

# Online Appendix—FilMINT: An Outer-Approximation-Based Solver for Convex Mixed Integer Nonlinear Programs

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## A.1. Cutting Planes, Preprocessing, and Primal Heuristics

Figures A.1 and A.2 show the performance profiles encapsulating the results of the computational experiments described in Section 2.1. In the figures, the lines MILPcuts, preprocess, and primal-heuristic graph the relative performance of the algorithm with only that feature enabled. The performance of the vanilla version of the algorithm is also depicted.

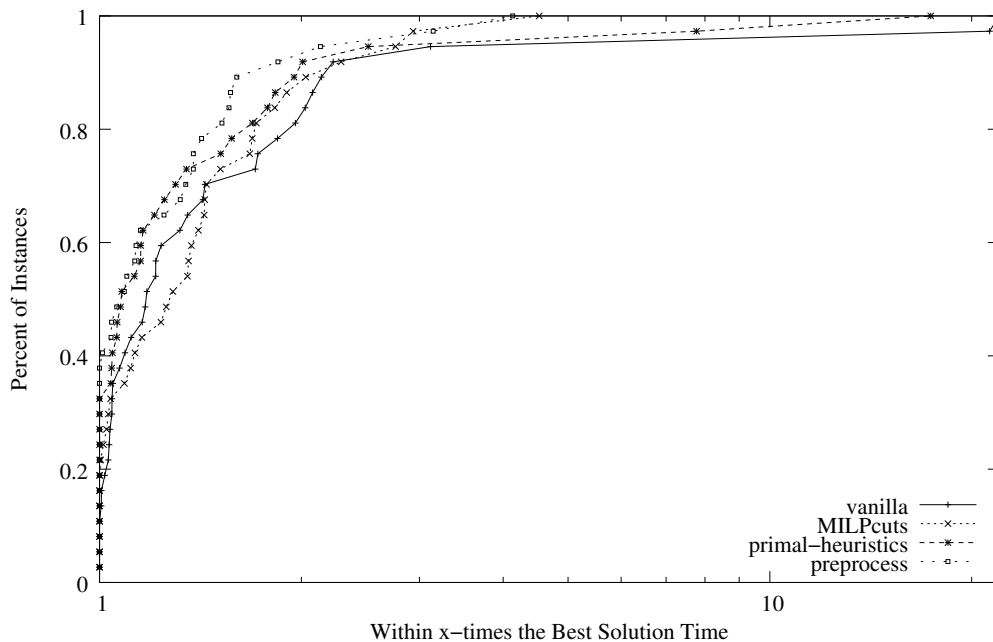


Figure A.1: Relative Performance of MILP techniques for moderate instances.

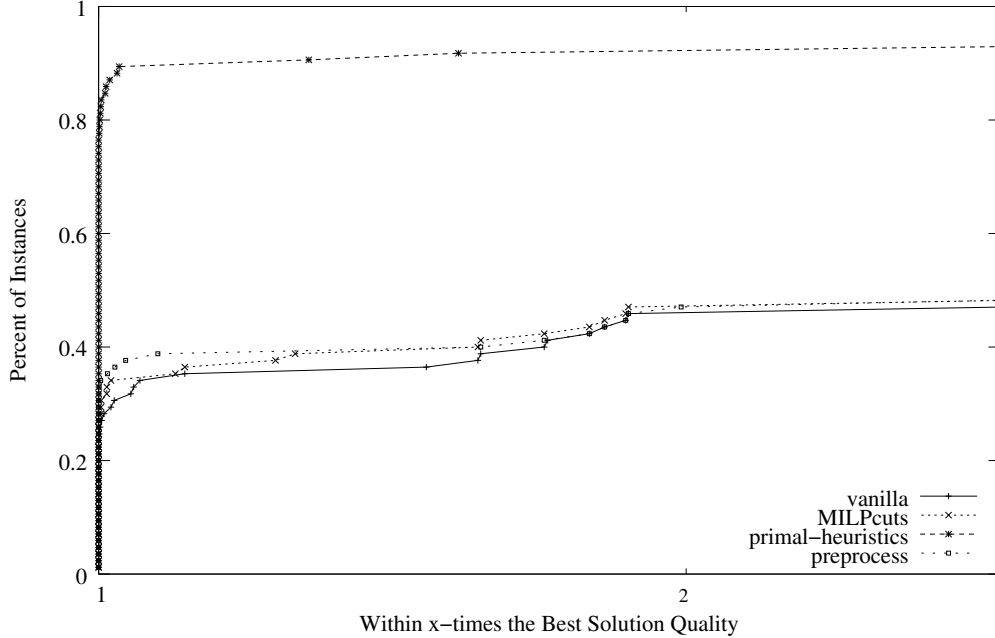


Figure A.2: Performance profile showing the effect of MILP cuts, preprocessing and heuristics for hard instances.

## A.2. Branching Rules

In Section 2.2, we described experiments aimed at determining the effect of advanced branching and node selection rules on solver performance. Performance profiles summarizing the results of the computational experiments for the different branching rules tested are shown in Figures A.3 and A.4. Profiles for the experiment on node selection rules are given in Figures A.5 and A.6.

Table A.1: Summary of Properties of Moderate Instances

Instance	Vars	Cont. Vars	Bin. Vars	Cons	NL. Cons	NL. Obj
m6	86	56	30	157	12	
BatchS101006M	278	149	129	1019	1	✓
BatchS121208M	406	203	203	1511	1	✓
BatchStorage10BM10	238	149	89	799	1	✓
BatchStorageBM10	238	149	89	799	1	✓
BatchStorageBM9964	205	125	80	647	1	✓
BatchStorageCH9964	527	447	80	1048	1	✓
CLay0304H	176	140	36	258	48	
CLay0304M	56	20	36	106	48	
CLay0305M	85	30	55	155	60	

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Instance	Vars	Cont. Vars	Bin. Vars	Cons	NL. Cons	NL. Obj
FLay04H	234	210	24	282	4	
FLay05M	62	22	40	65	5	
RSyn0805M02H	700	552	148	1045	6	
RSyn0805M04H	1400	1104	296	2438	12	
RSyn0810M02H	790	622	168	1188	12	
RSyn0810M04H	1580	1244	336	2784	24	
RSyn0810M	185	111	74	312	6	
RSyn0815M02H	898	710	188	1361	22	
RSyn0815M	205	126	79	347	11	
RSyn0820M02H	978	770	208	1500	28	
RSyn0830M02H	1172	924	248	1794	40	
RSyn0840M02H	1360	1072	288	2106	56	
SafetyLayout4Conv	182	150	32	228	32	
SafetyLayoutBM52	110	60	50	125	40	
SafetynorotationBM	168	108	60	162	6	✓
SLay06H	342	282	60	435	0	✓
SLay06M	102	42	60	135	0	✓
Syn10M03M	165	105	60	342	18	
Syn10M04M	220	140	80	516	24	
Syn15M02M	170	110	60	313	22	
Syn15M03M	255	165	90	537	33	
Syn20M04H	764	604	160	1452	56	
Syn30M03H	864	684	180	1485	60	
Syn30M04H	1152	912	240	2160	80	
Syn30M	100	70	30	167	20	
Syn40M02H	764	604	160	1212	56	
Syn40M03H	1146	906	240	1998	84	

Table A.2: Summary of Properties of Difficult Instances

Instance	Vars	Cont. Vars	Bin. Vars	Cons	NL. Cons	NL. Obj
csched2	400	92	308	137	0	✓
fo7	114	72	42	211	14	
fo72	114	72	42	211	14	
fo8	146	90	56	273	16	
fo9	182	110	72	343	18	
m7	114	72	42	211	14	
o7	114	72	42	211	14	
o72	114	72	42	211	14	

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Instance	Vars	Cont. Vars	Bin. Vars	Cons	NL. Cons	NL. Obj
tls12	812	144	656	384	12	
tls4	105	16	85	64	4	
tls5	161	25	131	90	5	
tls6	215	36	173	120	6	
tls7	345	49	289	154	7	
stockcycle	480	48	432	97	0	✓
trimloss12	812	156	656	384	12	
trimloss4	105	20	85	64	4	
trimloss5	161	30	131	90	5	
trimloss6	215	42	173	120	6	
trimloss7	345	56	289	154	7	
BatchS151208M	445	242	203	1781	1	✓
BatchS201210M	558	307	251	2327	1	✓
CLay0205H	260	210	50	365	40	
CLay0205M	80	30	50	135	40	
CLay0305H	275	220	55	395	60	
FLay05H	382	342	40	465	5	
FLay06H	566	506	60	693	6	
FLay06M	86	26	60	93	6	
RSyn0805H	720	424	296	1886	12	
RSyn0805M02M	360	212	148	769	6	
RSyn0805M03H	1050	828	222	1698	9	
RSyn0805M03M	540	318	222	1284	9	
RSyn0805M04M	720	424	296	1886	12	
RSyn0810H	820	484	336	2140	24	
RSyn0810M02M	410	242	168	866	12	
RSyn0810M03H	1185	933	252	1935	18	
RSyn0810M03M	615	363	252	1452	18	
RSyn0810M04M	820	484	336	2140	24	
RSyn0815H	940	564	376	2430	44	
RSyn0815M02M	470	282	188	981	22	
RSyn0815M03H	1347	1065	282	2217	33	
RSyn0815M03M	705	423	282	1647	33	
RSyn0815M04H	1796	1420	376	3190	44	
RSyn0815M04M	940	564	376	2430	44	
RSyn0820H	1020	604	416	2676	56	
RSyn0820M02M	510	302	208	1074	28	
RSyn0820M03H	1467	1155	312	2448	42	
RSyn0820M03M	765	453	312	1809	42	
RSyn0820M04H	1956	1540	416	3528	56	
RSyn0820M04M	1020	604	416	2676	56	

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Instance	Vars	Cont. Vars	Bin. Vars	Cons	NL. Cons	NL. Obj
RSyn0820M	215	131	84	371	14	
RSyn0830H	1240	744	496	3192	80	
RSyn0830M02M	620	372	248	1272	40	
RSyn0830M03H	1758	1386	372	2934	60	
RSyn0830M03M	930	558	372	2151	60	
RSyn0830M04H	2344	1848	496	4236	80	
RSyn0830M04M	1240	744	496	3192	80	
RSyn0830M	250	156	94	425	20	
RSyn0840H	1440	864	576	3728	112	
RSyn0840M02M	720	432	288	1480	56	
RSyn0840M03H	2040	1608	432	3447	84	
RSyn0840M03M	1080	648	432	2508	84	
RSyn0840M04H	2720	2144	576	4980	112	
RSyn0840M04M	1440	864	576	3728	112	
RSyn0840M	280	176	104	484	28	
Safety3	259	161	98	294	0	✓
safetyrotationCH	408	348	60	462	6	✓
SLay07H	476	392	84	609	0	✓
SLay07M	140	56	84	189	0	✓
SLay08H	632	520	112	812	0	✓
SLay08M	184	72	112	252	0	✓
SLay09H	810	666	144	1044	0	✓
SLay09M	234	90	144	324	0	✓
SLay10H	1010	830	180	1305	0	✓
SLay10M	290	110	180	405	0	✓
Syn15M04M	340	220	120	806	44	
Syn20M02M	210	130	80	406	28	
Syn20M03M	315	195	120	699	42	
Syn20M04M	420	260	160	1052	56	
Syn30M02M	320	200	120	604	40	
Syn30M03M	480	300	180	1041	60	
Syn30M04M	640	400	240	1568	80	
Syn40M02M	420	260	160	812	56	
Syn40M03M	630	390	240	1398	84	
Syn40M04M	840	520	320	2104	112	
Syn40M	130	90	40	226	28	

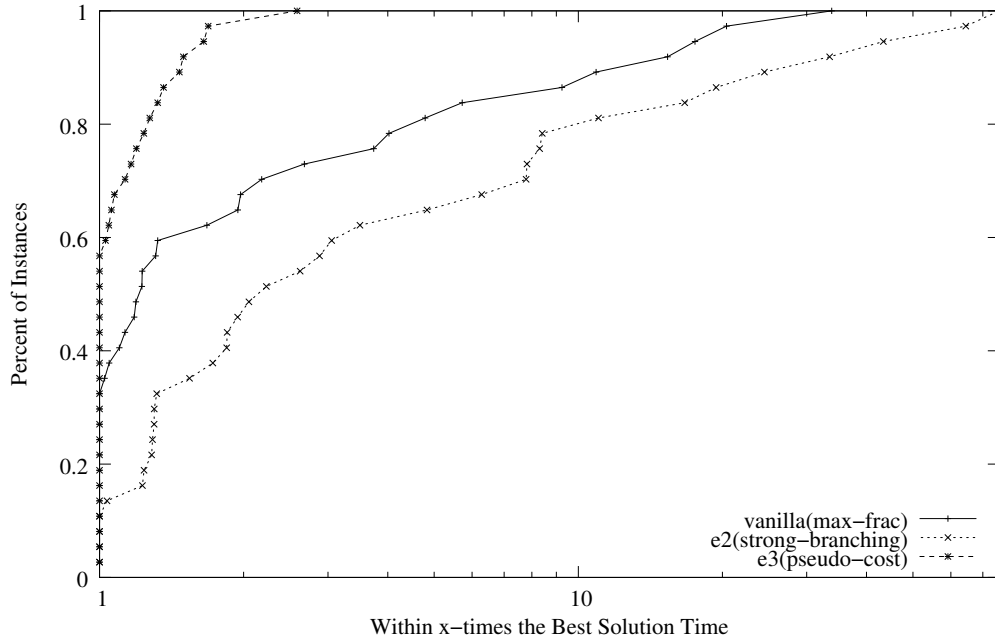


Figure A.3: Relative Performance of Branching Rules for Moderate Instances.

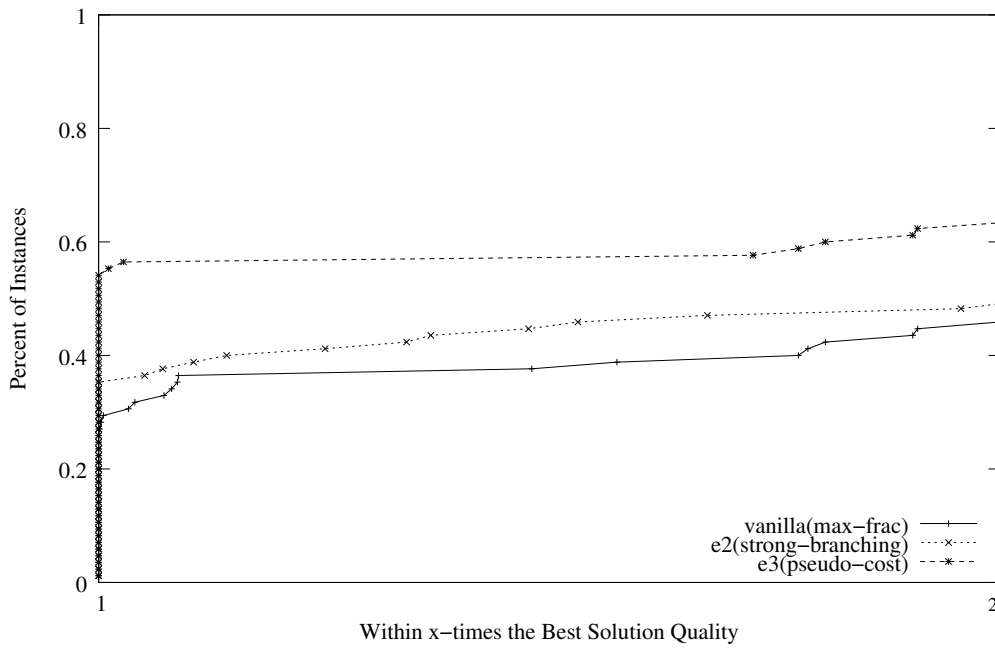


Figure A.4: Relative Performance of Branching Rules for Difficult Instances.

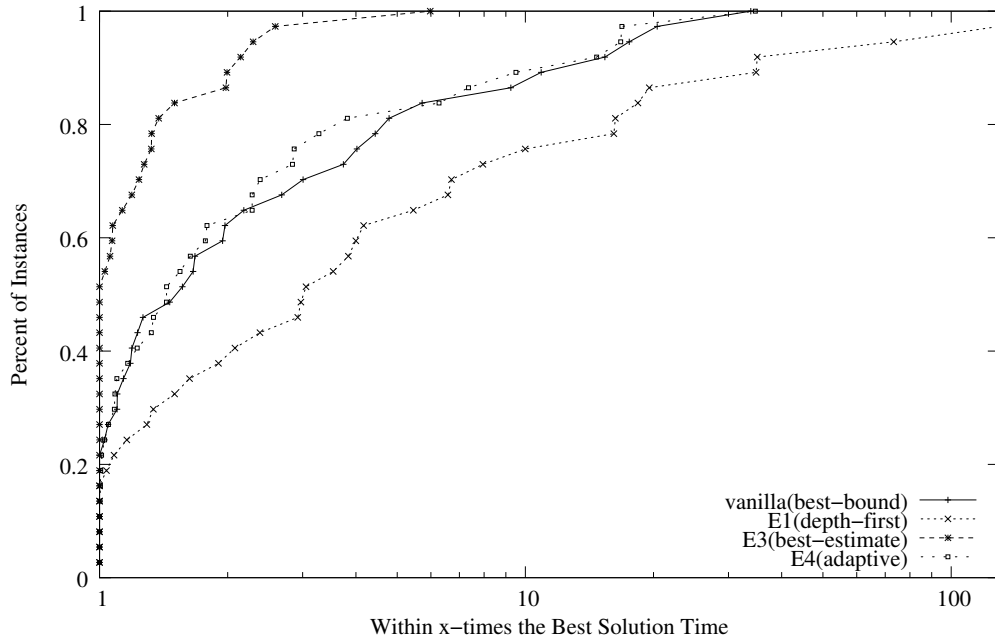


Figure A.5: Relative Performance of Node Selection Rules for Moderate Instances.

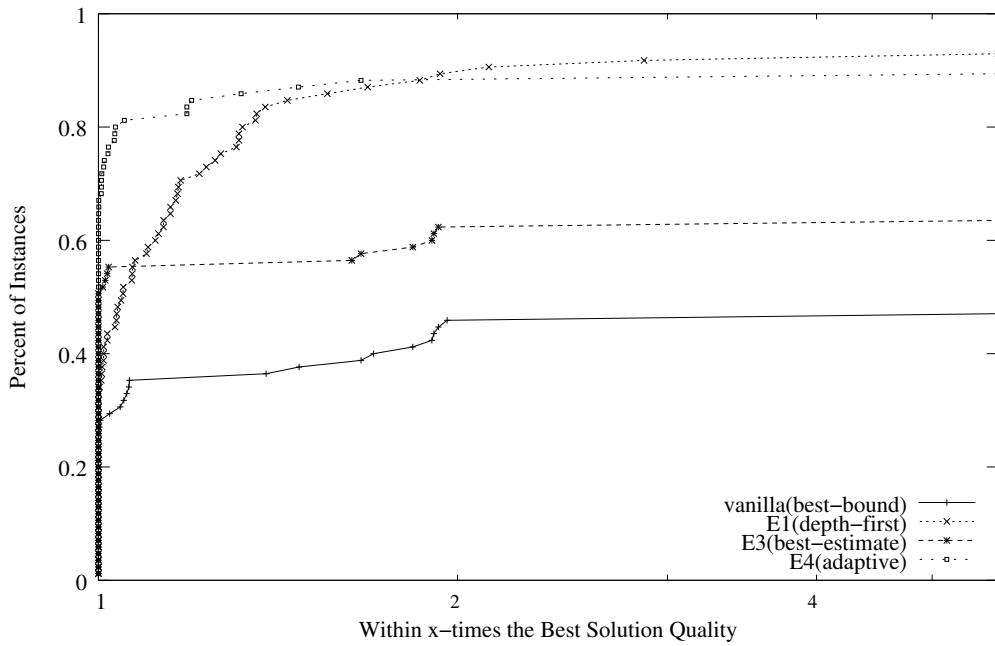


Figure A.6: Relative Performance of Node Selection Rules for Difficult Instances.

Table A.3: CPU Time Required for Moderate Instances

Instance	MINLP-BB	vanilla	FilMINT
m6	394.3	640.5	36.6
BatchS101006M	163.6	3209.6	15.2
BatchS121208M	1436.5	5785.4	36.7
BatchStorage10BM10	1316.5	7125.4	540
BatchStorageBM10	1730.4	6894.1	887.2
BatchStorageBM9964	60.7	333.4	46
BatchStorageCH9964	474	458.8	78.7
CLay0304H	394.3	692.9	30.2
CLay0304M	256.7	540.2	20.6
CLay0305M	298.1	927.9	43.8
FLay04H	77.1	45.8	13.7
FLay05M	324.4	10084.3	823.7
RSyn0805M02H	1398.5	16.3	3.1
RSyn0805M04H	798.0	12.7	7.7
RSyn0810M02H	843.6	12.2	3.9
RSyn0810M04H	1248.0	33.9	15.1
RSyn0810M	448.5	402.7	13.9
RSyn0815M02H	541.1	10.5	3.1
RSyn0815M	402.7	143.9	16
RSyn0820M02H	2446.4	38	4.6
RSyn0830M02H	2000.0	20.2	5.6
RSyn0840M02H	1122.2	18.5	10.7
SafetyLayout4Conv	276.2	46.3	5.8
SafetyLayoutBM52	921	8	1.7
SafetynorotationBM	83.7	183.5	36.4
SLay06H	1962.8	2077.4	10.2
SLay06M	84.3	39.4	1.0
Syn10M03M	261.8	21.3	3.2
Syn10M04M	2089.7	59.5	7.1
Syn15M02M	394.1	20.4	1.2
Syn15M03M	2148.3	73.7	4.2
Syn20M04H	62.0	6.1	3.3
Syn30M03H	130.2	10.4	3.7
Syn30M04H	360.9	23.8	9.0
Syn30M	226.4	563.6	6.3
Syn40M02H	125.8	4.5	2.1
Syn40M03H	615.5	32.9	9.5

In Table A.4, a \* next to the solution value implies that the solver proved the optimality of the solution within the time limit.

Table A.4: Value of Best Solution Found and CPU Time for Difficult Instances

Instance	MINLP-BB		Vanilla		FilMINT	
	Value	Time	Value	Time	Value	Time
csched2	-166102.0*	370	-158733.8		-164027.6	
fo7	45.6		$\infty$		20.7*	2237
fo72	40.6		$\infty$		17.7*	21
fo8	36.1		$\infty$		22.4*	2866
fo9	$\infty$		$\infty$		23.5*	8175
m7	220.5		$\infty$		106.8*	173
o7	164		$\infty$		131.7*	4145
o72	132.7		$\infty$		116.9*	5542
tls12	$\infty$		$\infty$		$\infty$	
tls4	$\infty$		$\infty$		10.3	
tls5	$\infty$		$\infty$		12	
tls6	$\infty$		$\infty$		$\infty$	
tls7	$\infty$		$\infty$		$\infty$	
stockcycle	119948.7*	322	206468.2		159828.7	
trimloss12	$\infty$		$\infty$		$\infty$	
trimloss4	11.3		$\infty$		8.3*	903
trimloss5	12.6		$\infty$		11.2	
trimloss6	$\infty$		$\infty$		18.9	
trimloss7	$\infty$		$\infty$		$\infty$	
BatchS151208M	1543472.4		1543472.4		1543472.4	
BatchS201210M	2295348.8*	9041	$\infty$		2295348.8*	223
CLay0205H	8092.5*	3994	8092.5*	1448	8092.5*	70
CLay0205M	25611.3		8092.5*	397	8092.5*	19
CLay0305H	8092.5*	3948	8497.5		8092.5*	129
FLay05H	64.5*	14380	64.5		64.5*	2031
FLay06H	66.9		66.9		66.9	
FLay06M	66.9*	167	66.9		66.9*	11703
RSyn0805H	-6573.3		$\infty$		-7174.2*	706
RSyn0805M02M	-2168		$\infty$		-2238.4*	127
RSyn0805M03H	-3022.1		-3068.9*	23	-3068.9*	6
RSyn0805M03M	-2607.9		$\infty$		-3068.9*	486
RSyn0805M04M	-6573.3		$\infty$		-7174.2*	707
RSyn0810H	$\infty$		$\infty$		-6581.9*	8295
RSyn0810M02M	-1129.3		$\infty$		-1741.4*	2094
RSyn0810M03H	-2722.4*	4107	-2722.4*	52	-2722.4*	9

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	MINLP-BB		Vanilla		FilMINT	
Instance	Value	Time	Value	Time	Value	Time
RSyn0810M03M	-2196.2		$\infty$		-2722.5*	7142
RSyn0810M04M	$\infty$		$\infty$		-6581.9*	8201
RSyn0815H	$\infty$		$\infty$		-3353.4	
RSyn0815M02M	-493.7		$\infty$		-1774.4*	4632
RSyn0815M03H	-2807.9		-2827.9*	112	-2827.9*	11
RSyn0815M03M	-2284.7		$\infty$		-2827.6	
RSyn0815M04H	-3375.9		-3398.7		-3405.8	
RSyn0815M04M	$\infty$		$\infty$		-3353.4	
RSyn0820H	$\infty$		$\infty$		-2411.5	
RSyn0820M02M	-154.7		$\infty$		-1026.9	
RSyn0820M03H	-2028.8*	11288	-2028.8*	824	-2022.6	
RSyn0820M03M	-1014.6		$\infty$		-1988.9	
RSyn0820M04H	$\infty$		-2450.8		99.4	
RSyn0820M04M	$\infty$		$\infty$		-2433.8	
RSyn0820M	-1022		-1150.3		-1146.3	
RSyn0830H	$\infty$		$\infty$		-2385.9	
RSyn0830M02M	-121.3		$\infty$		-716.5	
RSyn0830M03H	-1543.1		-1543.1		229.3	
RSyn0830M03M	-1249.3		$\infty$		-1469.3	
RSyn0830M04H	$\infty$		-2527.3		102.9	
RSyn0830M04M	$\infty$		$\infty$		-2385.9	
RSyn0830M	-402.4		$\infty$		-506.7	
RSyn0840H	$\infty$		$\infty$		-2448.7	
RSyn0840M02M	-58.7		$\infty$		-696	
RSyn0840M03H	-2742.6		-2742.6		-1031.9	
RSyn0840M03M	$\infty$		$\infty$		-2674.1	
RSyn0840M04H	$\infty$		-2564.5		143.3	
RSyn0840M04M	$\infty$		$\infty$		-2448.7	
RSyn0840M	-52.9		$\infty$		-289.1	
Safety3	195782.3		38435		27803.0*	5515
safetyrotationCH	28380.7*	6600	28380.7*	5018	28380.7*	736
SLay07H	64748.8*	5545	64748.8		64748.8*	60
SLay07M	77483.4		64748.8*	338	64748.8*	2
SLay08H	86744.6		90110.7		84960.2*	553
SLay08M	312377.7		85115		84960.2*	9
SLay09H	135952.1		116955.3		107805.8*	2994
SLay09M	438359.2		114444.9		107805.8*	481
SLay10H	194758.1		$\infty$		138730.4	
SLay10M	682209.3		$\infty$		129579.9	
Syn15M04M	-4909.5		-4937.5*	503	-4937.5*	4

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	<b>MINLP-BB</b>		<b>Vanilla</b>		<b>FilMINT</b>	
Instance	Value	Time	Value	Time	Value	Time
Syn20M02M	-1703		-1752.1*	1239	-1752.1*	27
Syn20M03M	-2532.9		-1394.6		-2647.0*	335
Syn20M04M	-1695.1		-1056.3		-3532.7*	2326
Syn30M02M	-352.4		-13.1		-399.7*	1486
Syn30M03M	-610.1		-17.1		-648.7	
Syn30M04M	-798.2		-31.1		-852.2	
Syn40M02M	-321.7		-13.1		-374.5	
Syn40M03M	-270.3		-17.1		-358.9	
Syn40M04M	-837.5		-31.1		-795.7	
Syn40M	-39.7		-63.9		-67.7*	85