplink.com/maplink/maplink/wholesale_terms.html

- OTT's Discount Art Supply waives shipping charges on backorders.

  If the total is more than $30, the company will keep the backorder active for 30 days in hopes the out of stock items arrive. These items will be shipped at the company's expense.

  http://www.otts.com/faq.htm

- 1bookstreet.com waives shipping charges on backorders.

  ... occasionally items become no longer available or go on backorder. If an item is on backorder, you will receive notification of the estimated date we expect to ship the backordered item ... it will be shipped to you via parcel post at no extra charge.

  http://www.1bookstreet.com/faq.asp

- IntelliHome offers a 10% discount on out of stock items:

  If the delivery time for the backorder exceeds 3 weeks, then we will give you a reduction of - 10% as a compensation for the late delivery.

  http://www.intellihome.be/pages/track.htm

IntelliHome
Intellihome
Shop Here
What's New
Software
Hot Deals
Manuals

DELIVERY TIME?

WITHIN THE EUROPEAN UNION : 1 BUSINESS DAY

CHECK FOR DETAILS UNDER POINT 1.

1. What about delivery time?

Orders received before 2.30 pm Central European Time, will be shipped the same day.

Shipment

- **Within European Union** : 1 business day by DHL express service.
- **Outside EU** : 2-5 business days by DHL Express (in some cases by EMS).

2. How to track your order?

- The day we ship your order, you will receive an e-mail notification with tracking number. With this number you can check the status of your shipment at **DHL.com**.

3. Our Backorder policy?

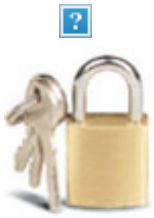
- We do NOT charge your card until we ship the goods. If something is part shipped, you are only charged for what is in stock.
- If there are backorders in your order that can be shipped within 3 working days, we may decide to keep your order on hold. You will of course be notified in advance.
- If the delivery time for the backorder exceeds 3 weeks, then we will give you a reduction of - 10% as a compensation for the late delivery.

4. Shipping costs?

Look here for your situation :

Country	Order > 149 €	Order < 149 €
Albania *	69,95 Euro	69,95 Euro

Country	Order >149 €	Order <149 €
Luxembourg	0 Euro	15,45 Euro



Wholesale Customer Center

Wholesale Discount Table

AVERAGE ORDER SIZE	CUSTOMER CLASS	DISCOUNT LEVEL (ITEM DISCOUNT CATEGORY)						
		1	2	3	4	5	6	7
> $200	L	30%	41%	44%	50%	56%	20%	0%
>$100, but < $200	I	30%	40%	42%	47%	53%	20%	0%
>$50, but < $100	N	30%	39%	41%	44%	50%	20%	0%
< $50	K	30%	39%	40%	42%	45%	20%	0%

Discount Adjustment (Explained)

B = B/O	2.00%
E = EDI	3.00%
D = Simplified Electronic Ordering	1.00%
C = Central Distribution	3.00%
G = Growth Incentive	1.00%
P = Prepaid Freight	-3.00%
T = Net 60	-1.00%
U = Net 90	-2.00%
S = Stock Balancing	-2.00%

Customer Class Explained
All existing customers are classified based on the previous 12-month history of orders with Map Link. This is reviewed and automatically adjusted annually. The customer class for new customers is based on the total retail value of the first order, and agreed-upon projections. Customer class is determined by average order size (dollar value). This is calculated based on a customer's total invoiced sales over a 12-month period divided by the number of orders in that period. This average order size separates customer class into the above four tiers.

Wholesale Shipping Services Map Link's preferred carrier is UPS. However, shipping via FedEx and FedEx Ground Service (formerly RPS), US Postal Service and other couriers is available upon request. We charge published rates for these services. We charge $1.00 per package for handling. International shipments (outside the U.S.) incur a $3.00 per order document preparation fee. Same day rush processing is available for reasonably sized orders placed before 2:00 p.m. Pacific Time. The rush fee for orders under $200.00 is $10.00. For orders in excess of $200.00 the rush fee is 5% of the total order of cost.

A $5.00 fee is charged for drop shipments from resellers to their customers.

Discount Level (Item Discount Category) Explained Wholesale Shipping Services
This allows us to pass along the widely varying discounts offered to us by publishers. Each item in our catalog is assigned a discount level from 1-7 which is listed in the catalog next to the retail price.

1 = Short discount items i.e., COM and other street atlases

2 = Slightly lower discount than industry norm i.e., TB, M, and MSC
3 = The bulk of our catalog i.e., DELO, SW, RM SF, RM EF, HAG, and MEA
4 = Somewhat higher discount i.e., RM, UMN, FB, NEL, ITM, PP, NG, and BB
5 = Domestic, folded paper maps i.e., COMP, GTR, G, HAG TV, and some UMN
6 = Academic Titles
7 = Fixtures are sold at cost and do NOT receive any discount.

Discount Adjustment Explained

Backorder - Those customers wishing us to maintain backorders will be given an additional 2% discount on all items ordered. We maintain backorders for 90 days from the date of order entry, after which they are automatically deleted. Backorders, once in stock, are shipped with subsequent orders unless they are requested sooner by the customer. Individual items may be removed from backorder providing that the number of cancellations is less than 10% of the total backorders in a given yearly review period. The backorder option may be canceled at any time and the discount will be discontinued.

EDI - We offer an additional 3% discount on all orders which meet the volume and technical specifications of our Information Systems Department.

SEO - Additionally, we offer a simplified electronic ordering system (S.E.O.) which consists of less elaborate programming but enables submission of orders computer to computer. This earns 1.0% discount. Both E.DI. and S.E.O. are offered contingent upon our respective computer departments ability to arrange the technicalities. Should these arrangements require an extraordinary degree of programming, there may be a hookup charge. If you select this option please fill out the discount adjustments box on the application for credit.

Central Distribution - For those customers with six or more retail locations, an additional 3.0% discount is available if we ship to a single distribution center. Orders for maps from individual store locations within the group will still be accepted but will not receive the distribution discount.

Growth Incentive - Those customers who can increase their purchases (at retail) from us during a calendar year by 10% will receive an additional 1.0% discount for the following calendar year. This incentive is determined by Map Link.

Prepaid Freight - For those North American customers wishing to receive their maps prepaid freight (free freight), a reduction of discount of 3.0% will be made on all items ordered for the yearly review period. Special shipping of rush orders will be billed as always at the full carrier rate plus the usual rush handling charge. Sorry, but we cannot extend this offer to customers outside of North America.

Net 60/90-Day Terms - We charge 1.0% for each 30 days over net 30. If you choose 60-day terms, this will decrease the discount we offer on all items by 1.0%. 90-day terms decreases discount by 2.0%. This offer is subject to approval by our credit department.

Stock Balancing - We offer returnability on vendor-returnable or resalable merchandise for a 2.0% reduction in discount. On those vendors offering Map Link full return we pass this along to our customers with the vendors conditions. For items where vendors offer no returns to us we will accept up to 10% of the items purchased during the one year review period. These items must be in saleable condition. Stock balancing does not affect returns due to Map Link error or product damaged in transit. For detailed guidelines as to vendor conditions and Map Link requirements please phone customer service. Also, all books are returnable as is the industry standard. Distributors, internet website accounts, and individual orders of case quantities and larger of single titles should call for terms.

Q: How are you able to ship so fast?

A: We are folks just like you. When we place order, we want them as soon as possible. While other companies have long "processing" times...we simply print your order, our warehouse pulls and packs it and we ship in many instances, the same day!

Q: How do you offer such LOW prices?

A: Simple, we operate off a smaller margin and we buy in bulk -- our huge warehouse can hold a lot of art supplies!

Q: I've ordered an item and it is not in stock, what happens?

A: If the TOTAL is more than $30, we will keep the backorder active for 30 days in hopes the out-of-stock items arrive. They will be shipped at our expense. If your back-order is less than $30, we will credit your account and send you notice. at that point, you can either keep a credit with us until your next order or request a refund check.

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FIND THINGS

Monthly Newsletters

New Acquisitions (w/Cart)

Backlist by Genre

Advanced Search

INFO

HOW TO ORDER

CONTACT US

SPOOL INFO

last updated:
01.2004

Ordering Information

VERGE · music distribution

87 FRANKLIN STREET, UXBRIDGE, ON.
L9P 1J5, CANADA
PHONE/FAX 905-852-9745
eMail: info@*****music.com (replace '*****' with 'verge'; we are trying to prevent spam-bots)

You may place your order to Verge by phone, fax, mail or eMail. You may pay by credit card, international money order, postal order or bank draft.

If you are paying by credit card you must get your number and exp date to us safely by phone, fax or regular mail. We do not recommend including your credit card information on eMail. Once we have your credit card on file you may order by any means you wish without quoting it. Remember to inform us of any change in your expiry date or credit card. Please include a phone number.

If you are going to pay by money order, postal order, or bank draft you may send your order by eMail or by phone or fax. We will eMail your total back to you. We require that you confirm your order after recieving your total and we will hold your order for payment and ship when it is received

BACKORDERS
We encourage you to select the backorder option on your order form thus allowing us to guarantee sales to our suppliers on reorders. The hardest thing for many small labels is selling their backlist as many distributors concentrate exclusively on new titles. Verge, on principle, endeavours to continue to make available great recordings regardless of when they were produced. Because we will go out of stock of products from time to time, we will occasionally not be able to complete an order. If you choose the backorder option and your order is over $50 before postage we will give you a 5% discount. Backorders will take between 2-3 weeks for North American based labels up to 5-7 weeks for some labels overseas. We will cancel backorders after **six** months. Unfortunately this discount cannot be combined with backlist sale discount, however if you backorder any item on the backlist and it is currently not in stock it is still to your advantage to backorder since

B Technical Details and Proofs

Proof. (Lemma 2) Let p^* be the optimal price to Eq. (8) (i.e., total profit per unit time of the Hybrid operation with the condition $hT_1 = bT_2$). Then,

$$\begin{aligned}\pi_1^* &= \max_p \left[(p-c)d(1-G(p)) - \sqrt{2\left(\frac{b}{h+b}A\right)hd(1-G(p))}\right] \\ &= (p^*-c)d(1-G(p^*)) - \sqrt{2\left(\frac{b}{h+b}A\right)hd(1-G(p^*))}. \end{aligned} \tag{12}$$

Suppose $(p_1^*, \hat{p}_2^*, T_1^*, T_2^*)$ is the optimal solution to the Hybrid operation (Eq. (1)). Then,

$$\begin{aligned}\pi &= \frac{T_1(p_1-c)d(1-G(p_1))}{T_1+T_2} + \frac{T_2(\hat{p}_2-c)d(1-G(\hat{p}_2))}{T_1+T_2} \\ &\quad - \left[\frac{A}{T_1+T_2} + \frac{hT_1^2 d(1-G(p_1))}{2(T_1+T_2)} + \frac{bT_2^2 d(1-G(\hat{p}_2))}{2(T_1+T_2)}\right] \\ &= \frac{T_1^*}{T_1^*+T_2^*}(p_1^*-c)d(1-G(p_1^*)) + \frac{T_2^*}{T_1^*+T_2^*}(\hat{p}_2^*-c)d(1-G(\hat{p}_2^*)) \\ &\quad - \frac{1}{T_1^*+T_2^*}\left[A + \frac{h(T_1^*)^2 d(1-G(p_1^*))}{2} + \frac{b(T_2^*)^2 d(1-G(\hat{p}_2^*))}{2}\right].\end{aligned}$$

We write A as $\frac{b}{h+b}A + \frac{h}{h+b}A$, then we can express the total profit of the Hybrid operation as

$$\begin{aligned}\pi &= \frac{T_1^*}{T_1^*+T_2^*}\left[(p_1^*-c)d(1-G(p_1^*)) - \left[\frac{\frac{b}{h+b}A}{T_1^*} + \frac{hT_1^* d(1-G(p_1^*))}{2}\right]\right] \\ &\quad + \frac{T_2^*}{T_1^*+T_2^*}\left[(\hat{p}_2^*-c)d(1-G(\hat{p}_2^*)) - \left[\frac{\frac{h}{h+b}A}{T_2^*} + \frac{bT_2^* d(1-G(\hat{p}_2^*))}{2}\right]\right] \\ &\le \frac{T_1^*}{T_1^*+T_2^*}\left[(p_1^*-c)d(1-G(p_1^*)) - \sqrt{2\left(\frac{b}{h+b}A\right)hd(1-G(p_1^*))}\right] \\ &\quad + \frac{T_2^*}{T_1^*+T_2^*}\left[(\hat{p}_2^*-c)d(1-G(\hat{p}_2^*)) - \sqrt{2\left(\frac{h}{h+b}A\right)bd(1-G(\hat{p}_2^*))}\right]. \end{aligned} \tag{13}$$

The last step is derived by using the EOQ model (i.e., the minimum inventory cost is $\sqrt{2Ahd(1-G(p))}$). Let Eq. (13) be $\bar{\pi}$.

Now we show that the optimal profit of Eq. (8) is always greater than that of Eq. (1). Toward this end,

$$\begin{aligned}\pi_1^* - \bar{\pi} &= \frac{T_1^*}{T_1^*+T_2^*}\left[(p^*-c)d(1-G(p^*)) - \sqrt{2\left(\frac{b}{h+b}A\right)hd(1-G(p^*))}\right. \\ &\quad \left. - \left[(p_1^*-c)d(1-G(p_1^*)) - \sqrt{2\left(\frac{b}{h+b}A\right)hd(1-G(p_1^*))}\right]\right]\end{aligned}$$

$$
\begin{aligned}
&+\frac{T_2^*}{T_1^*+T_2^*}\left[(p^*-c)d(1-G(p^*))-\sqrt{2\left(\frac{b}{h+b}A\right)hd(1-G(p^*))}\right.\\
&\left.-\left[(\hat{p}_2^*-c)d(1-G(\hat{p}_2^*))-\sqrt{2\left(\frac{h}{h+b}A\right)bd(1-G(\hat{p}_2^*))}\right]\right]\\
\geq\ & 0
\end{aligned}
$$

Therefore, the proof is completed. ■

Proof. (Lemma 3) Let p^* and $\bar{p}$ be the smallest and another feasible solution of Eq. (9) (i.e., $p^* \leq \bar{p}$). We prove Lemma 3 by showing that $\pi(p^*) - \pi(\bar{p}) \geq 0$. To this end, first, we rewrite Eq. (8) as

$$
\pi = \max_p \left[d(1-G(p)) \left(p - c - \sqrt{\frac{2\left(\frac{b}{h+b}A\right)h}{d(1-G(p))}} \right) \right].
$$

Next we write the inverse of Eq. (9) to get

$$
\frac{1-G(p)}{g(p)} = \left(p - c - \sqrt{\frac{\left(\frac{b}{h+b}A\right)h}{2d(1-G(p))}} \right). \tag{14}
$$

The LHS of this equation is the inverse of the hazard rate function. Therefore, the condition of Lemma 3 is that $\frac{\partial}{\partial x}\left(\frac{1-G(x)}{g(x)}\right) \leq 0$. This assumption implies that

$$
\left(p^* - c - \sqrt{\frac{\left(\frac{b}{h+b}A\right)h}{2d(1-G(p^*))}} \right) \geq \left(\bar{p} - c - \sqrt{\frac{\left(\frac{b}{h+b}A\right)h}{2d(1-G(\bar{p}))}} \right).
$$

Since $\sqrt{\frac{\left(\frac{b}{h+b}A\right)h}{2d(1-G(p))}}$ is non-decreasing on p, we subtract $\sqrt{\frac{\left(\frac{b}{h+b}A\right)h}{2d(1-G(p^*))}}$ from LHS (respectively, $\sqrt{\frac{\left(\frac{b}{h+b}A\right)h}{2d(1-G(\bar{p}))}}$ from RHS) to get

$$
\left(p^* - c - \sqrt{\frac{2\left(\frac{b}{h+b}A\right)h}{d(1-G(p^*))}} \right) \geq \left(\bar{p} - c - \sqrt{\frac{2\left(\frac{b}{h+b}A\right)h}{d(1-G(\bar{p}))}} \right).
$$

Combining the above inequality and $d(1-G(p^*)) \geq d(1-G(\bar{p}))$, one can easily see

$$
\begin{aligned}
\pi(p^*) - \pi(\bar{p}) \ &= \ d(1-G(p^*)) \left(p^* - c - \sqrt{\frac{2\left(\frac{b}{h+b}A\right)h}{d(1-G(p^*))}} \right) \\
&\quad -d(1-G(\bar{p})) \left(\bar{p} - c - \sqrt{\frac{2\left(\frac{b}{h+b}A\right)h}{d(1-G(\bar{p}))}} \right) \geq 0.
\end{aligned}
$$

This completes the proof. ∎

Proof. (Theorem 3) We wish to prove that the stockout compensation policy generates greater revenues and lower inventory costs than the wait-free and stockless operations. Let R^c (R^w) be the optimal revenue per unit time for the stockout compensation policy (respectively, the wait-free operation). Then $R^w = (p^w - c)d(1 - p^w)$ and $R^c = (p^c - c)d(1 - p^c)$, where p^c and p^w are the respective optimal prices. Writing $p^w - p^c = \epsilon$ (> 0 by Part 1 of Theorem 2), we see that $R^c - R^w = d\epsilon(2p^c - 1 - c + \epsilon) > 0$, hence the stockout compensation policy generates greater revenue per unit time.

To compute the effect on inventory costs, let I^c and I^w be the optimal inventory costs under the two policies. Then, $I^w = \sqrt{2Ahd(1-p^w)}$, $I^c = \sqrt{\frac{b}{h+b}}\sqrt{2Ahd(1-p^c)}$, and

$$I^w - I^c = \sqrt{2Ahd}\left(\sqrt{(1-p^w)} - \sqrt{\frac{b}{h+b}}\sqrt{(1-p^c)}\right). \tag{15}$$

Note that the optimal prices satisfy

$$\frac{1}{2}h(T_1^c)^2 d(1-p^c) = \left(\frac{b}{h+b}\right)A \tag{16}$$

$$\frac{1}{2}h(T^w)^2 d(1-p^w) = A \tag{17}$$

where T_1^c and and T^w are the optimal lengths of the in-stock period under stockout compensation and wait-free operation, respectively. From Eq. (16) and Eq. (17), we get $\sqrt{\frac{b}{h+b}} = \frac{T_1^c}{T^w}\frac{\sqrt{1-p^c}}{\sqrt{1-p^w}}$, and substituting this into Eq.(15), we get

$$I^w - I^c = \frac{\sqrt{2Ahd}}{T^w\sqrt{1-p^w}}\left(T^w(1-p^w) - T_1^c(1-p^c)\right).$$

Substituting $p^w - p^c = \epsilon$, and applying the first-order conditions for p^w, p^c, T^w and T_1^c (see Eq. (2)) to the case where G is uniform, and simplifying we derive

$$I^w - I^c = \frac{\sqrt{2Ahd}}{T^w\sqrt{1-p^w}}\left(\epsilon\frac{4}{h}\right)\left((1 - \frac{1+c}{2} - \frac{hT_1^c}{4} - \frac{hT^w}{4}\right).$$

We note that $p^* > \frac{2+c}{3}$ corresponds to negative profit. Hence, setting $p^* < \frac{2+c}{3}$ and employing first order conditions for the optimal prices, we have

$$\frac{hT_1^c}{4} + \frac{hT^w}{4} < 2\left(\frac{2+c}{3} - \frac{1+c}{2}\right) = \frac{1-c}{3}.$$

By substituting this result into the previous equation, we get

$$I^w - I^c > \frac{\sqrt{2Ahd}}{T^w\sqrt{1-p^w}}\left(\epsilon\frac{4}{h}\right)\left(\frac{1-c}{6}\right) > 0.$$

∎

C Extensions: Model Formulation and Proofs

Exact Waiting Time & Dynamic Pricing: We formulate the case where the firm offers a dynamic price $p_2(t)$ during the stockout period and announces the exact waiting time to each customer; the case of single fixed price can be derived by substituting $p_2(t) = p_2$. Under this policy, the retailer's profit is

$$\begin{aligned}\pi &= \frac{T_1(p_1-c)d(1-G(p_1))}{T_1+T_2} + \frac{d\int_0^{T_2}[(p_2(t)-c)(1-G(p_2(t)+b(T_2-t)))]dt}{T_1+T_2} \\ &\quad - \left[\frac{A}{T_1+T_2} + \frac{hT_1^2 d(1-G(p_1))}{2(T_1+T_2)}\right]. \end{aligned} \tag{18}$$

Solving for the optimal solutions, we set the first derivatives with respect to p_1 to zero, to get

$$\frac{g(p_1)}{1-G(p_1)} = \frac{1}{p_1 - c - \frac{hT_1}{2}}. \tag{19}$$

To derive the optimal price scheme during the stockout period, we take a derivative of $(p_2(t) - c)(1-G(p_2(t)+b(T_2-t)))$ with respect to $p_2(t)$. This is possible due to the fact that the price at t (i.e., $p_2(t)$) only affects the fraction of customers at time t, but does not affect the other period's demand. Setting the derivative to 0 and rearranging, we have

$$\frac{g(\hat{p}_2(t))}{1-G(\hat{p}_2(t))} = \frac{1}{\hat{p}_2(t) - b(T_2-t) - c} \tag{20}$$

where $\hat{p}_2(t) = p_2(t) + b(T_2 - t)$.

Setting first derivatives of the profit function with respect to T_1 and T_2 to zero, we get

$$\begin{aligned}&(p_1-c)d(1-G(p_1)) - hT_1 d(1-G(p_1)) \\ &= (\hat{p}_2(T_2)-c)d(1-G(\hat{p}_2(T_2))) - bd\int_0^{T_2}(1-G(\hat{p}_2(t)))dt. \end{aligned} \tag{21}$$

Therefore, the optimal pricing scheme under the dynamic pricing satisfies the above optimality conditions (i.e., Eqs. (19), (20), and (21)).

First, we use the above optimality conditions to show that under the average waiting time announcement assumption, the two-price policy is optimal. For this, we substitute $b(T_2-t)$ by $\frac{bT_2}{2}$ in (18)-(21). Then Eqs. (19), (20), and (21) reduce to (2), (3), and (6), respectively. This implies that $p_2^*(t) = p_2^* = p_1^* - \frac{bT_2}{2}$ should be the optimal price charged during the stockout period. Note that $p_2^*(t)$ is independent of t, implying that the two-price policy is optimal under the average waiting time announcement assumption.

Next, we consider the case where the firm offers a single price p_2 during stockout period but the customers are aware of the exact waiting time. By substituting p_2 for $p_2(t)$, replacing $G(p)$ by p (under uniform customer valuations), and integrating over t, we see that (assuming only one price during stockout) the retailer's expected profits, prices, and cycle lengths are identical under both average waiting time and exact waiting time announcement. In other words, let P_{ave} and P_{exact} denote the optimal (single) price under the average and exact waiting time announcement, respectively. When customer valuations are uniformly distributed, we see $P_{ave} = P_{exact}$.

Heterogeneous Waiting Costs: We consider the case where customers are heterogeneous in their waiting cost per unit time. We assume that the customers valuations are uniformly distributed (i.i.d.) between 0 and 1 (i.e., $U \sim U[0,1]$). Suppose the customers unit waiting costs are uniformly distributed (i.i.d.) between 0 and $\bar{b}$ (i.e., $B \sim U[0,\bar{b}]$).

The firm does not know the true waiting cost of any customer, so it plans its policy using the average waiting cost ($E[B] = \frac{\bar{b}}{2}$). The firm announces its prices and (during the stockout period) each customer makes her purchase decision using her actual waiting cost. It is interesting to note that stockout compensation policy is superior to the Wait-free strategy even in this case of heterogeneous waiting costs.

We prove the result under the assumption that the effective price of the highest waiting-sensitive customers is less than 1 (i.e., $p_2^* + \frac{\bar{b}T_2^*}{2} \leq 1$ where p_2^*, T_2^* are optimal solutions of the retailer's profit with average waiting cost). The proof of this assumption is available from the authors.

Using the customer's average unit waiting cost $E[B]$ $(= \frac{\bar{b}}{2})$, planned total demand during the stockout period is

$$d\left[1 - \left(p_2^* + \frac{E[B]T_2^*}{2}\right)\right] = d\left[1 - \left(p_2^* + \frac{\frac{\bar{b}}{2}T_2^*}{2}\right)\right].$$

Under the these optimal solutions, the expected actual demand during the stockout period is

$$d \cdot Pr\left(U - \frac{BT_2^*}{2} \geq p_2^*\right) = d\int_0^{\bar{b}} \left[1 - \left(p_2^* + \frac{bT_2^*}{2}\right)\right] \frac{1}{\bar{b}} db = d\left[1 - \left(p_2^* + \frac{\frac{\bar{b}}{2}T_2^*}{2}\right)\right].$$

This completes the proof.