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# Online Supplement to the paper: Modeling Two-Dimensional Guillotine Cutting Problems via Integer Programming

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## 1. Redundant-Cut reduction

Conditions 1 and 2 of Remark 3 are checked during the enumeration of plates and variables through Procedure 1. In order to check these conditions, we associate four flags to each plate. The flags can assume values -1, 0, 1 (denoting that the plate is obtained through a trim cut only, it is not obtained through a trim cut with the same orientation, and it is obtained without a trim cut, respectively):

- Flag $sh$ indicates the status of the plate with respect to a cut with orientation $h$;
- Flag $sv$ indicates the status of the plate with respect to a cut with orientation $v$;
- Flag $fh$ indicates the status of flags $sh$ for all the father plates of the plate;
- Flag $fv$ indicates the status of flags $sv$ for all the father plates of the plate.

At step 3 of Procedure 1 we select plate $j$, and check if it can be eventually obtained (again) by further cuts from plates in $J$. If this is the case, we cannot safely remove redundant cuts, otherwise, at step 6:

- if one (or more) of the flags of plate $j$ has value -1, do not perform trim cuts on $j$ in the flag orientation.

At step 7, if a *new* plate $j_1 \notin J$ is obtained from $j$ through a trim cut with orientation $h$:

- set the flag $sh$ of $j_1$ to -1;
- set the flag $sv$ of $j_1$ to 0;
- set the flags $fh$ and $fv$ equal to flags $sh$ and $sv$ of $j$;

if a *new* plate $j_1 \notin J$ is obtained from $j$ through a trim cut with orientation $v$:

- set the flag $sv$ of $j_1$ to -1;
- set the flag $sh$ of $j_1$ to 0;
- set the flags $fh$ and $fv$ equal to flags $sh$ and $sv$ of $j$;

if an *existing* plate $j_1 \in J$ is obtained from $j$ through a trim cut:

- if $sh$ of $j$ is larger than -1, set the flag $fh$ of $j_1$ to 1;
- if $sv$ of $j$ is larger than -1, set the flag $fv$ of $j_1$ to 1;

if a plate (new or existing) $j_1$ is obtained from $j$ without a trim cut:

- set all flags of $j_1$ to 1.

## 2. Computational experiments - additional results

Table 1 reports the features for the small-size instances, i.e., those instances having less than 200,000 variables in the *Complete PP-G2KP Model*.

Table 2 reports the results obtained by applying the proposed solution procedure to the small-size instances.

| | *instance features* | | | | *Complete PP-G2KP Model* | | | |
|---|---|---|---|---|---|---|---|---|
| *name* | *opt* | *n* | $\bar{n}$ | $\rho$ | *vars* | *plates* | *t* | *gap* |
| *cgcut1* | 244 | 7 | 16 | 10.0 | 801 | 140 | 0.1 | 0.0 |
| *CHL5* | 390 | 10 | 18 | 20.0 | 2,858 | 345 | 0.6 | 0.0 |
| *Hchl8s* | 911 | 10 | 18 | 49.0 | 13,997 | 896 | *tl* | 1.8 |
| *OF2* | 2,690 | 10 | 24 | 10.0 | 37,261 | 2,110 | 66.0 | 0.0 |
| *cgcut3* | 1,860 | 19 | 62 | 6.4 | 38,485 | 1,860 | 58.6 | 0.0 |
| *wang20* | 2,721 | 19 | 42 | 6.4 | 38,485 | 1,860 | 60.7 | 0.0 |
| *3* | 1,860 | 20 | 62 | 6.4 | 38,485 | 1,860 | 81.6 | 0.0 |
| *3s* | 2,721 | 20 | 62 | 6.4 | 38,485 | 1,860 | 60.7 | 0.0 |
| *W* | 2,721 | 20 | 62 | 6.4 | 38,485 | 1,860 | 56.5 | 0.0 |
| *OF1* | 2,737 | 10 | 23 | 10.0 | 38,608 | 2,098 | 53.9 | 0.0 |
| *gcut1* | 48,368 | 10 | 10 | 3.8 | 39,896 | 4,429 | 8.8 | 0.0 |
| *A1* | 2,020 | 20 | 62 | 5.6 | 45,333 | 2,040 | 85.4 | 0.0 |
| *A1s* | 2,950 | 20 | 62 | 5.6 | 45,333 | 2,040 | 82.3 | 0.0 |
| *cgcut2* | 2,892 | 10 | 23 | 10.0 | 52,590 | 2,017 | 58.4 | 0.0 |
| *2* | 2,892 | 10 | 23 | 10.0 | 52,590 | 2,017 | 55.2 | 0.0 |
| *2s* | 2,778 | 10 | 23 | 10.0 | 52,590 | 2,017 | 57.2 | 0.0 |
| *CHL2* | 2,326 | 10 | 19 | 6.1 | 57,567 | 2,348 | 104.1 | 0.0 |
| *CHL2s* | 3,279 | 10 | 19 | 6.1 | 57,567 | 2,348 | 110.4 | 0.0 |
| *A2* | 2,505 | 20 | 53 | 5.0 | 61,047 | 2,276 | 236.6 | 0.0 |
| *A2s* | 3,535 | 20 | 53 | 5.0 | 61,047 | 2,276 | 131.0 | 0.0 |
| *STS2* | 4,620 | 30 | 78 | 8.5 | 118,036 | 3,383 | 289.8 | 0.0 |
| *STS2s* | 4,653 | 30 | 78 | 8.5 | 118,036 | 3,383 | 385.7 | 0.0 |
| *Hchl9* | 5,240 | 35 | 76 | 7.6 | 130,738 | 3,666 | 612.5 | 0.0 |
| *A3* | 5,451 | 20 | 46 | 5.7 | 134,164 | 3,752 | 435.5 | 0.0 |
| *HH* | 11,586 | 5 | 18 | 7.5 | 141,167 | 5,486 | *tl* | 4.5 |
| *A4* | 6,179 | 20 | 35 | 10.0 | 179,759 | 4,860 | 1,259.0 | 0.0 |

**Table 1 Small-size instances.**

| | | | | linear relaxation | | Priced PP-G2KP Model | |
|---|---|---|---|---|---|---|---|
| *name* | $t_{tot}$ | $t_{gen(\%)}$ | $t_{LB(\%)}$ | $t_{LP(\%)}$ | $gap_{LP}$ | $t_{(\%)}$ | *gap* |
| *cgcut1* | 0.6 | 0.0 | 96.8 | 1.6 | 0.2 | 1.6 | 0.0 |
| *CHL5* | 1.4 | 0.7 | 86.2 | 5.1 | 8.3 | 8.0 | 0.0 |
| *Hchl8s* | 3,638.2 | 0.0 | 1.1 | 0.0 | 22.0 | 98.9* | 0.8 |
| *OF2* | 4.3 | 0.7 | 69.7 | 27.7 | 7.8 | 1.8 | 0.0 |
| *cgcut3* | 13.9 | 0.3 | 53.8 | 7.6 | 13.5 | 38.3 | 0.0 |
| *wang20* | 1.9 | 2.7 | 58.9 | 37.3 | 5.0 | 1.1 | 0.0 |
| *3* | 16.3 | 0.2 | 63.2 | 7.0 | 14.5 | 29.6 | 0.0 |
| *3s* | 1.9 | 2.6 | 58.1 | 38.7 | 5.0 | 0.5 | 0.0 |
| *W* | 1.8 | 2.2 | 43.1 | 53.6 | 3.6 | 1.1 | 0.0 |
| *OF1* | 2.1 | 2.3 | 34.6 | 58.9 | 1.5 | 4.2 | 0.0 |
| *gcut1* | 0.3 | 3.1 | 78.1 | 9.4 | 4.7 | 9.4 | 0.0 |
| *A1* | 9.8 | 0.4 | 84.8 | 14.5 | 15.1 | 0.3 | 0.0 |
| *A1s* | 1.5 | 3.4 | 33.1 | 62.8 | 0.0 | 0.7 | 0.0 |
| *cgcut2* | 10.0 | 0.5 | 78.6 | 20.7 | 12.5 | 0.2 | 0.0 |
| *2* | 9.2 | 0.4 | 84.4 | 14.9 | 12.5 | 0.3 | 0.0 |
| *2s* | 9.6 | 0.5 | 79.2 | 19.2 | 13.0 | 1.0 | 0.0 |
| *CHL2* | 65.7 | 0.1 | 10.0 | 3.1 | 6.8 | 86.8 | 0.0 |
| *CHL2s* | 23.9 | 0.2 | 23.7 | 6.3 | 6.6 | 69.8 | 0.0 |
| *A2* | 47.9 | 0.1 | 19.8 | 3.2 | 15.7 | 76.9 | 0.0 |
| *A2s* | 2.4 | 2.9 | 45.0 | 50.4 | 3.4 | 1.7 | 0.0 |
| *STS2* | 41.7 | 0.9 | 47.7 | 31.2 | 3.0 | 20.3 | 0.0 |
| *STS2s* | 29.1 | 1.2 | 52.1 | 45.7 | 1.8 | 1.0 | 0.0 |
| *Hchl9* | 121.6 | 0.4 | 75.4 | 21.4 | 9.9 | 2.7 | 0.0 |
| *A3* | 25.1 | 0.8 | 28.1 | 22.4 | 2.4 | 48.7 | 0.0 |
| *HH* | 3,654.0 | 0.0 | 1.5 | 0.1 | 12.1 | 98.5* | 4.4 |
| *A4* | 297.9 | 0.2 | 43.2 | 9.0 | 5.8 | 47.6 | 0.0 |

**Table 2 Solution of the small-size instances.**