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Modeling and Optimization of a Spatial Detection System

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Oil and gas companies are drilling and developing fields in the Arctic Ocean, which has an environment with ice floes. These companies must protect their platforms from ice floe collisions. One proposal is to use a system that consists of autonomous underwater vehicles (AUVs) and docking stations. The AUVs measure the under-water topography of the ice floes, while the docking stations launch the AUVs and recharge their batteries. Given resource constraints, we optimize locations and quantities for the docking stations and the AUVs, as well as the AUV scheduling policies, in order to maximize security of the platform. We model the system using a multi-stage stochastic facility location problem in order to optimize the docking station locations, the AUV allocations, and the scheduling policies of the AUVs. A two-stage stochastic facility location problem, and two efficient online scheduling heuristics, provide lower bounds and upper bounds for the multi-stage model. Even though the model is motivated by an oil industry project, most of the modeling and optimization methods apply more broadly to two-dimensional radial detection.

Key words: spatial detection; queues with abandonments; stochastic programming; multi-stage stochastic facility location problem; scheduling heuristics

1. Online Supplement

Algorithm 1

Input: Set I, J, G_i, current system time t, $L_{i,j}$, $U_{i,j}$, $s_{i,j,t}$, $\forall i,j,t$;

J_i^c, na_k, $\forall k \in G_i$, $\forall i \in I$; $\bar{\imath}_j$, $\bar{k}_j$, st_j, $\forall j \in J_i^c$;

Output: J^d: set of ice floe abandonments;

J_i^c, $\forall i \in I$: set of floes that have been served or are awaiting service by docking station i;

na_k, $\forall k \in G_i$, $i \in I$: next available time for each AUV;

$\bar{\imath}_j$, $\bar{k}_j$, st_j, $\forall j \notin J^d$: station $\bar{\imath}_j$ dispatches AUV $\bar{k}_j$ at time st_j to serve floe j if it is not abandoned.

Subroutines: Assign$(t_1, j, st_j, s_{i,j,t_1}, J_i^c)$: $\{st_j = t_1,\ t_1 = t_1 + s_{i,j,t_1},\ J_i^c = J_i^c \cup \{j\}\}$;

Abandon$(\bar{\imath}, j, I, J^d, A(j), t, s_{i,j,t}, U_{i,j})$: $\{$if $\arg\min_{\{i\in I, i\neq\bar{\imath}\}}\{s_{i,j,t} | U_{i,j} \geq t\} = \emptyset,\ J^d = J^d \cup \{j\}$, $A(j) = A(j) + 1\}$.

Step 0: Initialize $A(j) = 0$, $st_j = 0$, $\forall j \in J$; $J^d = \emptyset$, $J^z = \emptyset$, $j = 1$.

Step 1: Assign floe j to station $\bar{\imath}_j \in \arg\min_{i\in I}\{s_{i,j,t} | U_{i,j} \geq t\}$. If $t > U_{i,j}$, $\forall i \in I$, then $J^d = J^d \cup \{j\}$ and go to step 4.

Step 2: $\bar{k}_j \in \arg\min_{k\in G_{\bar{\imath}_j}}\{na_k\}$; if $na_{\bar{k}_j} \leq U_{\bar{\imath}_j,j}$, then call Assign$(na_{\bar{k}_j}, j, st_j, s_{\bar{\imath}_j,j,na_{\bar{k}_j}}, J_{\bar{\imath}_j}^c)$ and go to step 4; else, let $\hat{z} \in \arg\max_{z\in J_{\bar{\imath}_j}^c}\{s_{\bar{\imath}_j,z,st_z} | st_z \geq t\}$, loop$(k \in G_{\bar{\imath}_j})$: $\{pna_k = na_k$, $pna_k = \min_{q\in J_{\bar{\imath}_j}^c}\{st_q | \bar{k}_q = k, st_q \geq st_{\hat{z}}\}\}$.

Step 3:

Loop$(q \in J_{\bar{\imath}_j}^c | st_q > st_{\hat{z}})$: $\{\bar{k}_q \in \arg\min_{k\in G_{\bar{\imath}_j}}\{pna_k\},\ pst_q = pna_{\bar{k}_q},\ pna_{\bar{k}_q} = pna_{\bar{k}_q} + s_{\bar{\imath}_j,q,pst_q}\}$, $\bar{k}_j \in \arg\min_{k\in G_{\bar{\imath}_j}}\{pna_k\}$; if $pna_{\bar{k}_j} \leq U_{\bar{\imath}_j,j}$, then call Assign$(pna_{\bar{k}_j}, j, st_j, s_{\bar{\imath}_j,j,pna_{\bar{k}_j}}, J_{\bar{\imath}_j}^c)$, loop$(k \in G_{\bar{\imath}_j}, q \in J_{\bar{\imath}_j}^c | st_q > st_{\hat{z}}, q \neq j)$: $\{na_k = pna_k,\ st_q = pst_q\}$, $J_{\bar{\imath}_j}^c = J_{\bar{\imath}_j}^c \backslash \{\hat{z}\}$, $A(\hat{z}) = 1$, $J^z = J^z \cup \{\hat{z}\}$, and call Abandon$(\bar{\imath}_j, \hat{z}, I, J^d, A(\hat{z}), t, s_{i,\hat{z},t}, U_{i,\hat{z}})$; else, $A(j) = A(j) + 1$ and call Abandon$(\bar{\imath}_j, j, I, J^d, A(j), t, s_{i,j,t}, U_{i,j})$.

Step 4: If $j = |J|$, then go to step 5; else, $j = j + 1$ and go to step 1.

Step 5:

Loop$(j \in J \cup J^z \,|\, A(j) = 1)$: $\{$assign floe j to station $\hat{\imath}_j \in \arg\min_{\{i\in I, i\neq\bar{\imath}_j\}}\{s_{i,j,t} | U_{i,j} \geq t\}$; let $\bar{\imath}_j = \hat{\imath}_j$ and $\bar{k}_j \in \arg\min_{k\in G_{\bar{\imath}_j}}\{na_k\}$. If $na_{\bar{k}_j} \leq U_{\bar{\imath}_j,j}$, then call Assign$(na_{\bar{k}_j}, j, st_j, s_{\bar{\imath}_j,j,na_{\bar{k}_j}}, J_{\bar{\imath}_j}^c)$; else, $J^d = J^d \cup \{j\}\}$.

Algorithm 2

Input: Set I, J, G_i, current system time t, $L_{i,j}$, $U_{i,j}$, $s_{i,j,t}$, $\forall i,j,t$;

J_i^c, na_k, $\forall k \in G_i$, $\forall i \in I$; $\bar{\imath}_j$, $\bar{k}_j$, st_j, $\forall j \in J_i^c$;

Output: J^d: set of ice floe abandonments;

J_i^c, $\forall i \in I$: set of floes that have been served or are awaiting service by docking station i;

na_k, $\forall k \in G_i$, $i \in I$: next available time for each AUV;

$\bar{\imath}_j$, $\bar{k}_j$, st_j, $\forall j \notin J^d$: station $\bar{\imath}_j$ dispatches AUV $\bar{k}_j$ at time st_j to serve floe j if it is not abandoned.

Subroutines: Assign() and Abandon() from Algorithm 1;

Move$(j, A(j), J^z, J_i^c)$: $\{A(j) = 1,\ J^z = J^z \cup \{j\},\ J_i^c = J_i^c \backslash \{j\}\}$.

Step 0: Initialize $A(j) = 0$, $st_j = 0$, $\forall j \in J$; $J^d = \emptyset$, $J^z = \emptyset$, $j = 1$.

Step 1: Assign floe j to station $\bar{\imath}_j \in \arg\min_{i \in I}\{s_{i,j,t} | U_{i,j} \geq t\}$. If $t > U_{i,j}$, $\forall i \in I$, then $J^d = J^d \cup \{j\}$ and go to step 4. Loop$(k \in G_{\bar{\imath}_j})$:$\{na_k = \max\{t, \min_{q \in J^c_{\bar{\imath}_j}}\{st_q | \bar{k}_q = k, st_q \geq t\}\}\}$, $J^c_{\bar{\imath}_j} = J^c_{\bar{\imath}_j} \cup \{j\}$, $st_j = t$, loop$(q \in J^c_{\bar{\imath}_j} | st_q \geq t)$: {order the floes according to EDD}, $J^s = \emptyset$.

Step 2:

Loop$(k \in G_{\bar{\imath}_j})$:{loop$(q \in J^c_{\bar{\imath}_j} \mid q \notin J^s,\ st_q \geq t)$:{If $na_k \leq U_{\bar{\imath}_j,q}$, then $\bar{k}_q = k$ and call Assign$(na_k, q, st_q, s_{\bar{\imath}_j,q,na_k}, J^s)$. If $na_k > U_{\bar{\imath}_j,q}$, then $\hat{z} \in \arg\max_{z \in J^s}\{s_{\bar{\imath}_j,z,st_z} | \bar{k}_z = k\}$, $pna_k = st_{\hat{z}}$ and loop$(r \in J^s \mid \bar{k}_r = k,\ st_r > st_{\hat{z}})$: $\{pst_r = pna_k,\ pna_k = pna_k + s_{\bar{\imath}_j,r,pst_r}\}$;
if $pna_k \leq U_{\bar{\imath}_j,q}$, then call Assign$(pna_k, q, st_q, s_{\bar{\imath}_j,q,pna_k}, J^s)$, $na_k = pna_k$, loop$(r \in J^s \mid \bar{k}_r = k,\ st_r > st_{\hat{z}})$: $\{st_r = pst_r\}$, $J^s = J^s \backslash \{\hat{z}\}$, and if $k = |G_{\bar{\imath}_j}|$, call Move$(\hat{z}, A(\hat{z}), J^z, J^c_{\bar{\imath}_j})$ and Abandon$(\bar{\imath}_j, \hat{z}, I, J^d, A(\hat{z}), t, s_{i,\hat{z},t}, U_{i,\hat{z}})$; if $pna_k > U_{\bar{\imath}_j,q}$ and $k = |G_{\bar{\imath}_j}|$, then call Move$(q, A(q), J^z, J^c_{\bar{\imath}_j})$ and Abandon$(\bar{\imath}_j, q, I, J^d, A(q), t, s_{i,q,t}, U_{i,q})$.}}

Step 3: If $j = |J|$, then go to step 4; else, $j = j + 1$ and go to step 1.

Step 4:

Loop$(j \in J \cup J^z \mid A(j) = 1)$: {assign floe j to station $\hat{\imath}_j \in \arg\min_{\{i \in I, i \neq \bar{\imath}_j\}}\{s_{i,j,t} | U_{i,j} \geq t\}$; let $\bar{\imath}_j = \hat{\imath}_j$ and $\bar{k}_j \in \arg\min_{k \in G_{\bar{\imath}_j}}\{na_k\}$. If $na_{\bar{k}_j} \leq U_{\bar{\imath}_j,j}$, then call Assign$(na_{\bar{k}_j}, j, st_j, s_{\bar{\imath}_j,j,na_{\bar{k}_j}}, J^c_{\bar{\imath}_j})$; else, $J^d = J^d \cup \{j\}\}$.