Online Supplement to: "Constraint-based Local Search for Inventory Control under Stochastic Demand and Lead time"

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The work "Constraint-based Local Search for Inventory Control under Stochastic Demand and Lead time" addresses the general multi-period production/inventory problem with non-stationary stochastic demand and supplier lead time under service-level constraints. Two hybrid algorithms that blend Constraint Programming and Local Search for computing near-optimal policy parameters are proposed. This document presents tabular data for the extensive computational results conducted in our research in order to compare both the heuristic techniques with each other and with an existing exact approach available in the literature.

Key words: inventory control; demand uncertainty; supplier lead-time uncertainty; constraint-based local search; heuristics.
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1. Experimental results

In this section we provide a full overview on the results of our computational experience (Section 7 of the article).

The results obtained by running the exact approach in Rossi et al. [1] over the test bed discussed in the article are shown in Table 1 and Table 2. In Table 3, we compare the cost of the solutions found by the two heuristics to the optimal ones. In Table 4, we present the run-times for the two heuristics.

It might be argued that the exact approach in Rossi et al. [1] may converge quickly to good solutions and therefore be used as a heuristic approach by simply limiting the run time and collecting the best solution found within the given time limit. Since the maximum run time observed for our heuristic methods over the given test bed is 8.5 seconds, we allocated a run time of 10 seconds to the exact approach and observed the cost difference between the optimal solution produced by the exact approach without a time limit and the best solution that this approach could find in the given time limit of 10 seconds. In Table 5 we present the observed cost differences, in percentage of optimal costs.

In order to assess the scalability of the two heuristics, we now run tests over a 15-period planning horizon. In addition, we also consider all the 5 possible discrete probability density functions for the lead time originally presented in the article, and we vary the remaining model parameters. By doing so, we obtain a total of 240 instances. In Table 6 and Table 7 we compare, respectively, the run times and the cost of the solutions found by the two heuristics for this new set of instances. Since these instances are intractable for the exact approach, we will only compare the two heuristics among each other.

References

[1] Rossi, R., S. A. Tarim, B. Hnich, S. Prestwich. 2010. Computing the non-stationary replenishment cycle inventory policy under stochastic supplier lead-times. *International Journal of Production Economics* **127** 180–189.

		$a = 100$				$a = 175$				$a = 250$			
		$c_v = 0.2$		$c_v = 0.3$		$c_v = 0.2$		$c_v = 0.3$		$c_v = 0.2$		$c_v = 0.3$	
Set	LT	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$
1	1	620	682	640	728	770	833	797	893	920	983	947	1043
	2	677	764	686	826	834	914	836	976	970	1042	986	1126
	3	703	820	738	869	853	967	888	1020	970	1042	1030	1138
2	1	715	783	745	847	895	973	937	1051	1045	1123	1087	1201
	2	803	961	838	1028	1028	1131	1057	1215	1187	1281	1207	1365
	3	903	1025	953	1103	1057	1243	1105	1317	1207	1383	1255	1467
3	1	772	854	798	924	966	1052	1016	1148	1116	1202	1166	1298
	2	836	1032	886	1098	1061	1182	1100	1280	1214	1332	1250	1430
	3	889	1087	950	1162	1065	1278	1137	1362	1215	1378	1287	1504
4	1	530	590	560	650	680	740	710	800	808	886	860	950
	2	576	662	588	716	726	811	738	866	808	886	874	994
	3	545	678	600	734	695	811	750	884	808	886	874	994

Table 1: Exact approach, optimal policy costs, 6-period planning horizon.

		$a = 100$				$a = 175$				$a = 250$			
		$c_v = 0.2$		$c_v = 0.3$		$c_v = 0.2$		$c_v = 0.3$		$c_v = 0.2$		$c_v = 0.3$	
Set	LT	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$
1	1	20	46	15	47	0.98	2.5	0.86	3.7	0.39	0.72	0.52	1.2
	2	210	582	142	793	4.4	16	3.2	24	0.66	1.9	0.80	4.5
	3	3188	15570	4258	18496	27	243	35	318	0.66	2.8	2.0	19
2	1	210	583	212	934	7.2	20	9.4	37	1.1	2.9	1.4	4.8
	2	1611	14409	1993	23781	150	521	148	935	16	43	13	76
	3	46082	88934	62789	132963	794	16092	1224	24845	49	537	82	943
3	1	571	1800	436	2413	28	69	39	153	3.3	8.3	4.3	16
	2	1866	16488	2486	20712	340	1176	379	2016	17	115	14	234
	3	70805	216835	104318	380934	2630	49724	6767	79378	110	1261	208	5472
4	1	3.6	6.1	3.1	9.5	0.42	1.2	0.69	1.9	0.19	0.53	0.44	1.1
	2	52	211	32	292	1.4	12	1.2	15	0.19	0.53	0.49	1.4
	3	90	2684	304	4139	2.0	40	7.7	90	0.20	0.53	0.47	1.4

Table 2: Exact approach, run times (secs), 6-period planning horizon.

		$a = 100$								$a = 175$								$a = 250$							
		$c_v = 0.2$				$c_v = 0.3$				$c_v = 0.2$				$c_v = 0.3$				$c_v = 0.2$				$c_v = 0.3$			
		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$	
Set	LT	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2
	1	3.4	0.48	0.44	0	3.1	0.63	1.4	0.55	3.4	0.39	0.12	0.36	4.6	0.38	0.11	0.67	2.7	0.33	0.10	0.31	4	0.32	0	0.58
1	2	5.0	0.44	0.52	0.79	5.4	0.87	0.85	0.97	3.2	0.48	0.44	0.66	4.4	0.72	0.82	0.82	4.2	0	0.19	0	4	0.61	0	0.71
	3	22	0	0.12	3.3	8.0	0.54	0.12	4.1	5.0	0	0.21	0	4.6	0.45	0.10	4.2	7.6	0	0.19	0	5	0	0.09	0
	1	4.2	0.56	0.77	0.51	6.7	0.27	2.7	0.71	4.9	0.67	0.31	0.31	7.5	0.64	0.10	0.29	4.2	0.57	0.27	0.27	6	0.55	0.08	0.25
2	2	14	0.50	1.2	0.94	14	1.1	1.6	0.58	5.6	0.39	0.27	0.18	6.0	0.85	0.25	0.49	4.1	0.17	0.23	0.16	5	0.75	0.22	0.44
	3	2.3	0	6.9	0.68	4.0	0.10	7.7	1.1	13	0	0.16	0.80	7.1	0	0	1.2	4.8	0	0.14	0	6	0	0.07	1.1
	1	2.8	0.78	0.35	0.70	3.9	0.75	0.22	0.65	5.1	0.62	1.2	0.57	3.6	0.30	0.26	0.52	4.4	0.54	0.17	0.50	5	0.26	0.23	0.46
3	2	15	2.0	0.29	0.78	8.9	2.6	2.3	0.82	6.4	1.0	0.25	0.68	4.7	0.91	0.47	0.31	5.4	0.66	0.23	0.60	4	0.80	0.28	0.28
	3	13	4.3	4.0	6.0	10	2.4	4.5	6.3	8.8	3.5	0.16	1.9	8.9	0	0.22	1.8	7.8	3.0	0.15	0	7	0	0.07	0
	1	5.7	0.38	0.34	1.0	9.3	1.1	1.2	1.2	4.4	0.29	0.27	0.81	6.6	0.85	0.12	1.0	6.4	0	0.56	0	5	0.70	0.11	0.84
4	2	5.2	0.69	0.76	1.2	9.4	1.0	0.42	1.1	4.1	0.55	0.49	0	6.1	0.81	0.35	0.92	10	0	0.45	0	7	0	0.10	0
	3	21	0	1.2	8.6	9.5	0	0.82	8.2	10	0	0.37	0	10	0	0.68	4.0	10	0	0.34	0	12	0	0.10	0

Table 3: Additional cost, in % of the cost of the optimal policy, incurred if Heuristic I (H1) or Heuristic II (H2) are used, 6-period planning horizon.

		$a = 100$								$a = 175$								$a = 250$							
		$c_v = 0.2$				$c_v = 0.3$				$c_v = 0.2$				$c_v = 0.3$				$c_v = 0.2$				$c_v = 0.3$			
		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$	
Set	LT	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2
	1	3.5	0.72	2.4	0.16	1.2	0.14	2.8	0.16	1.2	0.11	1.7	0.13	0.88	0.34	1.8	0.20	1.3	0.39	1.2	0.36	0.91	0.69	1.4	0.14
1	2	3.9	0.72	3.5	0.33	3.0	0.56	5.2	0.41	1.8	0.19	2.4	0.27	2.3	0.45	1.5	0.27	1.6	0.30	1.8	0.70	2.0	1.0	1.8	0.67
	3	3.4	0.36	4.4	1.3	3.5	1.2	2.2	1.1	2.3	0.27	2.2	1.8	2.8	0.49	2.1	1.27	1.5	0.70	1.4	0.69	1.3	0.77	1.2	1.5
	1	2.8	0.45	3.2	0.17	2.4	0.20	3.2	0.20	1.8	0.17	2.4	0.55	2.4	0.17	2.6	0.20	1.8	0.52	2.1	0.44	2.0	0.16	2.5	0.50
2	2	3.8	0.89	2.9	0.66	4.7	0.77	3.2	1.5	3.5	0.70	3.4	0.95	3.6	0.38	3.4	1.1	2.2	0.83	2.2	0.94	2.3	0.39	2.6	1.0
	3	3.3	0.83	3.7	2.0	4.4	1.4	4.3	1.7	3.1	1.5	2.7	2.5	3.8	1.5	3.4	1.6	3.0	0.97	2.0	0.88	2.7	1.5	3.1	1.3
	1	2.7	0.53	3.8	0.78	2.4	0.16	4.4	0.20	3.4	1.2	3.7	0.58	1.5	0.55	3.6	0.70	2.3	0.41	2.0	0.31	2.4	0.47	4.6	0.34
3	2	4.3	0.80	6.4	1.7	3.2	0.88	5.7	1.0	3.8	1.1	4.3	0.67	2.9	0.78	3.9	1.2	4.1	0.69	2.5	0.91	3.0	0.61	4.1	0.94
	3	7.7	1.1	5.5	2.4	6.3	1.4	5.5	3.4	4.0	2.6	4.5	1.6	4.8	1.4	3.5	1.1	3.7	1.2	2.1	1.3	4.0	0.36	5.6	1.3
	1	2.5	0.33	2.6	0.39	2.8	0.33	3.0	0.38	2.4	0.16	2.1	0.31	1.3	0.23	2.7	0.39	1.3	0.31	1.4	0.34	0.7	0.09	1.7	0.30
4	2	3.2	0.59	4.0	0.74	3.9	0.70	3.0	0.58	2.7	0.16	2.9	0.50	1.7	0.72	3.9	0.70	2.0	0.50	2.1	0.55	2.1	0.36	2.3	0.17
	3	3.8	0.49	8.5	0.61	3.2	0.63	4.8	2.3	2.6	0.16	2.7	1.1	4.0	0.55	2.9	1.2	1.4	0.16	1.3	0.78	1.8	0.45	3.3	0.47

Table 4: Heuristic I (H1) and Heuristic II (H2) run times (secs), 6-period planning horizon.

		$a = 100$				$a = 175$				$a = 250$			
		$c_v = 0.2$		$c_v = 0.3$		$c_v = 0.2$		$c_v = 0.3$		$c_v = 0.2$		$c_v = 0.3$	
Set	LT	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$	$\alpha = 0.85$	$\alpha = 0.95$
1	1	0	0	0	2.0	0	0	0	0	0	0	0	0
	2	1	2.3	0	0.85	0	0	0	0	0	0	0	0
	3	12	8.7	6.7	10	0.59	0	0	4.2	0	0	0	0
2	1	4.2	5.1	5.6	6.3	0	0	0	0	0	0	0	0
	2	42	28	45	31	1.3	15	22	17	0	2.4	0	1.6
	3	26	20	28	22	15	5.2	17	8.4	7.1	0	9.2	2.4
3	1	5.7	7	8.5	8	0	1.1	0	0	0	0	0	0
	2	12	17	7	17	2.8	3.9	0	11	0	3.4	0	1.6
	3	25	12	24	14	12	1.9	12	4.9	2.5	0	5.6	0
4	1	0	0	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	2.9	0	0	0	0	0	0	0	0
	3	11	0	5	4.5	0	0	0	0	0	0	0	0

Table 5: Exact approach with a run time limited to 10 seconds. Additional cost, in % of the cost of the optimal policy.

		$a = 100$								$a = 175$								$a = 250$							
		$c_v = 0.2$				$c_v = 0.3$				$c_v = 0.2$				$c_v = 0.3$				$c_v = 0.2$				$c_v = 0.3$			
		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$	
Set	LT	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2
1	1	1.5	0	0.5	0	1.7	0	0	0.21	0.68	0	0	0.45	1.2	0	0	0.59	1.4	0	0	0.50	1.7	0	0	0.54
	2	2.0	0	0	0.18	1.1	0	0.04	0	1.3	0	0	0.84	1.5	0	0	0.91	1.5	0	0	0.37	2.2	0	0	0.85
	3	1.6	0	1.4	0	2.7	0	0	0.12	1.5	0	0	0	0.76	0	0	0.71	2.2	0	0	0.93	1.4	0	0	0.97
	4	9.8	0	3.4	0	11	0	2.6	0	11	0	1.0	0	5.0	0	2.3	0	2.6	0	1.3	0	1.6	0	0.33	0
	5	18	0	6.1	0	5.1	0	2.1	0	16	0	2.4	0	9.9	0	0	0.94	8.9	0	1.3	0	9.6	0	0.53	0
2	1	0.9	0	0	0.04	1.6	0	0	0	1.3	0	0	0.43	0.82	0	0	0.5	0.88	0	0	0.09	1.1	0	0	0.49
	2	6.2	0	0.5	0	4.0	0	0.9	0	3	0	0	0.48	2.4	0	0	0	3.0	0	0	0.55	0.77	0	0	0.58
	3	5.4	0	4.3	0	1.0	0	1.1	0	1.0	0	2.5	0	2.0	0	0.65	0	1.9	0	2.1	0	1.5	0	0.44	0
	4	2.9	0	5.7	0	1.7	0	2.5	0	3.5	0	2.9	0	3.2	0	3.5	0	2.5	0	2.6	0	3.0	0	2.88	0
	5	9.4	0	6.9	0	1.1	0	0	3.3	14	0	0	0.29	0.09	0	0	0.19	8.1	0	1.6	0	3.7	0	0	0.84
3	1	3.0	0	0	0.44	4.0	0	0	0	2.4	0	0	0	4.8	0	0.25	0	4.5	0	0	0.51	5.3	0	0	0.73
	2	8.6	0	0.5	0	5.5	0	0.42	0	6.0	0	0	0.54	2.4	0	0	0	3.6	0	0	0.48	4.3	0	0.32	0
	3	8.1	0	0	1.4	8.5	0	0	0	6.2	0	0.45	0	6.1	0	0	3.5	5.2	0	1.7	0	5.9	0	0	3.4
	4	15	0	3.3	0	11	0	0	0	6.0	0	2.4	0	9.6	0	0	0.32	5.3	0	2.2	0	9.5	0	0	0.27
	5	6.4	0	0.8	0	1.8	0	0	0	14	0	1.6	0	4.3	0	0	2.8	7.5	0	1.5	0	4.1	0	0	1.4
4	1	2.7	0	0.0	0.65	3.5	0	0	0.82	0.82	0	0	0.47	2.1	0	0	0.71	2.4	0	0	0.20	3.0	0	0	0.61
	2	2.2	0	1.9	0	3.7	0	0.55	0	2.6	0	0	0.70	3.1	0	0	0.86	2.3	0	0	0.44	2.7	0	0	0.81
	3	2.2	0	0.3	0	3.7	0	1.7	0	2.9	0	0	1.3	3.7	0	0	0.65	3.2	0	0	0.88	3.8	0	0	0.58
	4	14	0	7.0	0	8.0	0	3.9	0	5.0	0	6.2	0	1.5	0	5.3	0	5.5	0	3.0	0	3.1	0	2.85	0
	5	8.7	0	4.6	0	11	0	3.0	0	5.6	0	3.2	0	6.1	0	0.64	0	1.2	0	0	0.03	2.8	0	0	0.03

Table 6: Cost comparison between Heuristic I (H1) and Heuristic II (H2). A value of 0 is associated with the heuristic that has found the best solution, the other value denotes the cost difference — in percentage of the best solution — achieved by the other heuristic, 15-period planning horizon.

		$a = 100$								$a = 175$								$a = 250$							
		$c_v = 0.2$				$c_v = 0.3$				$c_v = 0.2$				$c_v = 0.3$				$c_v = 0.2$				$c_v = 0.3$			
		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 0.85$		$\alpha = 0.95$	
Set	LT	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2	H1	H2
	1	24	161	17	224	18	195	15	218	12	150	16	196	10	127	18	194	21	69	21	102	14	60	21	104
	2	26	446	33	420	14	311	28	404	23	327	30	421	12	275	27	405	28	210	36	360	19	153	30	355
1	3	26	602	33	763	24	568	43	741	29	588	36	762	19	560	36	742	40	437	49	730	28	397	46	702
	4	36	1096	53	1294	46	1088	51	1516	39	1098	43	1296	19	1089	41	1439	43	979	56	1291	38	935	46	1431
	5	48	1874	48	2178	37	1902	66	2391	30	1870	42	2190	31	1909	47	2391	54	1646	50	2191	50	1678	44	2402
	1	11	295	16	328	14	281	15	327	17	299	15	331	10	282	16	330	12	255	23	311	12	231	21	310
	2	25	520	29	669	18	472	28	648	25	524	35	674	18	480	29	653	24	523	29	673	16	473	37	651
2	3	28	959	35	1345	24	924	51	1219	23	961	35	1272	25	930	35	1220	26	961	40	1272	25	925	53	1217
	4	32	1837	99	2105	35	1826	27	2443	36	1836	59	2099	32	1825	38	2449	31	1843	71	2103	36	1825	84	2447
	5	37	3129	48	3470	46	3223	36	4206	38	3122	46	3479	38	3207	73	4221	30	3123	66	3475	33	3213	67	4267
	1	21	196	21	218	18	189	21	220	13	120	21	172	12	122	19	194	12	43	18	80	6	50	17	110
	2	35	359	32	442	27	342	39	440	28	323	26	438	19	318	25	439	20	178	24	308	17	165	22	357
3	3	33	629	33	793	36	605	38	811	25	587	34	792	25	576	26	809	22	360	30	689	22	366	24	761
	4	36	1219	42	1303	32	1202	29	1549	24	1207	30	1304	32	1190	31	1551	21	883	20	1283	27	853	28	1519
	5	30	1800	31	2076	30	1915	34	2422	26	1794	20	2070	28	1906	23	2421	19	1559	17	2044	22	1697	19	2418
	1	11	200	12	219	9	182	12	217	12	98	22	141	9	75	17	137	15	39	17	64	4	29	16	61
	2	18	343	22	400	17	322	27	387	26	313	29	401	21	283	26	385	20	153	24	276	18	128	22	270
4	3	13	547	27	722	21	530	39	706	19	515	29	721	26	493	30	705	21	290	29	616	21	278	27	605
	4	65	1018	153	1347	23	992	65	1353	30	1016	46	1345	32	1045	48	1351	33	784	35	1289	25	706	39	1288
	5	25	1709	40	2067	22	1720	52	2126	139	1702	46	2069	47	1717	37	2124	28	1465	41	2055	36	1471	53	2121

Table 7: Heuristic I (H1) and Heuristic II (H2) run times (secs), 15-period planning horizon.