

Online Supplement to “Solving the Capacitated Local-Access Network Design Problem”

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A Small Example to Illustrate the SOR Algorithm

We illustrate the algorithm on a small example with three source nodes and three cable types. The input data are given in Figure 1.

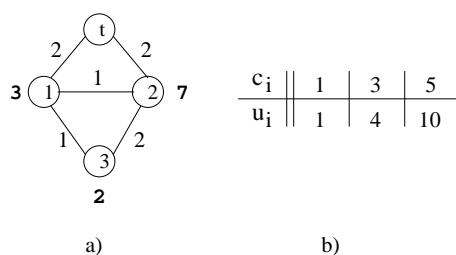


Figure 1: In part a), numbers on the edges are edge lengths. Demands are next to source nodes. In part b), cable costs per unit length and capacities are given for each of three cable types.

We first calculate the cable cost function $C(f)$ for $0 \leq f \leq 12$, which is shown in Figure 2a). Then, we approximate $C(f)$ by a convex piecewise linear function $h_0(f)$ as seen in Figure 2b).

At the root node of the branch-and-bound tree we use $h_0(f)$ to approximate cable costs on every edge. Function $h_0(f)$ has two pieces with slopes 0.5 and 1, and ranges 10 and 2. We split the flow variable f_e into variables $f_{e,1}$ and $f_{e,2}$ to represent these two linear pieces. We solve the following minimum-cost flow problem.

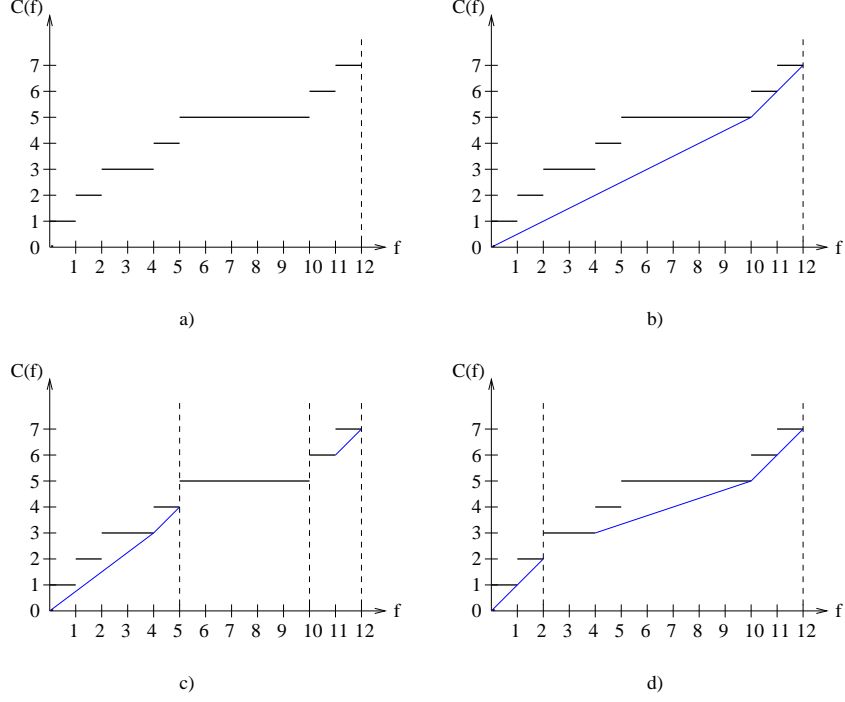


Figure 2: The cable cost function and its approximations in the algorithm.

$$\begin{aligned}
\min \quad & \sum_{e \in E} l_e (0.5 f_{e,1} + f_{e,2}) \\
s.t. \quad & f_{1t} + f_{12} + f_{13} - f_{t1} - f_{21} - f_{31} = 3 \\
& f_{2t} + f_{21} + f_{23} - f_{t2} - f_{12} - f_{32} = 7 \\
& f_{31} + f_{32} - f_{13} - f_{23} = 2 \\
& f_{t1} + f_{t2} - f_{1t} - f_{2t} = -12 \\
& f_{ij} + f_{ji} - f_{e,1} - f_{e,2} = 0 \quad \forall e = (i, j) \in E \\
& f_{e,1} \leq 10 \quad \forall e = (i, j) \in E \\
& f_{e,2} \leq 2 \quad \forall e = (i, j) \in E \\
& f_{e,1}, f_{e,2}, f_{ij}, f_{ji} \geq 0 \quad \forall e = (i, j) \in E
\end{aligned}$$

We obtain a lower bound 13 by sending the demand of source nodes 1 and 2 directly to t and the demand of 3 via node 1. That is, $f_{1t} = 5$, $f_{2t} = 7$, and $f_{31} = 2$. This flow solution has cost $C(2) + 2C(5) + 2C(7) = 20$ when cables are chosen optimally.

To determine which edge to branch on we use the first rule and for each edge e with positive flow, we calculate $(C(\bar{f}_e) - h_0(\bar{f}_e))l_e$.

$$\text{Edge } (1, t) : (4 - 2.5)2 = 3$$

$$\text{Edge } (2, t) : (5 - 3.5)2 = 3$$

$$\text{Edge } (1, 3) : (2 - 1)1 = 1$$

Both $(1, 2)$ and $(2, t)$ have the maximum deviation. We break ties by choosing the edge with more flow, i.e. $(2, t)$. The piece of $C(f)$ for $f_{2,t} = 7$ has flow range $(5, 10]$. Thus, we branch by dividing the flow range of $(2, t)$ as follows.

$$1) 0 \leq f_{2,t} + f_{t,2} \leq 5$$

- 2) $5 < f_{2,t} + f_{t,2} \leq 10$
- 3) $10 < f_{2,t} + f_{t,2} \leq 12$.

Using the cost approximation shown in Figure 2c), we solve the three new relaxations. The solutions are shown under nodes 1, 2 and 3 in Figure 3.

Next, we choose node 2 of the branch-and-bound tree and branch on flow range of edge $(1, t)$. Since $f_{1t} = 2$ corresponds to a breakpoint in $C(f)$, we branch into two:

- 1) $0 \leq f_{1,t} + f_{t,1} \leq 2$
- 2) $2 < f_{1,t} + f_{t,1} \leq 12$

We use the cost approximation for edge $(1, t)$ given in Figure 2d). The solutions for the two branches are given under nodes 4 and 5 in Figure 3. The algorithm proceeds in this fashion.

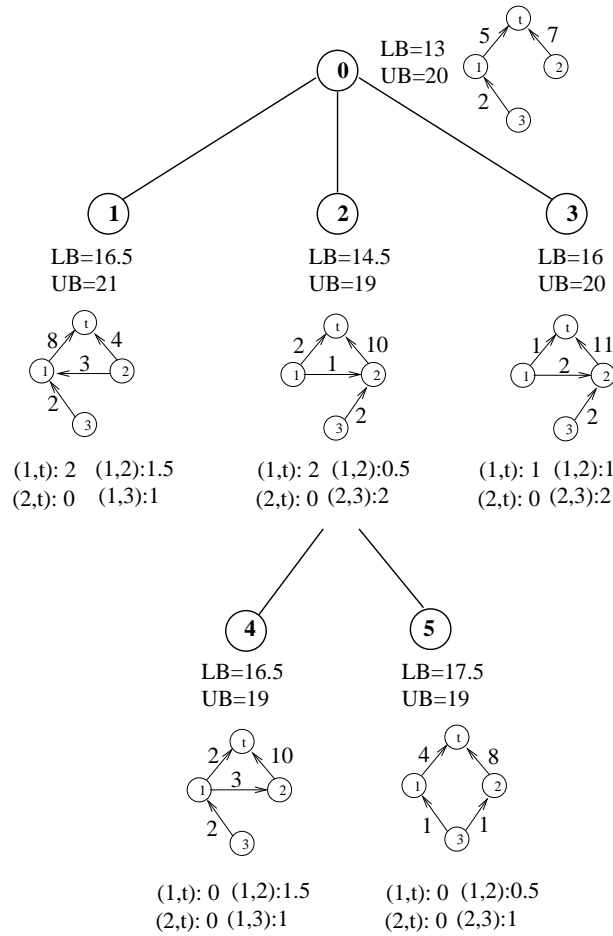


Figure 3: The partial branch-and-bound tree for the example. At each node of the tree, the flow values in the optimal relaxation solution are sketched and under the sketches the cost deviations $(C(\bar{f}_e) - h(\bar{f}_e))l_e$ are given for each edge e .