Online Supplement for "Branch-and-Price for Large-Scale Capacitated Hub Location Problems with Single Assignment "

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GRASP heuristic for obtaining the initial set of columns.

The GRASP metaheuristic was originally proposed in Feo and Resende (1995). It is an iterative procedure that consists of two phases: the first phase constructs a feasible solution that is improved by a local search phase in the second phase.

Each of the two phases that take place at any given iteration of the GRASP heuristic focuses on one of the decision levels of the CHLPSA. The constructive phase concentrates on obtaining a feasible subset of hub nodes and an initial allocation pattern of the non-hub nodes within the obtained subset of hub nodes. The local search phase is mainly devoted to the assignment subproblem, although the initial subset of hub nodes may change at the end of this phase.

In what follows, solutions are represented by pairs of the form $h = (M, a)$ where $M \subseteq N$ denotes the set of selected sites to locate the hubs and $a : N \to M$ is the assignment mapping, i.e. $a(i) = k$ if non-hub node $i \in N$ is assigned to hub node $k \in N$. For any feasible assignment, h_k denotes the available capacity of hub k (i.e. $h_k = b_k - \sum_{i:a(i)=k} d_i$).

Construction phase

Let $s = (M, a)$ be a partial solution where some customers are not yet allocated, and let NA denote the set of non allocated nodes. Partial assignments are always feasible with respect to the capacity constraints and non assigned nodes are ordered by increasing values of O_i. For each candidate hub node $k \in NA$ we compute the greedy function

$$\delta(k) = \frac{f_k + \sum_{j \in Nodes_k} (O_j + D_j) c_{jk}}{|Nodes_k|}$$

where $Nodes_k$ is the set of unassigned nodes that fit in hub k, assuming that nodes in $Nodes_k$ are assigned to hub k according to the ordering described above. Note that, for all $k \in NA$,

$\hat{\imath} \in Nodes_k$ where $\hat{\imath} = \arg\max \{O_i : i \in NA\}$. That is, at each step of the Construction Phase the unassigned node with the biggest outgoing flow, $\hat{\imath}$, is assigned to the selected candidate hub node k. Several other nodes may also be assigned to hub node k. In order to achieve a tradeoff between quality and diversity, a partially randomized greedy procedure is considered. The choice of the next node to become a hub node is determined by randomly selecting one element from a Restricted Candidate List (RCL). The RCL is updated at each iteration of the construction phase and contains the best candidates with respect to a threshold value. This threshold value is computed in the following way. Let $\delta(k)_{min} = min\{\delta(k) : k \in NA\}$ and $\delta(k)_{max} = max\{\delta(k) : k \in NA\}$. Then, $RCL = \{k : k \leq \delta(k)_{min} + \alpha(\delta(k)_{max} - \delta(k)_{min})\}$, where $0 \leq \alpha \leq 1$. The construction phase terminates when all nodes are assigned to an open hub node.

The randomized greedy algorithm is depicted in Algorithm 6.

Algorithm 6 Construction phase of GRASP

Let M be the subset of selected hub nodes and NA the set of nodes that have not been assigned
$M \leftarrow \varnothing$
$NA \leftarrow N$
while $(NA \neq \varnothing)$ **do**
  $\forall k \in NA$
  • Define $Nodes_k$
  • Evaluate $\delta(k)$
  $RCL \leftarrow \{k : k \leq \delta(k)_{min} + \alpha(\delta(k)_{max} - \delta(k)_{min})\}$
  Select randomly $k^* \in RCL$
  $M \leftarrow M \cup k^*$
  $NA \leftarrow NA \setminus Nodes_k$
end while

Local Search Phase

The Local Search Phase procedure explores two types of neighborhood structures. The neighborhood structures of the first type are related to the assignment of nodes within the set of selected hubs, whereas the neighborhoods of the second type are related to solutions where the set of open hubs changes. In all cases, we consider solutions within the feasible domain.

The *shift* neighborhood considers all solutions that can be reached from the current one by changing the assignment of exactly one node, whereas the *swap* neighborhood contains all solution that differ from the current one in the assignment of two nodes. Let $s = (S, a)$ be the current solution, then

$$N_{shift}(s) = \left\{s' = (M, a') : \exists! i \in N, a'(i) \neq a(i)\right\}$$

and

$$N_{swap}(s) = \{s' = (M, a') : \exists i_1, i_2, a'(i_1) = a(i_2), a'(i_2) = a(i_1), a'(i) = a(i), \forall i \neq i_1, i_2\}$$

For exploring N_{shift} we consider all pairs of the form (i, j) where $a(j) \neq i$ and $h_i \geq d_j$. Also, for exploring N_{swap} we consider all pairs of the form (i_1, i_2) where $a(i_1) \neq a(i_2), h_{a(i_1)} + d_{i_1} \geq d_{i_2}$ and $h_{a(i_2)} + d_{i_2} \geq d_{i_1}$. In both cases we perform the first improving move.

We explore two additional neighborhood structures of the second type. They affect the current set of open hubs. The first one considers a subset of feasible solutions that are obtained from the current one by opening a new hub and by assigning some clients to it. Then,

$$N_{open}(s) \subset \{s' = (M', a') : M' = M \cup \{k\}; \forall j, a'(j) = r \in M' s.t. \sum_{j:a'(j)=r} O_j \leq b_r, \forall r \in M'\}$$

To explore $N_{open}(s)$ all nodes $k \in N \setminus M$ are considered. Again, let $s = (M, a)$ denote the current solution, $a'(k)$ the new assignment and $\widehat{h}_r$ the available capacity of hub r. Initially, $a'(p) = a(p)$ for all $p \in N$ and $\widehat{h}_r = h_r$ for all $r \in M$. For each potential hub node (i.e. $k \in N \setminus M$), we consider nodes by decreasing order of their O_i values. Node j is reassigned to hub k if $c_{jk} \leq c_{a(j),j}$ and $\widehat{h}_k \geq O_j$. If node j is reassigned to hub k we update its assignment and the available capacity of hubs k and $a(j)$.

The last neighborhood structure of the second type, considers a subset of feasible solutions that are obtained from the current one by closing a hub and reassigning its assigned nodes to other open hubs. Then,

$$N_{close}(s) \subset \{s' = (M', a') : M' = M \setminus k; \forall j, a'(j) = r \in M' s.t. \sum_{j:a'(j)=r} O_j \leq b_r, \forall r \in M'\}$$

To explore $N_{close}(s)$ all hub nodes $k \in M$ are considered. Again, let $s = (M, a)$ denote the current solution, $a'(k)$ the new assignment and $\widehat{h}_r$ the available capacity of hub r. Initially, $a'(p) = a(p)$ for all $p \in N$ and $\widehat{h}_r = h_r$ for all $r \in M$. For each hub node (i.e. $k \in M$), we consider its assigned nodes by decreasing order of their O_i values. Node j is reassigned to hub $\widehat{m}$, where $c_{j\widehat{m}} = min\left\{c_{jm} : \widehat{h}_m - O_j \geq 0, m \in M \setminus k\right\}$. If node j is reassigned to hub $\widehat{m}$ we update its assignment and the available capacity of hubs $\widehat{m}$ and $a(j)$.

The local search procedure is depicted in Algorithm 7.

Algorithm 7 Local Search phase of GRASP

$StopCriterion \leftarrow$ **false**
while ($StopCriterion =$ **false**) **do**
  Explore N_{shift}
  if (solution has not been updated in N_{shift}) **then**
    Explore N_{swap}
  end if
  if (solution has not been updated in N_{shift} and N_{swap}) **then**
    Explore N_{open}
  end if
  if (solution has not been updated in N_{shift}, N_{swap} and N_{open}) **then**
    Explore N_{close}
  end if
  if (solution has not been updated) **then**
    $StopCriterion \leftarrow$ **true**
  end if
end while

Finally, for the initial set $S = \{S_{ij}\}_{i,j \in N}$ in Algorithm 3.4 we define $S_{ij} = \bigcup_{i=1}^{k} M_k$ for all $i, j \in N$, where $h_1 = (M_1, a_1), \ldots, h_k = (M_k, a_k)$ denote the k best solutions found by the GRASP heuristic associated with different sets of open hubs.

References

Feo, T.A., M.G.C. Resende. 1995. Greedy randomized adaptive search procedures. *Journal of Global Optimization* **2** 1–27.