

Appendix

A. Detailed Results on the Hardest Solomon Instances

Table 5 shows the detailed results on the 14 hardest Solomon instances. In this table, column **Ins** gives the name of the instance. The next five columns describe the instance characteristics: **Q**, the vehicle capacity; **d**, the average customer demand; **H**, the time horizon; $\bar{t}w$, the average customer time window width; and **s**, the average customer service time. The next two columns, **UB** and **Ksol**, present the cost of the best known solution and the number of vehicles used in this solution, respectively. The next five columns provide results about the root-node solution process: **RLB**, the lower bound obtained using only robust cuts; **LB**, the lower bound improved by the addition of non-robust rank-1 and elementary cuts; **C**, the number of active rank-1 and elementary cuts (i.e., those with non-zero dual variables) in the root-node solution; **ELB**, the improved root-node lower bound obtained if enumeration succeeds (from this point, only elementary routes are considered, full-memory rank-1 and elementary cuts as well as clique cuts are separated); and **T**, the total time (in seconds) for the root node. The next three columns concern the complete solution process: **B**, the total number of nodes in the branch-and-bound search tree (including the root node); **TT** the total computational time (in seconds); and **OPT**, the optimal value. Finally, column **BMR11** report the computational time obtained by Baldacci et al. (2011). Note that the number of vehicles used in the optimal solutions are the same as those provided in column **Ksol**.

Ins	Q	d	H	$\bar{t}w$	s	UB	Ksol	RLB	LB	C	ELB	T	B	TT	OPT	BMR11
C203	700	18.1	3390	1715.8	90.0	588.7	3	588.7	588.7	0	-	365	1	365	588.7	32
C204	700	18.1	3390	2493.6	90.0	588.1	3	588.1	588.1	0	-	1648	1	1648	588.1	182
RC204	1000	17.2	960	718.1	10.0	783.5	4	776.5	780.9	56	783.4	1167	1	1167	783.5	1052
RC207	1000	17.2	960	350.5	10.0	962.9	6	941.6	957.3	111	962.9	347	1	347	962.9	26903
RC208	1000	17.2	960	472.9	10.0	776.1	4	763.6	774.3	93	776.1	599	1	599	776.1	1193
R202	1000	14.6	1000	329.8	10.0	1029.6	8	1016.5	1026.5	40	1029.3	316	1	316	1029.6	3406
R203	1000	14.6	1000	542.7	10.0	870.8	6	859.7	869.0	40	870.8	565	1	565	870.8	2139
R204	1000	14.6	1000	752.3	10.0	731.3	5	720.2	729.7	89	731.1	1495	1	1495	731.3	216367
R206	1000	14.6	1000	423.4	10.0	875.9	5	860.0	873.9	105	875.7	546	1	546	875.9	6675
R207	1000	14.6	1000	604.0	10.0	794.0	4	780.5	791.9	79	794.0	728	1	728	794.0	1401
R208	1000	14.6	1000	784.3	10.0	701.0	4	691.1	698.1	150	-	13828	5	64105	701.0	-
R209	1000	14.6	1000	350.5	10.0	854.8	5	837.2	851.7	141	854.3	599	1	599	854.8	4496
R210	1000	14.6	1000	384.3	10.0	900.5	6	881.3	897.1	157	900.5	729	1	729	900.5	39711
R211	1000	14.6	1000	472.9	10.0	746.7	4	731.1	745.9	216	746.7	1320	1	1320	746.7	10993

Table 5 Detailed results on the hardest Solomon instances.

B. Detailed Results on 200-Customer Gehring and Homberger Instances

Tables 6 to 10 report detailed results for all 60 Gehring and Homberger instances with 200 customers, grouped by class. The labels in these tables are the same as in Table 5. An underlined value in column **OPT** indicates that it differs from the upper bound given in column **UB**. A time limit of four days was allowed before aborting the execution of the algorithm. For the instances that could not be solved within this time frame, only the root-node results are presented. Note that, for instance RC2-2-4, we used $NG = 10$ because, with $NG = 20$, the solution process stalled before adding non-robust cuts.

Ins	Q	d	H	$t\bar{w}$	s	UB	Ksol	RLB	LB	C	ELB	T	B	TT	OPT
C1-2-1	200	17.6	1351	61.4	90.0	2698.6	20	2698.6	2698.6	0	-	20	1	20	2698.6
C1-2-2	200	17.6	1351	348.3	90.0	2694.3	20	2694.3	2694.3	0	-	44	1	44	2694.3
C1-2-3	200	17.6	1351	634.1	90.0	2675.8	20	2659.7	2663.8	55	-	179	7	626	2675.8
C1-2-4	200	17.6	1351	920.1	90.0	2625.6	19	2619.7	2625.4	118	2625.6	340	1	340	2625.6
C1-2-5	200	17.6	1351	120.3	90.0	2694.9	20	2694.9	2694.9	0	-	24	1	24	2694.9
C1-2-6	200	17.6	1351	158.8	90.0	2694.9	20	2694.9	2694.9	0	-	26	1	26	2694.9
C1-2-7	200	17.6	1351	181.0	90.0	2694.9	20	2694.9	2694.9	0	-	22	1	22	2694.9
C1-2-8	200	17.6	1351	240.3	90.0	2684.0	20	2684.0	2684.0	0	-	28	1	28	2684.0
C1-2-9	200	17.6	1351	361.0	90.0	2639.6	19	2639.4	2639.6	1	-	84	1	84	2639.6
C1-210	200	17.6	1351	480.7	90.0	2624.7	19	2619.7	2619.7	0	2624.7	113	1	113	2624.7

Table 6 Detailed results on 200-customer Gehring and Homberger instances of class C1.

Ins	Q	d	H	$\bar{t}w$	s	UB	Ksol	RLB	LB	C	ELB	T	B	TT	OPT
RC1-2-1	200	17.8	634	31.0	10.0	3516.9	20	3459.6	3499.1	368	3500.7	690	25	2530	3516.9
RC1-2-2	200	17.8	634	166.1	10.0	3221.6	19	3176.7	3204.4	387	3208.4	2487	71	11427	3221.6
RC1-2-3	200	17.8	634	301.7	10.0	3001.4	18	2949.5	2980.7	488	-	2536	59	65745	3001.4
RC1-2-4	200	17.8	634	437.1	10.0	2845.2	18	2809.1	2832.8	417	-	6031	5	23001	2845.2
RC1-2-5	200	17.8	634	65.7	10.0	3325.6	19	3271.4	3310.0	462	3314.6	5925	15	8584	3325.6
RC1-2-6	200	17.8	634	61.0	10.0	3300.7	19	3254.5	3289.0	408	3295.8	1144	3	1606	3300.7
RC1-2-7	200	17.8	634	92.4	10.0	3177.8	19	3112.3	3150.0	471	-	5345	179	188414	3177.8
RC1-2-8	200	17.8	634	120.2	10.0	3060.0	18	3017.4	3044.8	419	3051.5	6953	11	10215	3060.0
RC1-2-9	200	17.8	634	121.0	10.0	3074.8	18	3000.7	3035.7	500	-	7633	-	-	-
RC1-210	200	17.8	634	151.0	10.0	2990.5	18	2932.6	2966.0	585	-	7994	-	-	-

Table 7 Detailed results on the 200-customer Gehring and Homberger instances of class RC1.

Ins	Q	d	H	$\bar{t}w$	s	UB	Ksol	RLB	LB	C	ELB	T	B	TT	OPT
R1-2-1	200	17.6	634	11.0	10.0	4667.2	23	4655.0	4665.3	18	4665.3	59	1	59	4667.2
R1-2-2	200	17.6	634	151.2	10.0	3919.9	20	3907.2	3919.9	135	-	162	1	162	3919.9
R1-2-3	200	17.6	634	291.0	10.0	3373.9	18	3320.5	3352.2	466	-	2807	157	93367	3373.9
R1-2-4	200	17.6	634	432.0	10.0	3047.6	18	3000.0	3032.0	448	-	2007	33	18026	3047.6
R1-2-5	200	17.6	634	31.0	10.0	4053.2	20	4006.8	4043.9	248	4044.6	207	3	445	4053.2
R1-2-6	200	17.6	634	166.2	10.0	3559.2	19	3492.4	3543.0	446	3547.9	4138	21	7201	<u>3559.1</u>
R1-2-7	200	17.6	634	301.0	10.0	3141.9	18	3099.2	3136.9	371	3139.6	1002	1	1002	3141.9
R1-2-8	200	17.6	634	437.0	10.0	2938.4	18	2901.7	2929.7	384	2934.3	1817	5	2499	2938.4
R1-2-9	200	17.6	634	61.1	10.0	3734.7	19	3676.9	3724.8	364	3727.9	484	5	771	3734.7
R1-210	200	17.6	634	125.5	10.0	3293.1	18	3243.9	3276.9	409	3281.0	1827	25	5010	3293.1

Table 8 Detailed results on the 200-customer Gehring and Homberger instances of class R1.

Ins	Q	d	H	$\bar{t}w$	s	UB	Ksol	RLB	LB	C	ELB	T	B	TT	OPT
C2-2-1	700	18.9	3598	161.0	90.0	1922.1	7	1915.0	1922.1	147	-	860	1	860	1922.1
C2-2-2	700	18.9	3598	986.0	90.0	1851.4	7	1839.4	1851.4	116	-	5377	1	5377	1851.4
C2-2-3	700	18.9	3598	1811.6	90.0	1763.4	7	1737.1	1752.1	194	-	30283	-	-	-
C2-2-4	700	18.9	3598	2635.3	90.0	1695.0	7	1652.3	1665.4	218	-	142466	-	-	-
C2-2-5	700	18.9	3598	321.0	90.0	1869.6	7	1860.7	1869.6	99	-	1428	1	1428	1869.6
C2-2-6	700	18.9	3598	507.3	90.0	1844.8	7	1840.7	1844.8	113	-	2043	1	2043	1844.8
C2-2-7	700	18.9	3598	631.5	90.0	1842.2	6	1835.6	1842.2	39	-	2673	1	2673	1842.2
C2-2-8	700	18.9	3598	641.0	90.0	1813.7	6	1808.2	1813.7	79	-	2542	1	2542	1813.7
C2-2-9	700	18.9	3598	852.6	90.0	1815.0	7	1801.2	1815.0	206	-	7284	1	7284	1815.0
C2-210	700	18.9	3598	881.0	90.0	1791.2	7	1770.3	1791.2	198	-	10560	1	10560	1791.2

Table 9 Detailed results on the Gehring and Homberger instances of class C2.

Ins	Q	d	H	$\bar{t}w$	s	UB	Ksol	RLB	LB	C	ELB	T	B	TT	OPT
RC2-2-1	1000	17.8	2535	121.0	10.0	2797.4	9	2789.1	2797.4	15	-	2149	1	2149	2797.4
RC2-2-2	1000	17.8	2535	709.4	10.0	2481.6	9	2458.9	2478.0	110	-	12101	3	17423	2481.6
RC2-2-3	1000	17.8	2535	1297.3	10.0	2227.7	8	2199.5	2227.7	280	-	48725	1	48725	2227.7
RC2-2-4	1000	17.8	2535	1885.6	10.0	1854.8	7	1785.5	1849.6	212	-	345490	-	-	-
RC2-2-5	1000	17.8	2535	280.1	10.0	2493.5	8	2478.9	2491.4	78	-	5117	1	5117	<u>2491.4</u>
RC2-2-6	1000	17.8	2535	241.0	10.0	2496.3	9	2472.4	2495.1	120	-	5584	1	5584	<u>2495.1</u>
RC2-2-7	1000	17.8	2535	370.8	10.0	2287.7	7	2270.6	2286.4	119	-	27734	3	29505	2287.7
RC2-2-8	1000	17.8	2535	486.4	10.0	2151.2	6	2138.4	2150.3	95	2151.2	83101	1	83101	2151.2
RC2-2-9	1000	17.8	2535	481.0	10.0	2086.6	6	2053.6	2072.0	121	-	344990	-	-	-
RC2-210	1000	17.8	2535	601.0	10.0	1989.2	5	1946.0	1966.7	110	-	357497	-	-	-

Table 10 Detailed results on the 200-customer Gehring and Homberger instances of class RC2.

Ins	Q	d	H	$\bar{t}w$	s	UB	Ksol	RLB	LB	C	ELB	T	B	TT	OPT
R2-2-1	1000	17.6	2535	122.2	10.0	3468.0	13	3462.5	3468.0	31	-	2083	1	2083	3468.0
R2-2-2	1000	17.6	2535	709.7	10.0	3008.2	11	3000.6	3008.2	30	-	4065	1	4065	3008.2
R2-2-3	1000	17.6	2535	1297.3	10.0	2537.5	9	2515.5	2537.5	188	-	21011	1	21011	2537.5
R2-2-4	1000	17.6	2535	1884.1	10.0	1928.5	5	1917.9	1925.0	112	-	141826	-	-	-
R2-2-5	1000	17.6	2535	241.0	10.0	3061.1	8	3029.0	3061.1	205	-	6316	1	6316	3061.1
R2-2-6	1000	17.6	2535	800.2	10.0	2675.4	8	2645.6	2675.0	144	2675.1	14672	1	14672	2675.4
R2-2-7	1000	17.6	2535	1358.3	10.0	2304.7	7	2277.7	2304.7	233	-	44174	1	44174	2304.7
R2-2-8	1000	17.6	2535	1915.0	10.0	1842.4	4	1810.7	1831.6	134	-	346022	-	-	-
R2-2-9	1000	17.6	2535	374.1	10.0	2843.3	9	2820.6	2843.3	117	-	5031	1	5031	2843.3
R2-210	1000	17.6	2535	477.5	10.0	2549.4	8	2526.5	2546.3	194	-	9678	3	11659	2549.4

Table 11 Detailed results on the 200-customer Gehring and Homberger instances of class R2.