Online Supplement for "Lot Sizing with Piecewise Concave Production Costs"

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Valid Inequalities Used in the Computational Study

In this section, we briefly describe the valid inequalities developed by Sanjeevi and Kianfar (2012) for the multi-module lot-sizing problem.

Let $k < l$ be two periods and $S \subseteq \{k, \ldots, l\}$. For each $i \in S$, define $S_i = S \cap \{k, \ldots, i\}$ and compute

$$n_i = \begin{cases} \min\{t : t \in S \setminus S_i\} & \text{if } S \setminus S_i \neq \emptyset, \\ l+1 & \text{if } S \setminus S_i = \emptyset. \end{cases}$$

Adding up the inventory balance equations in the formulation MC from $t = k$ to $t = n_i - 1$, we obtain

$$s_{k-1} + \sum_{t=k}^{n_i-1} \sum_{j=1}^{m} x_t^j = d_{k,n_i-1} + s_{n_i-1}.$$

As $s_{n_i-1} \geq 0$ and $x_t^j \leq b^j y_t^j$, we have

$$s_{k-1} + \sum_{t \in \{k,\ldots,n_i-1\} \setminus S_i} \sum_{j=1}^{m} x_t^j + \sum_{j=1}^{m} \sum_{t \in S_i} b^j y_t^j \geq d_{k,n_i-1}.$$

Now, let $I \subseteq S$. Then $s_{k-1} + \sum_{t \in \{k,\ldots,n_{|I|}-1\} \setminus S} \sum_{j=1}^{m} x_t^j \geq s_{k-1} + \sum_{t \in \{k,\ldots,n_i-1\} \setminus S_i} \sum_{j=1}^{m} x_t^j$ for all $i \in I$. By letting $z_i^j = \sum_{t \in S_i} y_t^j$, one obtains the relaxation

$$s_{k-1} + \sum_{t \in \{k,\ldots,n_{|I|}-1\} \setminus S} \sum_{j=1}^{m} x_t^j + \sum_{j=1}^{m} b^j z_i^j \geq d_{k,n_i-1} \qquad i \in I.$$

Note that $s_{k-1} + \sum_{t \in \{k,\ldots,n_{|I|}-1\} \setminus S} \sum_{j=1}^{m} x_t^j \in \mathbb{R}_+$ and $z_i^j \in \mathbb{Z}_+$ for all $i \in I$ and $j = 1, \ldots, m$. Sanjeevi and Kianfar (2012) generate mixed n-step MIR inequalities based on this relaxation when the coefficients satisfy some conditions. Like Sanjeevi and Kianfar (2012), we consider all possible pairs k and l. We let $S = \{k, \ldots, l\}$, $S = \{t \in \{k, \ldots, l\} : \bar{y}_t^j > 0$ for some $j \in$

$\{1,\ldots,m\}\}$ and $S = \{t \in \{k,\ldots,l\} : \bar{y}_t^j < 1$ for some $j \in \{1,\ldots,m\}\}$ where $(\bar{x},\bar{y},\bar{s})$ is the LP optimum. For these choices of S, we consider all possible two-element subsets I, i.e., $|I| = 2$, and add the resulting inequality if it is violated. We apply this cutting phase at the root node. Then we drop the inactive cuts and give the strengthened formulation to the solver.

Results of the Computational Study

In this section, we provide the detailed results of our computational study in Tables 1-3.

Table 1: Results for $n = 40$ and $m = 2$

instance			MC			MC-CUTS			DP	
(f^1, f^2)	(c^1, c^2)	(b^1, b^2)	BUB	LPGap	FGap	BUB	LPGap	FGap (Time)	OPT	Time
1	1	1	69644.6	2.61	1.60	69737.1	1.37	1.22	69620.3	159.68
		2	63556.1	5.05	3.96	63779.3	3.64	3.63	63474.5	161.31
		3	57745.4	5.91	4.35	57888.0	4.02	3.90	57651.9	152.53
	2	1	78676.3	2.31	1.29	78762.6	1.15	0.61	78660.8	162.50
		2	72588.6	4.42	3.30	73285.9	3.10	1.92	72515.0	158.43
		3	66835.4	5.11	3.72	66801.9	3.44	1.33	66692.4	158.39
	3	1	79678.2	3.51	1.29	79707.4	1.51	0.76	79642.0	161.67
		2	73610	5.71	3.22	73604.6	3.00	2.38	73504.9	150.65
		3	67908.3	6.79	3.47	67967.6	3.54	2.64	67890.7	146.49
	4	1	87701.3	2.07	1.24	87781.3	1.09	0.53	87701.3	162.23
		2	81676.2	3.93	3.13	81786.8	2.83	2.72	81555.5	154.98
		3	75872.8	4.50	3.37	75797.0	3.06	2.76	75732.9	147.92
2	1	1	48332.1	6.34	2.79	48260.3	1.29	(35)	48260.3	161.80
		2	44261.8	9.22	4.67	44280.0	2.43	0.73	44261.8	152.92
		3	40523.5	10.74	5.06	40519.3	2.97	0.81	40514.8	146.48
	2	1	65963	4.03	1.31	65941.3	1.19	(315)	65941.3	161.30
		2	61905.3	5.87	2.64	61924.1	2.35	1.41	61892.8	151.16
		3	58118.9	6.64	2.72	58142.0	2.84	1.58	58098.0	144.96
	3	1	57755	5.90	2.93	57642.0	0.78	(4)	57642.0	161.41
		2	53595.3	8.01	4.27	53504.9	1.17	(3)	53504.9	153.65
		3	49913.7	9.40	4.70	49890.7	1.48	(6)	49890.7	149.81
	4	1	66363.1	4.61	1.91	66341.3	0.94	(13)	66341.3	161.61
		2	62360.1	6.55	3.36	62346.6	1.73	0.43	62342.8	153.27
		3	58616.1	7.43	3.53	58609.3	2.06	0.53	58595.8	146.49
3	1	1	69620.3	2.61	1.45	69620.3	1.30	(1)	69620.3	160.77
		2	63491.7	5.05	3.61	63552.7	3.50	2.58	63474.5	150.25
		3	57730.1	5.91	4.21	57810.7	3.98	3.42	57651.9	147.68
	2	1	78660.8	2.31	1.22	78660.8	1.15	(1)	78660.8	158.88
		2	72566.5	4.42	3.29	72626.0	3.06	2.50	72515.0	150.50
		3	66721.5	5.11	3.57	66700.1	3.44	2.74	66692.4	145.53
	3	1	87701.3	2.07	1.12	87701.3	1.03	(1)	87701.3	160.51
		2	81618.3	3.93	2.93	82461.0	2.78	2.64	81555.5	151.99
		3	75816.5	4.50	3.24	76412.7	3.04	2.56	75732.9	144.93
	4	1	87722.5	2.07	1.18	87848.6	1.03	0.51	87701.3	161.59
		2	81572.7	3.93	2.78	81679.4	2.72	2.30	81555.5	150.58
		3	75831.3	4.50	3.25	75827.0	3.03	2.50	75732.9	145.36

Table 2: Results for $n = 20$ and $m = 3$

instance			**MC**			**MC-CUTS**			**DP**	
(f^1, f^2, f^3)	(c^1, c^2, c^3)	(b^1, b^2, b^3)	BUB	LPGap	FGap (Time)	BUB	LPGap	FGap (Time)	OPT	Time
1	1	1	69160.1	3.85	3.40	69160.2	3.81	3.42	69159.7	172.7
		2	57167	3.01	2.37	57164.1	2.99	2.40	57137	149.5
	2	1	84084.2	3.26	2.35	84084.2	3.12	2.37	84084.2	173.8
		2	71544.9	2.41	0.94	71544.9	2.39	0.86	71544.9	150.3
	3	1	84216.4	3.55	2.26	84216.4	3.18	2.27	84216.3	171.8
		2	71544.9	2.41	0.42	71544.9	2.39	0.46	71544.9	150.7
	4	1	83842.1	3.50	2.62	83866.6	3.37	2.65	83836.2	171.4
		2	71902.8	2.89	1.62	71929.0	2.75	1.65	71902.8	150.7
	5	1	84107.3	3.81	2.57	84132.8	3.02	2.49	84107.2	171.1
		2	72194.2	3.28	1.47	72194.2	2.66	1.46	72194.1	150.8
	6	1	83913.8	3.57	2.56	83908.3	3.44	2.53	83901.2	176.8
		2	72017.5	3.05	1.52	72017.5	2.90	1.56	72017.5	155.4
	7	1	84153.1	3.86	2.45	84152.0	3.01	2.38	84151.9	172.5
		2	72307.4	3.44	1.31	72307.4	2.54	1.15	72307.4	154.4
2	1	1	51062.9	5.95	2.64	51063.6	2.87	1.38	51062.9	174.5
		2	42680.2	6.23	(181.7)	42680.2	3.00	(33)	42680.2	153.5
	2	1	70712.2	3.87	1.85	70712.3	2.42	1.57	70712.2	172.4
		2	62329.6	3.78	0.84	62329.6	2.36	0.57	62329.6	151.3
	3	1	67942.4	4.84	1.93	67942.4	1.72	(993)	67942.4	173.1
		2	59184.7	4.29	(27.8)	59184.7	2.21	(6)	59184.7	151.6
	4	1	70527.8	3.60	1.95	70512.2	2.67	1.89	70512.2	173.8
		2	61893.9	3.11	1.13	61899.6	2.59	1.18	61893.9	151.2
	5	1	65874.8	5.21	1.91	65865.5	1.43	(42)	65865.5	171.9
		1	57194.1	4.83	(19.8)	57194.1	2.16	(3)	57194.1	152.7
	6	1	67488.1	4.20	1.72	67488.1	2.42	1.25	67487.3	172.6
		2	59282.2	4.45	(984.1)	59282.2	2.36	(350)	59282.1	152.2
	7	1	65653.1	4.90	1.37	65652	1.95	0.33	65651.9	172.7
		2	57307.4	5.02	(116.4)	57307.4	1.95	(3)	57307.4	155.4
3	1	1	39063.6	5.43	2.22	39062.9	3.75	2.09	39062.9	174.4
		2	32633.1	5.66	(718.2)	32633.1	3.89	(410)	32633.1	149.8
	2	1	57392.8	3.45	1.80	57394.5	3.26	1.70	57392.8	173.2
		2	50557.0	3.18	(382.2)	50557.0	2.97	(624)	50557.0	153.1
	3	1	55942.4	4.24	0.76	55942.5	2.09	0.53	55942.4	171.9
		2	49926.6	5.04	(126.8)	49926.6	2.05	(14)	49926.6	154.3
	4	1	55409.6	3.99	1.88	55409.6	3.80	1.70	55409.6	172.3
		2	48651.2	3.94	(311.2)	48651.2	3.73	(360)	48651.2	151.3
	5	1	53865.5	4.67	(515.5)	53865.5	1.75	(40)	53865.5	172.0
		2	48132.8	6.11	(144.5)	48132.8	1.84	(3)	48132.8	151.4
	6	1	55050.8	3.36	1.96	55064.1	3.17	1.99	55050.8	174.8
		2	48190.1	3.01	0.50	48181.7	2.79	0.41	48181.6	150.1
	7	1	53652.0	4.29	1.09	53652.0	2.38	0.78	53652.0	171.6
		2	47492.5	4.84	(95.4)	47492.5	2.34	(9)	47492.5	151.3

Table 3: Results for $n = 15$ and $m = 4$

instance			MC			MC-CUTS			DP	
f	c	b	BUB	LPGap	FGap (Time)	BUB	LPGap	FGap (Time)	OPT	Time
1	1	1	52206.9	5.41	3.84	52206.9	5.16	3.80	52206.9	438.5
		2	40213.2	5.47	3.02	40213.2	5.43	2.97	40213.2	411.2
		3	34224.8	8.77	5.06	34224.8	8.20	(733.5)	34224.8	406.6
	2	1	52242.4	5.58	4.42	52242.4	5.42	4.34	52242.4	441.7
		2	40404.6	5.92	3.51	40404.6	5.71	2.89	40404.6	406.9
		3	34412.3	9.26	5.53	34412.3	8.41	4.20	34412.3	401.3
	3	1	52242.4	5.58	4.15	52242.4	5.42	3.97	52242.4	440.2
		2	40404.6	5.92	2.50	40404.6	5.71	2.05	40404.6	409.2
		3	34412.3	9.26	3.79	34412.3	8.41	(802.3)	34412.3	401.8
	4	1	52242.4	5.58	3.92	52242.4	5.42	3.75	52242.4	439.7
		2	40404.6	5.92	0.73	40404.6	5.71	(706.6)	40404.6	416.6
		3	34412.3	9.26	(654.6)	34412.3	8.41	(164.7)	34412.3	400.3
2	1	1	49312.8	5.63	3.93	49320	5.13	3.88	49312.8	446.4
		2	38762.2	5.73	3.28	38762.2	5.51	3.03	38762.2	410.9
		3	33269.9	8.87	5.60	33269.9	7.99	5.02	33269.9	408.2
	2	1	47276.4	5.87	2.91	47276.4	5.01	2.79	47276.4	443.1
		2	37321	6.03	1.03	37321	5.61	(641.1)	37321.0	409.7
		3	31912.3	9.24	(867.6)	31912.3	8.33	(230)	31912.3	406.0
	3	1	48242.4	5.57	4.37	48242.4	5.40	4.27	48242.4	442.1
		2	37404.6	5.94	1.38	37404.6	5.71	(464.7)	37404.6	412.3
		3	31912.3	9.24	(777.6)	31912.3	8.32	(142.7)	31912.3	411.1
	4	1	47276.4	5.87	3.26	47276.4	5.01	3.13	47276.4	441.7
		2	37321	6.03	(786.4)	37321	5.61	(214.1)	37321.0	438.7
		3	31912.3	9.24	(236.9)	31912.3	8.33	(53.6)	31912.3	408.2

References

Sanjeevi, S., K. Kianfar. 2012. Mixed n-step MIR inequalities: Facets for the n-mixing set. *Discrete Optim.* **9** 216 –235.