Risk averse shortest paths: A computational study
Online Supplement

Renaud Chicoisne
University of Colorado Denver
renaud.chicoisne@gmail.com
Fernando Ordóñez
Universidad de Chile
fordon@dii.uchile.cl
Daniel Espinoza
Gurobi Inc.
espinoza@gurobi.com

A NP-hardness of general risk-averse shortest path problems

CVaR and the entropic risk measure reduce to a quadratic form if $c^\top x$ follows a normal distribution. With this observation we can prove that problem (1)

$$(P_\rho) \qquad w^* = \min_{x\in X} \rho\left(c^\top x\right) \ . \tag{1}$$

is NP-hard by reducing the set partition problem. Given a set of N positive integers $(a_i)_{i\in\{1,...,N\}}$, we want to find a partition $\{N_1, N_2\}$, $N_1 \cap N_2 = \emptyset$, $N_1 \cup N_2 = \{1, ..., N\}$ such that

$$\sum_{i\in N_1} a_i = \sum_{i\in N_2} a_i.$$

Assuming that the a_i are "deterministic random variables", finding the shortest path in the network depicted

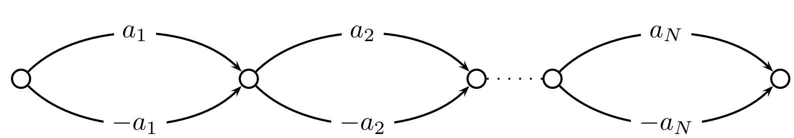

Figure 1: Reduction to Set Partition Problem

in Figure 1 with respect to the quadratic risk measure $\left(c^\top x\right)^2$ solves the set partition problem. Indeed, for this construction we have that this shortest path problem has an optimal value 0 if and only if the parameters $(a_i)_{i\in\{1,...,N\}}$ define a solvable instance of the partition problem. Further, if the instance is not solvable then the optimal path defines the partition minimizing the difference of weights between N_1 and N_2.

B Worsening stochastic upper bound for the entropic risk measure

Considering some path x, in Figure 2 we illustrate the effect on the stochastic upper bound $U_{S'}$ of adding more scenarios to the out of sample. The gray dots are a scatterplot of the values $(c^s)^\top x$ for each scenario

s and the black curve represents the stochastic upper bound returned by x when considering only the first s scenarios. We can see that the stochastic upper bound for the Entropic risk measure is very sensitive to the bad realizations as we can observe jumps in the black curve whenever a new worst case scenario appears.

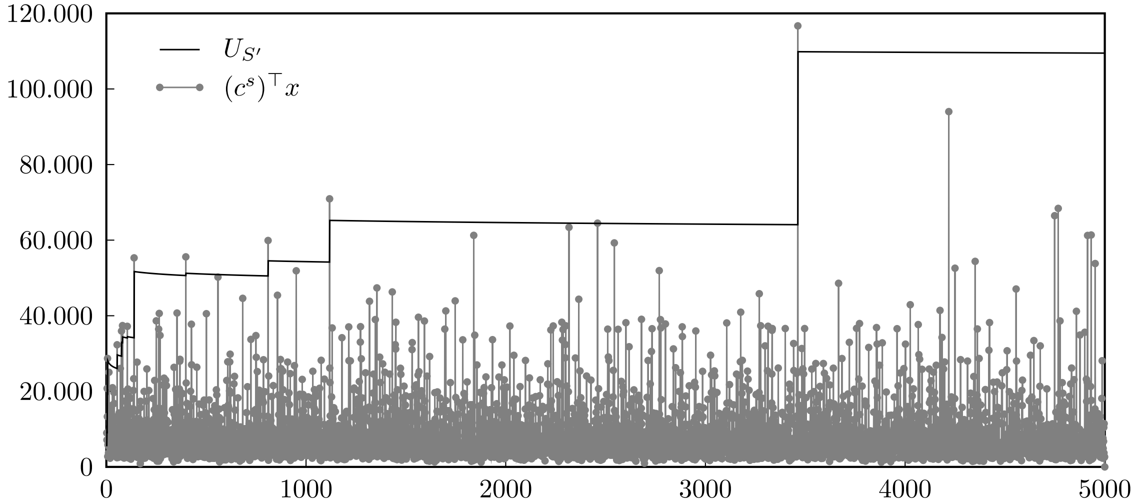

Figure 2: Example of upper bound for entropy and scenario realizations $(c^s)^\top x$ Vs. S'