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Online Supplement
An Improved Branch-Cut-and-Price Algorithm for Parallel Machine Scheduling Problems

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This Online Supplement contains accompanying detailed computational results associated with the paper An Improved Branch-Cut-and-Price Algorithm for Parallel Machine Scheduling Problems, accepted for publication in the INFORMS Journal on Computing, 2018. All instances are available at: `https://github.com/danieloliveira56/schedWT`

1. Detailed computational results

Table A1: Root relaxation and cut separation results

					BCP-PMWT Root			BCP-PMWT-OTI Root*			BCP-PMWT-OTI Root		
n	m	Inst	UB	1st LB	LB	Gap	Time	LB	Gap	Time	LB	Gap	Time
40	2	1	606	584	606		63.50	606		36.42	606		21.90
40	2	6	3886	3875	3886		11.80	3886		6.50	3886		6.70
40	2	11	9617	9592	9617		6.40	9617		3.87	9617		5.10
40	2	16	38356	38279	38356		41.80	38356		591.89	38356		32.50
40	2	31	3812	3758	3812		29.20	3812		20.23	3812		5.80
40	2	36	10713	10662	10713		50.70	10703	0.093%	153.65	10713		8.10
40	2	56	1279	1272	1279		8.30	1279		6.90	1279		5.70
40	2	61	11488	11311	11394	0.818%	684.80	11388	0.870%	359.36	**11488**		11.10

Table A1: continued

					BCP-PMWT Root			BCP-PMWT-OTI Root*			BCP-PMWT-OTI Root		
n	m	Inst	UB	1st LB	LB	Gap	Time	LB	Gap	Time	LB	Gap	Time
40	2	66	35279	35130	35197	0.232%	200.60	35195	0.238%	157.92	**35233**	0.130%	556.10
40	2	71	47952	47935	47952		0.90	47952		1.97	47952		6.50
40	2	81	571	452	571		67.60	571		46.90	571		18.50
40	2	86	6048	5996	6048		31.90	6042	0.099%	76.58	6048		5.20
40	2	96	66116	66111	66116		1.20	66116		1.05	66116		4.30
40	2	111	17936	17898	17936		3.50	17936		4.98	17936		6.00
40	2	116	25870	25786	25870		44.40	25870		44.17	25870		9.80
40	2	121	64516	64507	64516		1.00	64516		0.98	64516		4.40
40	4	1	439	438	439		1.70	439		1.29	439		3.90
40	4	6	2374	2372	2374		2.30	2374		1.15	2374		3.70
40	4	11	5737	5735	5737		0.50	5737		0.66	5737		3.20
40	4	16	21493	21484	21490	0.014%	19.90	21493		19.93	21493		24.50
40	4	31	2525	2496	2500	0.990%	39.30	2500	0.990%	11.61	**2507**	0.713%	29.70
40	4	36	6420	6355	6364	0.872%	26.50	6364	0.872%	29.21	**6367**	0.826%	16.40
40	4	41	17685	17634	17637	0.271%	24.30	17637	0.271%	11.97	**17639**	0.260%	14.10
40	4	56	826	798	**817**	1.090%	45.50	814	1.453%	16.06	815	1.332%	29.50
40	4	61	7357	7316	7322	0.476%	22.80	7322	0.476%	27.69	**7327**	0.408%	40.10
40	4	66	20251	20247	20251		1.70	20251		1.04	20251		2.90
40	4	81	564	550	560	0.709%	37.90	560	0.709%	20.81	560	0.709%	19.60
40	4	86	4725	4719	4725		6.80	4725		3.12	4725		6.10
40	4	91	15569	15557	15562	0.045%	27.00	15562	0.045%	13.28	15562	0.045%	17.90
40	4	111	11263	11212	11219	0.391%	58.60	11217	0.408%	15.79	**11222**	0.364%	17.30
40	4	116	15566	15539	15545	0.135%	19.70	15545	0.135%	19.41	**15547**	0.122%	19.20
40	4	121	35751	35739	35741	0.028%	39.10	35741	0.028%	9.86	35741	0.028%	15.60
50	2	1	1268	1232	1268		296.60	1268		433.80	1268		28.80
50	2	6	14272	14261	14272		63.10	14272		42.35	14272		10.00
50	2	11	23028	23000	23028		21.30	23028		10.76	23028		10.90
50	2	16	46072	46011	46072		25.10	46072		21.19	46072		17.70
50	2	21	111069	111067	111069		0.70	111069		0.79	111069		6.80
50	2	31	5378	5289	5349	0.539%	1060.20	5349	0.539%	223.79	**5378**		21.10
50	2	36	18956	18895	18956		26.80	18956		26.62	18956		9.10
50	2	41	38058	37968	38050	0.021%	341.60	38034	0.063%	1615.95	**38058**		58.00
50	2	46	82105	82085	82105		31.50	82105		26.33	82105		31.40
50	2	56	761	730	761		232.70	761		78.36	761		13.90
50	2	61	13682	13589	13619	0.460%	614.00	13617	0.475%	469.34	**13682**		22.60
50	2	66	40907	40904	40907		0.60	40907		0.67	40907		6.30
50	2	81	542	538	542		5.30	542		2.34	542		8.90
50	2	86	12557	12277	12427	1.035%	946.70	12415	1.131%	323.63	**12557**		17.80
50	2	91	47349	47294	47330	0.040%	492.30	47330	0.040%	649.79	**47349**		8.70
50	2	96	92822	92803	92822		6.30	92822		5.87	92822		7.30
50	2	111	15564	15544	15564		3.50	15564		5.60	15564		9.30
50	2	116	19608	19524	19573	0.178%	453.30	19572	0.184%	712.41	**19608**		47.10
50	4	1	785	777	785		24.80	785		33.58	785		29.00
50	4	6	8317	8298	8304	0.156%	110.80	8304	0.156%	35.30	8304	0.157%	33.70
50	4	11	12879	12871	12875	0.031%	71.20	12875	0.031%	39.81	**12876**	0.023%	34.60
50	4	16	25376	25375	25376		8.00	25376		5.96	25376		9.20
50	4	36	10796	10794	10796		5.00	10796		1.55	10796		5.60
50	4	41	21806	21783	21785	0.096%	44.90	21786	0.092%	79.51	21786	0.092%	59.10
50	4	46	44455	44452	**44455**		15.10	44453	0.004%	25.61	44453	0.004%	17.10
50	4	56	570	538	541	5.088%	45.80	541	5.088%	26.29	540	5.556%	14.20
50	4	61	7898	7850	7857	0.519%	81.80	7856	0.532%	44.74	7857	0.522%	49.30
50	4	71	42645	42625	42628	0.040%	138.40	42628	0.040%	41.61	42627	0.042%	19.00
50	4	81	495	478	495		64.90	495		49.95	495		26.30

Table A1: continued

					BCP-PMWT Root			BCP-PMWT-OTI Root*			BCP-PMWT-OTI Root		
n	m	Inst	UB	1st LB	LB	Gap	Time	LB	Gap	Time	LB	Gap	Time
50	4	86	8369	8330	8336	0.394%	163.30	8336	0.394%	52.30	8336	0.396%	69.90
50	4	91	26551	26546	26548	0.011%	111.60	26551		9.69	26551		12.40
50	4	96	50326	50312	50317	0.018%	91.00	50316	0.020%	54.30	**50325**	0.002%	77.10
50	4	111	10069	10049	10051	0.179%	63.20	10051	0.179%	27.43	**10052**	0.169%	39.90
50	4	116	11552	11520	11523	0.251%	54.40	11523	0.251%	26.05	11523	0.252%	27.00
50	4	121	23792	23769	23775	0.071%	58.50	23775	0.071%	34.82	23775	0.072%	49.00
100	2	1	3339	3314	3322	0.509%	814.60	3317	0.659%	273.99	**3339**		285.80
100	2	6	30665	30644	30665		328.10	30665		502.72	30665		200.10
100	2	11	93894	93894	93894		17.20	93894		0.00	93894		40.10
100	2	16	209100	209062	209100		5086.70	209100		362.96	209100		86.90
100	2	21	457836	457814	457836		24.30	457836		131.59	457836		215.10
100	2	31	12729	12725	12729		210.50	12729		101.87	12729		104.10
100	2	36	56671	56576	56590	0.143%	804.70	56604	0.118%	1352.29	**56671**		347.60
100	2	41	237964	237773	237841	0.052%	19685.20	237858	0.045%	4597.80	**237884**	0.034%	2201.70
100	2	46	422831	422804	422830	0.000%	1525.30	422827	0.001%	1034.05	**422831**		171.90
100	2	56	5047	4983	5047		796.90	4996	1.011%	366.49	5047		152.40
100	2	61	45573	45424	45481	0.202%	6818.20	45473	0.219%	2369.51	**45573**		275.50
100	2	66	126513	126408	126477	0.028%	6969.70	126475	0.030%	5139.77	**126503**	0.008%	1976.30
100	2	71	327305	327300	327305		3.10	327305		5.77	327305		57.60
100	2	81	908	792	830	8.590%	968.90	856	5.727%	1959.78	**908**		198.90
100	2	86	36581	36218	36322	0.708%	3234.00	36316	0.724%	3998.60	**36581**		262.80
100	2	91	129929	129619	129752	0.136%	3617.10	129766	0.125%	9699.51	**129804**	0.096%	6258.50
100	2	96	254194	254141	254194		400.20	254194		1144.89	254194		185.90
100	2	111	84220	84005	84097	0.146%	6578.00	84097	0.146%	8333.18	**84220**		219.50
100	2	116	191186	191085	191173	0.007%	11118.60	191170	0.008%	4350.81	**191186**		2373.00
100	2	121	242018	241954	241997	0.009%	56938.90	241996	0.009%	5195.25	**242018**		181.10
100	4	1	2001	1990	2001		668.10	1993	0.400%	724.78	2001		156.10
100	4	11	50232	50199	50203	0.058%	951.30	50203	0.058%	500.44	50203	0.058%	315.40
100	4	16	110219	110114	110118	0.092%	950.60	110116	0.093%	309.06	**110120**	0.090%	249.50
100	4	21	237392	237389	237390	0.001%	1047.80	237388	0.002%	548.77	**237391**	0.000%	375.80
100	4	31	7130	7080	7082	0.673%	1156.70	7082	0.673%	362.58	7082	0.678%	311.20
100	4	36	30791	30773	**30783**	0.026%	1654.20	30782	0.029%	512.14	30782	0.029%	427.90
100	4	41	126185	126130	126144	0.032%	923.40	126146	0.031%	2518.48	**126147**	0.030%	1376.40
100	4	46	219536	219526	219529	0.003%	910.00	219529	0.003%	724.20	219529	0.003%	578.30
100	4	56	3076	3021	3030	1.495%	1102.50	3030	1.495%	655.17	3030	1.518%	586.30
100	4	61	24856	24806	24821	0.141%	1094.90	24820	0.145%	656.88	**24825**	0.125%	548.80
100	4	66	67970	67948	67954	0.024%	819.70	67955	0.022%	919.41	67955	0.022%	723.00
100	4	71	170691	170674	170675	0.009%	398.80	170674	0.010%	381.42	170675	0.009%	568.40
100	4	81	819	754	772	5.739%	1242.50	769	6.105%	740.06	**784**	4.464%	466.10
100	4	86	21286	21209	21218	0.319%	798.60	21220	0.310%	717.18	21220	0.311%	823.80
100	4	91	70608	70582	**70589**	0.027%	1640.70	70587	0.030%	567.63	70588	0.028%	1269.80
100	4	96	133587	133572	133575	0.009%	443.80	133574	0.010%	457.02	133575	0.009%	1028.70
100	4	111	46719	46616	46630	0.191%	1094.30	46631	0.188%	584.21	**46633**	0.184%	957.10
100	4	116	101551	101514	101520	0.031%	871.70	101520	0.031%	413.25	**101521**	0.030%	790.40
100	4	121	127619	127593	127597	0.017%	926.70	127597	0.017%	1051.62	**127598**	0.016%	904.90

Table B1: Comparison of Alternative Time-Indexed Formulations

			1st LP Time(s)				MIP Time (s)			
n	m	Inst	Cy	Fy	Cz	Fz	Cy	Fy	Cz	Fz
40	2	66	2.03	**1.01**	2.71	36.01	798.67	**161.91**	180.48	655.83
40	4	31	0.84	**0.35**	0.87	22.78	≥3600	370.62	**31.45**	196.45
40	4	36	0.79	**0.57**	0.81	5.69	406.71	283.05	**12.14**	28.19
40	4	41	0.65	**0.58**	0.99	7.57	111.60	31.11	**19.84**	41.58
40	4	56	1.30	**0.32**	2.24	9.75	41.84	**7.33**	137.12	190.69
40	4	61	0.22	**0.21**	0.49	2.72	45.04	11.47	**5.41**	14.11
40	4	81	0.05	**0.02**	0.06	0.13	0.04	**0.03**	0.21	0.12
40	4	91	0.05	**0.01**	0.02	0.03	0.26	0.16	**0.12**	0.27
40	4	111	0.31	**0.26**	0.50	3.51	26.07	6.40	**6.22**	7.69
40	4	116	0.30	**0.11**	0.34	1.74	19.10	**3.60**	4.29	9.24
40	4	121	0.30	**0.09**	0.25	0.62	2.89	1.92	1.38	**1.30**
50	4	6	3.13	**1.08**	1.93	93.73	≥3600	407.15	**15.59**	171.26
50	4	11	0.91	**0.19**	0.47	3.27	18.30	**6.02**	11.62	10.38
50	4	41	2.44	**0.83**	1.08	8.54	104.60	**9.00**	26.03	25.75
50	4	46	0.04	<**0.01**	0.01	0.04	0.16	**0.02**	0.06	0.04
50	4	56	3.12	**1.38**	9.80	146.49	≥3600	≥3600	**1757.61**	≥3600
50	4	61	1.89	1.32	**1.08**	31.59	≥3600	≥3600	**653.57**	1171.95
50	4	71	1.18	**0.61**	0.94	10.04	17.53	**2.88**	17.51	25.67
50	4	86	0.72	**0.23**	0.36	3.53	100.68	59.15	**10.36**	10.87
50	4	96	0.84	0.35	**0.34**	2.49	46.88	**15.97**	16.65	49.62
50	4	111	0.33	**0.14**	0.30	1.72	5.79	**2.19**	4.12	8.54
50	4	116	0.54	**0.25**	1.05	5.82	2644.23	1033.20	**217.34**	467.90
50	4	121	1.11	**0.68**	0.96	12.62	109.73	63.73	**15.99**	23.71
100	2	41	114.36	**72.24**	93.83	6429.75	≥3600	≥3600	≥3600	≥3600

Table B1: continued

			1st LP Time(s)				MIP Time (s)			
n	m	Inst	Cy	Fy	Cz	Fz	Cy	Fy	Cz	Fz
100	2	66	14.92	**6.94**	7.59	198.42	≥3600	1835.84	**605.78**	2958.81
100	2	91	96.38	**73.22**	198.79	3088.84	≥3600	≥3600	≥3600	≥3600
100	4	11	**32.24**	34.88	54.41	1571.94	≥3600	≥3600	≥3600	≥3600
100	4	16	73.86	55.26	**46.72**	1072.26	≥3600	≥3600	≥3600	≥3600
100	4	21	30.32	3.04	**2.00**	109.52	8.37	**0.92**	2.52	38.28
100	4	31	**30.00**	118.63	803.14	2314.03	≥3600	≥3600	≥3600	≥3600
100	4	36	19.04	**11.30**	43.85	570.91	≥3600	≥3600	**610.22**	≥3600
100	4	41	42.75	**29.76**	32.70	603.60	3019.47	**299.67**	1617.61	2749.60
100	4	46	15.94	10.76	**4.39**	69.70	81.13	40.73	**23.83**	189.39
100	4	56	**53.09**	90.65	775.33	6540.45	≥3600	≥3600	≥3600	≥3600
100	4	61	11.63	**10.05**	14.11	242.97	≥3600	≥3600	**1537.80**	≥3600
100	4	66	18.66	**8.30**	12.55	165.54	1023.58	**214.09**	483.20	1412.99
100	4	71	35.73	**9.94**	18.82	227.95	≥3600	**488.18**	≥3600	≥3600
100	4	81	**19.81**	19.91	127.50	599.83	≥3600	≥3600	≥3600	≥3600
100	4	86	16.03	**8.02**	28.11	239.84	≥3600	≥3600	≥3600	≥3600
100	4	91	15.75	**6.03**	16.21	67.08	2245.20	485.18	401.22	**298.60**
100	4	96	17.23	5.35	**4.99**	166.46	940.15	**95.42**	122.28	282.37
100	4	111	26.33	**6.53**	31.93	245.29	≥3600	≥3600	≥3600	≥3600
100	4	116	15.00	**5.30**	31.04	60.67	≥3600	≥3600	≥3600	≥3600
100	4	121	23.88	**5.43**	12.36	246.66	≥3600	≥3600	**815.18**	≥3600

Table B2: Effect of Variable Fixation in the Cz Time-Index Formulation

			Root Gap		1st LP Time (s)		MIP Time (s)	
n	m	Inst	Fix.	w/o Fix.	Fix.	w/o Fix.	Fix.	w/o Fix.
40	2	66	0.480%	0.499%	2.59	248.26	109.68	≥3600
40	4	31	1.485%	1.485%	0.75	40.95	19.97	2418.44
40	4	36	1.049%	1.073%	0.76	42.63	21.37	1630.34
40	4	41	0.316%	0.342%	0.61	54.26	3.23	348.15
40	4	56	3.487%	3.487%	2.11	38.67	19.69	≥3600
40	4	61	0.513%	0.710%	0.76	31.84	9.46	1442.72
40	4	81	2.117%	4.796%	0.04	107.80	0.12	348.03
40	4	91	0.083%	0.191%	0.02	44.28	0.06	175.97
40	4	111	0.685%	0.736%	0.72	51.04	15.05	2251.39
40	4	116	0.245%	0.265%	0.23	25.68	2.31	1062.55
40	4	121	0.045%	0.045%	0.17	13.06	1.12	98.50
50	4	6	0.239%	0.239%	3.28	184.80	10.74	≥3600
50	4	11	0.069%	0.070%	0.53	125.02	4.58	1331.04
50	4	41	0.112%	0.120%	0.70	99.44	7.05	854.78
50	4	46	0.007%	0.009%	0.01	21.12	0.04	18.10
50	4	56	5.678%	5.678%	9.08	125.55	547.65	≥3600
50	4	61	0.776%	0.776%	1.03	272.35	854.85	≥3600
50	4	71	0.056%	0.062%	0.82	39.84	12.79	465.97
50	4	86	0.561%	0.848%	0.53	154.74	6.20	1014.25
50	4	96	0.036%	0.036%	0.50	34.59	12.24	412.73
50	4	111	0.277%	0.289%	0.43	203.12	5.18	2715.11
50	4	116	0.342%	0.353%	0.64	71.30	1101.00	≥3600
50	4	121	0.118%	0.118%	0.92	69.34	14.24	509.27
100	2	41	0.096%	0.096%	128.67	8597.65	≥3600	≥3600

Table B2: continued

			Root Gap		1st LP Time (s)		MIP Time (s)	
n	m	Inst	Fix.	w/o Fix.	Fix.	w/o Fix.	Fix.	w/o Fix.
100	2	66	0.105%	0.110%	10.67	6186.86	2267.02	≥3600
100	2	91	0.287%	0.289%	203.85	8342.06	≥3600	≥3600
100	4	11	0.076%	0.076%	38.98	1537.35	≥3600	≥3600
100	4	16	0.106%	0.107%	56.98	1014.23	≥3600	≥3600
100	4	21	0.002%	0.002%	2.79	275.85	1.18	271.68
100	4	31	0.701%	0.701%	856.73	1392.17	≥3600	≥3600
100	4	36	0.061%	0.061%	32.25	1608.79	556.24	≥3600
100	4	41	0.047%	0.049%	31.69	1709.76	2626.01	≥3600
100	4	46	0.005%	0.005%	5.31	467.65	237.06	1745.52
100	4	56	1.790%	1.790%	677.43	1510.31	≥3600	≥3600
100	4	61	0.248%	0.248%	4.97	1706.10	1653.76	≥3600
100	4	66	0.048%	0.049%	9.40	810.20	1778.90	≥3600
100	4	71	0.009%	0.012%	20.86	577.71	≥3600	≥3600
100	4	81	7.997%	7.997%	115.28	1357.05	2657.21	≥3600
100	4	86	0.499%	0.499%	38.87	2271.43	≥3600	≥3600
100	4	91	0.048%	0.049%	5.01	1421.21	499.05	≥3600
100	4	96	0.020%	0.020%	10.83	675.04	59.06	≥3600
100	4	111	0.239%	0.241%	41.39	2014.67	≥3600	≥3600
100	4	116	0.040%	0.042%	18.67	610.95	≥3600	≥3600
100	4	121	0.022%	0.022%	10.18	1259.12	1962.12	≥3600

Table B3: Effect of Projected Cuts in the Cz Time-Indexed Formulation

				ATIF		TIF				
n	m	Inst	Heu UB	Root LB	Root Gap	1st LP LB	1st LP Gap	Root LB	Root Gap	Gap Improv.
40	2	66	35279	35233	0.130%	35110	0.481%	35131	0.421%	14.26%
40	4	31	2525	2507	0.713%	2488	1.487%	2496	1.162%	28.00%
40	4	36	6420	6367	0.826%	6353	1.055%	6355	1.023%	3.11%

Table B3: continued

				ATIF		TIF				
n	m	Inst	Heu UB	Root LB	Root Gap	1st LP LB	1st LP Gap	Root LB	Root Gap	Gap Improv.
40	4	41	17685	17639	0.260%	17630	0.312%	17634	0.289%	7.87%
40	4	56	826	815	1.332%	798	3.509%	798	3.509%	0.00%
40	4	61	7357	7327	0.408%	7320	0.505%	7324	0.451%	12.18%
40	4	81	564	560	0.709%	553	1.989%	553	1.989%	0.00%
40	4	91	15569	15562	0.045%	15557	0.077%	15559	0.064%	20.02%
40	4	111	11263	11222	0.364%	11186	0.688%	11212	0.455%	51.33%
40	4	116	15566	15547	0.122%	15528	0.245%	15539	0.174%	40.84%
40	4	121	35751	35741	0.028%	35735	0.045%	35739	0.034%	33.35%
50	4	6	8317	8304	0.156%	8298	0.229%	8298	0.229%	0.00%
50	4	11	12879	12876	0.023%	12871	0.062%	12871	0.062%	0.00%
50	4	41	21806	21786	0.092%	21782	0.110%	21783	0.106%	4.35%
50	4	46	44455	44453	0.004%	44452	0.007%	44452	0.007%	0.00%
50	4	56	570	540	5.263%	538	5.948%	538	5.948%	0.00%
50	4	61	7898	7857	0.519%	7837	0.778%	7850	0.611%	27.29%
50	4	71	42645	42627	0.042%	42622	0.054%	42625	0.047%	15.01%
50	4	86	8369	8336	0.394%	8323	0.553%	8333	0.432%	27.93%
50	4	96	50326	50325	0.002%	50308	0.036%	50312	0.028%	28.58%
50	4	111	10069	10052	0.169%	10042	0.269%	10049	0.199%	35.09%
50	4	116	11552	11523	0.251%	11513	0.339%	11520	0.278%	21.95%
50	4	121	23792	23775	0.071%	23764	0.118%	23769	0.097%	21.76%
100	2	41	237964	237884	0.034%	237734	0.097%	237772	0.081%	19.81%
100	2	66	126513	126503	0.008%	126380	0.105%	126411	0.081%	30.42%
100	2	91	129929	129804	0.096%	129557	0.287%	129618	0.240%	19.67%
100	4	11	50232	50203	0.058%	50194	0.076%	50198	0.068%	11.77%
100	4	16	110219	110120	0.090%	110102	0.106%	110113	0.096%	10.39%
100	4	21	237392	237391	0.000%	237388	0.002%	237388	0.002%	0.00%
100	4	31	7130	7082	0.673%	7080	0.706%	7080	0.706%	0.00%
100	4	36	30791	30782	0.029%	30773	0.058%	30773	0.058%	0.00%
100	4	41	126185	126147	0.030%	126126	0.047%	126130	0.044%	7.28%
100	4	46	219536	219529	0.003%	219525	0.005%	219526	0.005%	10.00%
100	4	56	3076	3030	1.495%	3021	1.821%	3021	1.821%	0.00%
100	4	61	24856	24825	0.125%	24795	0.246%	24806	0.202%	22.05%
100	4	66	67970	67955	0.018%	67938	0.047%	67947	0.034%	39.15%
100	4	71	170691	170675	0.009%	170675	0.009%	170678	0.008%	23.08%
100	4	81	819	784	2.686%	754	8.621%	754	8.621%	0.00%
100	4	86	21286	21220	0.310%	21180	0.500%	21208	0.368%	36.08%
100	4	91	70608	70588	0.025%	70575	0.047%	70582	0.037%	26.94%
100	4	96	133587	133575	0.009%	133561	0.019%	133572	0.011%	73.35%
100	4	111	46719	46633	0.184%	46608	0.238%	46615	0.223%	6.75%
100	4	116	101551	101521	0.025%	101510	0.040%	101513	0.037%	7.90%
100	4	121	127619	127598	0.016%	127590	0.023%	127592	0.021%	7.41%

Table C1: Detailed Results for $m = 2$ and $n = 40$ instances

	BCP-PMWT								BCP-PMWT-OTI							
		Root		BCP	CPLEX	Overall				Root		BCP	CPLEX	Overall		
Inst	UB	LB	Time	Time	Time	Best	Time	Opt	UB	LB	Time	Time	Time	Best	Time	Opt
1	606	606	80.9			Root	80.9	606	606	606	21.9			Root	21.9	606
6	3886	3886	27.7			Root	27.7	3886	3886	3886	6.7			Root	6.7	3886
11	9617	9617	22.3			Root	22.3	9617	9617	9617	5.1			Root	5.1	9617
16	38356	38356	59.1			Root	59.1	38356	38356	38356	32.5			Root	32.5	38356
31	3812	3812	49.2			Root	49.2	3812	3812	3812	5.8			Root	5.8	3812
36	10713	10713	65.7			Root	65.7	10713	10713	10713	8.1			Root	8.1	10713
56	1279	1279	24.3			Root	24.3	1279	1279	1279	5.7			Root	5.7	1279
61	11488	11394	705.7	2398.5	1994.0	MIP	2699.7	11488	11488	**11488**	11.1			Root	11.1	11488
66	35279	35197	220.4	307.8	1776.0	BCP	528.2	35279	35279	**35233**	556.1	248.4	180.48	MIP	736.6	35279
71	47952	47952	15.6			Root	15.6	47952	47952	47952	6.5			Root	6.5	47952
81	573	571	83.7			Root	83.7	571	**571**	571	18.5			Root	18.5	571
86	6048	6048	46.2			Root	46.2	6048	6048	6048	5.2			Root	5.2	6048
96	66116	66116	22.8			Root	22.8	66116	66116	66116	4.3			Root	4.3	66116
111	17936	17936	28.1			Root	28.1	17936	17936	17936	6.0			Root	6.0	17936
116	25874	25870	66.6			Root	66.6	25870	**25870**	25870	9.8			Root	9.8	25870
121	64516	64516	24.3			Root	24.3	64516	64516	64516	4.4			Root	4.4	64516

Table C2: Detailed Results for $m = 4$ and $n = 40$ instances

	BCP-PMWT								BCP-PMWT-OTI							
		Root		BCP	CPLEX	Overall				Root		BCP	CPLEX	Overall		
Inst	UB	LB	Time	Time	Time	Best	Time	Opt	UB	LB	Time	Time	Time	Best	Time	Opt
1	439	439	11.4			Root	11.4	439	439	439	3.9			Root	3.9	439
6	2374	2374	11.1			Root	11.1	2374	2374	2374	3.7			Root	3.7	2374
11	5737	5737	10.3			Root	10.3	5737	5737	5737	3.2			Root	3.2	5737
16	21493	21490	30.4			Root	30.4	21493	21493	**21493**	24.5			Root	24.5	21493
31	2525	2500	50.1	2932.6	12172.3	BCP	2982.7	2525	2525	**2507**	29.7	69.6	31.45	MIP	61.2	2525
36	6420	6364	37.3	18799.5	2829.7	MIP	2867.0	6420	6420	**6367**	16.4	12298.1	12.14	MIP	28.5	6420
41	17685	17637	36.1	1856.0	128.0	MIP	164.1	17685	17685	**17639**	14.1	603.7	19.84	MIP	33.9	17685
56	826	**817**	54.6	>86400	878.0	MIP	932.6	826	826	815	29.5	>86378.9	137.12	MIP	166.6	826
61	7357	7322	33.8	230.2	143.0	MIP	176.8	7357	7357	**7327**	40.1	279.7	5.41	MIP	45.5	7357

Table C2: Detailed Results for $m = 4$ and $n = 40$ instances

	BCP-PMWT								BCP-PMWT-OTI							
		Root		BCP	CPLEX	Overall				Root		BCP	CPLEX	Overall		
Inst	UB	LB	Time	Time	Time	Best	Time	Opt	UB	LB	Time	Time	Time	Best	Time	Opt
66	20251	20251	12.9			Root	12.9	20251	20251	20251	2.9			Root	2.9	20251
81	565	560	46.4	9.7	0.9	MIP	47.3	564	**564**	560	19.6	0.4	0.21	BCP	20.0	564
86	4725	4725	15.3			Root	15.3	4725	4725	4725	6.1			Root	6.1	4725
91	15569	15562	39.4	9.6	0.7	MIP	40.1	15569	15569	15562	17.9	0.6	0.12	BCP	18.5	15569
111	11263	11219	72.0	303.9	105.3	MIP	177.3	11263	11263	**11222**	17.3	148.8	6.22	MIP	23.5	11263
116	15566	15545	31.3	101.4	40.4	MIP	71.7	15566	15566	**15547**	19.2	52.3	4.29	MIP	23.5	15566
121	35751	35741	52.0	42.6	4.7	MIP	56.7	35751	35751	35741	15.6	32.7	1.38	MIP	17.0	35751

Table C3: Detailed Results for $m = 2$ and $n = 50$ instances

	BCP-PMWT								BCP-PMWT-OTI							
		Root		BCP	CPLEX	Overall				Root		BCP	CPLEX	Overall		
Inst	UB	LB	Time	Time	Time	Best	Time	Opt	UB	LB	Time	Time	Time	Best	Time	Opt
1	1268	1268	345.8			Root	345.8	1268	1268	1268	28.8			Root	28.8	1268
6	14272	14272	122.8			Root	122.8	14272	14272	14272	10.0			Root	10.0	14272
11	23028	23028	67.1			Root	67.1	23028	23028	23028	10.9			Root	10.9	23028
16	46072	46072	59.4			Root	59.4	46072	46072	46072	17.7			Root	17.7	46072
21	111069	111069	36.1			Root	36.1	111069	111069	111069	6.8			Root	6.8	111069
31	5378	5349	1110.2	11608.9	11997.5	BCP	12719.1	5378	5378	**5378**	21.1			Root	21.1	5378
36	18956	18956	73.3			Root	73.3	18956	18956	18956	9.1			Root	9.1	18956
41	38058	38050	381.5	16.7	43.5	BCP	398.2	38058	38058	**38058**	58.0			Root	58.0	38058
46	82105	82105	66.0			Root	66.0	82105	82105	82105	31.4			Root	31.4	82105
56	761	761	267.4			Root	267.4	761	761	761	13.9			Root	13.9	761
61	13682	13619	661.4	83122.8	36094.6	MIP	36756.0	13682	13682	**13682**	22.6			Root	22.6	13682
66	40907	40907	37.5			Root	37.5	40907	40907	40907	6.3			Root	6.3	40907
81	542	542	40.5			Root	40.5	542	542	542	8.9			Root	8.9	542
86	12557	12427	999.3	13817.7	23070.0	BCP	14817.0	12557	12557	**12557**	17.8			Root	17.8	12557
91	47349	47330	552.6	210.6	204.5	MIP	757.1	47349	47349	**47349**	8.7			Root	8.7	47349
96	92822	92822	60.9			Root	60.9	92822	92822	92822	7.3			Root	7.3	92822
111	15564	15564	66.6			Root	66.6	15564	15564	15564	9.3			Root	9.3	15564
116	19609	19573	508.8	2390.7	8539.5	BCP	2899.5	19608	**19608**	**19608**	47.1			Root	47.1	19608

Table C4: Detailed Results for $m = 4$ and $n = 50$ instances

	BCP-PMWT								BCP-PMWT-OTI							
		Root		BCP	CPLEX	Overall				Root		BCP	CPLEX	Overall		
Inst	UB	LB	Time	Time	Time	Best	Time	Opt	UB	LB	Time	Time	Time	Best	Time	Opt
1	785	785	56.4			Root	56.4	785	785	785	29.0			Root	29.0	785
6	8317	8304	147.1	>86400	78140.0	MIP	78287.1	8317	8317	8304	33.7	1330.2	15.59	BCP	1363.9	8317
11	12879	12875	98.9	5691.6	43.8	MIP	142.7	12879	12879	**12876**	34.6	40.2	11.62	MIP	46.2	12879
16	25376	25376	31.2			Root	31.2	25376	25376	25376	9.2			Root	9.2	25376
36	10796	10796	35.1			Root	35.1	10796	10796	10796	5.6			Root	5.6	10796
41	21806	21785	72.7	232.4	68.1	MIP	140.8	21806	21806	**21786**	59.1	179.7	26.03	MIP	85.1	21806
46	44455	**44455**	40.5			Root	40.5	44455	44455	44453	17.1	0.7	0.06	BCP	17.8	44455
56	570	**541**	67.8	>86400	18126.0	MIP	18193.8	570	570	540	14.2	>14393.6	1757.61	MIP	1771.8	570
61	7898	7857	113.7	>86400	27997.4	MIP	28111.1	7898	7898	7857	49.3	6283.0	653.57	MIP	702.9	7898
71	42645	**42628**	218.8	336.6	42.3	MIP	261.1	42645	42645	42627	19.0	91.3	17.51	MIP	36.5	42645
81	495	495	118.9			Root	118.9	495	495	495	26.3			Root	26.3	495
86	8369	8336	241.3	575.2	355.6	MIP	596.9	8369	8369	8336	69.9	130.1	10.36	MIP	80.3	8369
91	26552	26548	187.6			Root	187.6	26551	**26551**	**26551**	12.4			Root	12.4	26551
96	50326	50317	183.2	261.1	32.4	MIP	215.6	50326	50326	**50325**	77.1	59.7	16.65	MIP	93.7	50326
111	10069	10051	134.6	52.0	43.8	MIP	178.4	10069	10069	**10052**	39.9	46.4	4.12	MIP	44.0	10069
116	11552	11523	114.3	45693.5	4087.2	MIP	4201.5	11552	11552	11523	27.0	7944.5	217.34	MIP	244.3	11552
121	23792	23775	85.3	438.5	248.5	MIP	333.8	23792	23792	23775	49.0	132.0	15.99	MIP	65.0	23792

Table C5: Detailed Results for $m = 2$ and $n = 100$ instances

	BCP-PMWT								BCP-PMWT-OTI							
		Root		BCP	CPLEX	Overall				Root		BCP	CPLEX	Overall		
Inst	UB	LB	Time	Time	Time	Best	Time	Opt	UB	LB	Time	Time	Time	Best	Time	Opt
1	3339	3322	1606.0	>14400			>14400	≤3339	3339	**3339**	285.8			Root	285.8	3339
6	30665	30665	1001.5			Root	1001.5	30665	30665	30665	200.1			Root	200.1	30665
11	93894	93894	613.0			Root	613.0	93894	93894	93894	40.1			Root	40.1	93894
16	209100	209100	5603.6			Root	5603.6	209100	209100	209100	86.9			Root	86.9	209100
21	457836	457836	1248.7			Root	1248.7	457836	457836	457836	215.1			Root	215.1	457836
31	12729	12729	927.7			Root	927.7	12729	12729	12729	104.1			Root	104.1	12729
36	56671	56590	1435.7	>14400			>14400	≤56671	56671	**56671**	347.6			Root	347.6	56671
41	237964	237841	20436.3	>20436	120721.7	MIP	141158.0	237964	237964	**237884**	2201.7	5753.7	21358.19	BCP	7955.4	237964

Table C5: continued

	BCP-PMWT								BCP-PMWT-OTI							
		Root		BCP	CPLEX	Overall				Root		BCP	CPLEX	Overall		
Inst	UB	LB	Time	Time	Time	Best	Time	Opt	UB	LB	Time	Time	Time	Best	Time	Opt
46	422831	422830	2077.9	44.1	51.9	BCP	2122.0	422831	422831	**422831**	171.9			Root	171.9	422831
56	5047	5047	1582.3			Root	1582.3	5047	5047	5047	152.4			Root	152.4	5047
61	45573	45481	7353.8	>14400	>86400		>14400	≤45573	45573	**45573**	275.5			Root	275.5	45573
66	126522	126477	7526.5	23425.6	>86400	BCP	30952.1	126512	**126513**	**126503**	1976.3	902.4	2717.89	BCP	2878.7	126513
71	327305	327305	630.2			Root	630.2	327305	327305	327305	57.6			Root	57.6	327305
81	908	830	1299.2	>14400			>14400	≤908	908	**908**	198.9			Root	198.9	908
86	36581	36322	3905.2	>14400	>86400		>14400	≤36581	36581	**36581**	262.8			Root	262.8	36581
91	129931	129752	4259.4	>14400	>86400		>14400	≤129931	**129929**	**129804**	6258.5	>8267.3	≥86400		6258.5	≤129929
96	254194	254194	1030.5			Root	1030.5	254194	254194	254194	185.9			Root	185.9	254194
111	84274	84097	7424.3	>14400	>86400		>14400	≤84250	**84220**	**84220**	219.5			Root	219.5	84220
116	191198	191173	11740.2	>14400	32567.7	MIP	44307.9	191186	**191186**	**191186**	2373.0			Root	2373.0	191186
121	242022	241997	57559.6	>86400	4072.4	MIP	61632.0	242018	**242018**	**242018**	181.1			Root	181.1	242018

Table C6: Detailed Results for $m = 4$ and $n = 100$ instances

	BCP-PMWT								BCP-PMWT-OTI							
		Root		BCP	CPLEX	Overall				Root		BCP	CPLEX	Overall		
Inst	UB	LB	Time	Time	Time	Best	Time	Opt	UB	LB	Time	Time	Time	Best	Time	Opt
1	2001	2001	1337.3			Root	1337.3	2001	2001	2001	156.1			Root	156.1	2001
11	50236	50203	1439.4	>14400			>14400	≤50236	**50232**	50203	315.4	>14106.9	49519.51	MIP	49834.9	50232
16	110222	110118	1449.2	>14400	>86400		>14400	≤110222	**110219**	**110120**	249.5	>14164.0	≥86400		249.5	≤110219
21	237392	237390	1493.8	5.5	29.0	BCP	1499.3	237392	237392	**237391**	375.8	10.1	2.52	BCP	385.9	237392
31	7130	7082	1901.5	>14400			>14400	≤7130	7130	7082	311.2	>14242.3	≥86400		311.2	≤7130
36	30791	**30783**	2201.3	>14400			>14400	≤30791	30791	30782	427.9	>14035.1	610.22	MIP	1038.1	30791
41	126193	126144	1493.6	11041.3	41682.9	BCP	12534.9	126185	**126185**	**126147**	1376.4	6314.4	1617.61	MIP	2994.0	126185
46	219537	219529	1372.0	256.3	800.9	BCP	1628.3	219536	**219536**	219529	578.3	401.9	170.21	MIP	748.5	219536
56	3076	3030	1727.3	>14400			>14400	≤3076	3076	3030	586.3	>13884.5	≥86400		586.3	≤3076
61	24868	24821	1682.5	>14400	>86400		>14400	≤24868	**24856**	**24825**	548.8	>13936.3	1233.98	MIP	1782.8	24856
66	67979	67954	1267.7	2885.7	3738.2	BCP	4153.4	67967	**67970**	**67955**	723.0	2134.1	483.20	MIP	1206.2	67967
71	170699	170675	862.9	>14400	81028.1	MIP	81891.0	170689	**170691**	170675	568.4	>85907.8	3874.12	MIP	4442.5	170689
81	819	772	1594.4	>14400			>14400	≤819	819	**784**	466.1	767.9	5785.43	BCP	1234.0	819
86	21299	21218	1379.0	>14400	>86400		>14400	≤21299	**21286**	**21220**	823.8	>13608.3	15326.80	MIP	16150.6	21282
91	70612	**70589**	2119.4	1436.3	7318.0	BCP	3555.7	70606	**70608**	70588	1269.8	1716.1	401.22	MIP	1671.0	70606
96	133591	133575	878.6	3156.3	1930.3	MIP	2808.9	133587	**133587**	133575	1028.7	332.4	122.28	MIP	1151.0	133587
111	46763	46630	1704.6	>14400	>86400		>14400	≤46747	**46719**	**46633**	957.1	>13496.4	28468.37	MIP	29425.5	46704
116	101563	101520	1374.6	>14400	156440.1	MIP	157814.7	101546	**101551**	**101521**	790.4	55690.0	3608.02	MIP	4398.4	101546
121	127639	127597	1389.6	>14400	108063.9	MIP	109453.5	127618	**127619**	**127598**	904.9	17652.2	815.18	MIP	1720.1	127618